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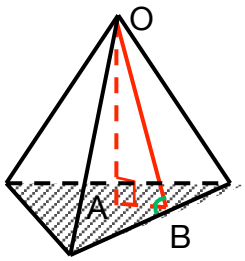
GEOMETILES®

High School Sample Lesson

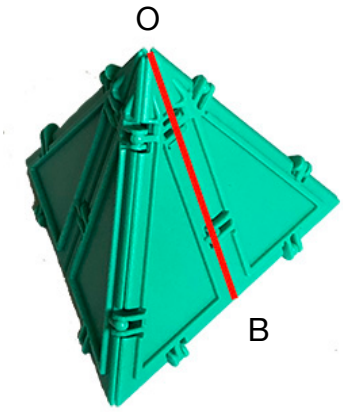
Topics covered: Right Triangles & Trigonometry; Geometric Measurement & Dimension

Compute the dihedral angles of a regular tetrahedron

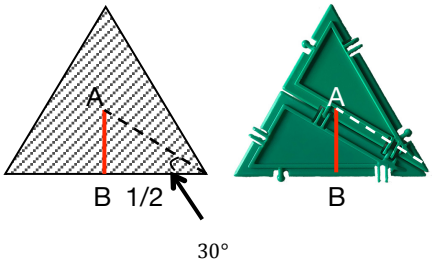
A **dihedral angle** is the angle between the faces of a solid. Since the tetrahedron is regular, there is just one angle to compute. We drop a perpendicular from one of the vertices to the opposite face.



The dihedral angle is equal to angle OBA in the picture above. Have the students construct a regular tetrahedron to model this picture.



Assuming the regular tetrahedron has sides of length 1 unit, side OB is $\sqrt{3}/_2$ units long. In order to determine the angles of triangle OBA we need to compute the length of AB or OA. AB is easier. The shaded triangle is the same one as in the first drawing above.



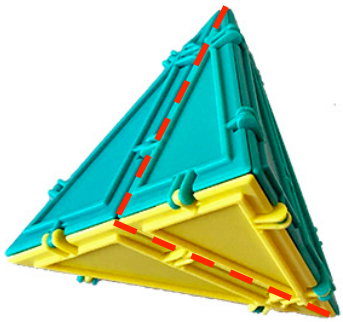
We see from the picture above that the length of AB is $\frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$ units long.

Therefore,

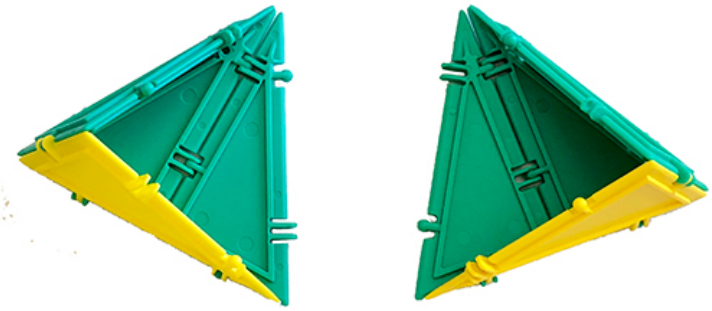
$$\text{angle OAB} = \arccos\left(\frac{\sqrt{3}/6}{\sqrt{3}/2}\right).$$
$$= \arccos\left(\frac{1}{3}\right).$$

The approximate measure of the angle is $\arccos\left(\frac{1}{3}\right) \cong 71°$.

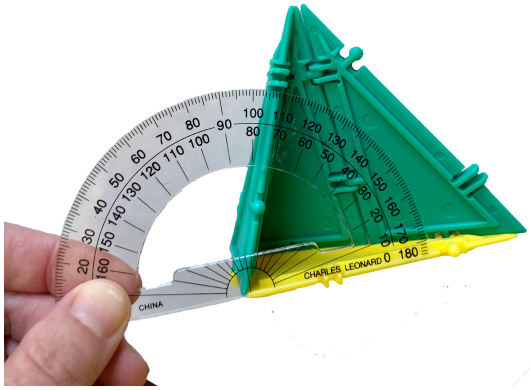
Now we can try to measure this angle with a protractor. We start out with a tetrahedron like this:



We need to measure the angle marked with the red dotted line. We can break the tetrahedron in two halves along this line

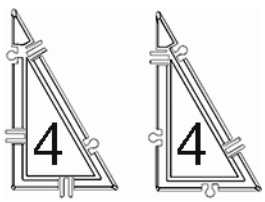


and measure the angle in one of the halves.



The angle shown by the protractor is about $70°$. This is close to the expected value.

Materials needed:

- 

- 4 inch protractor