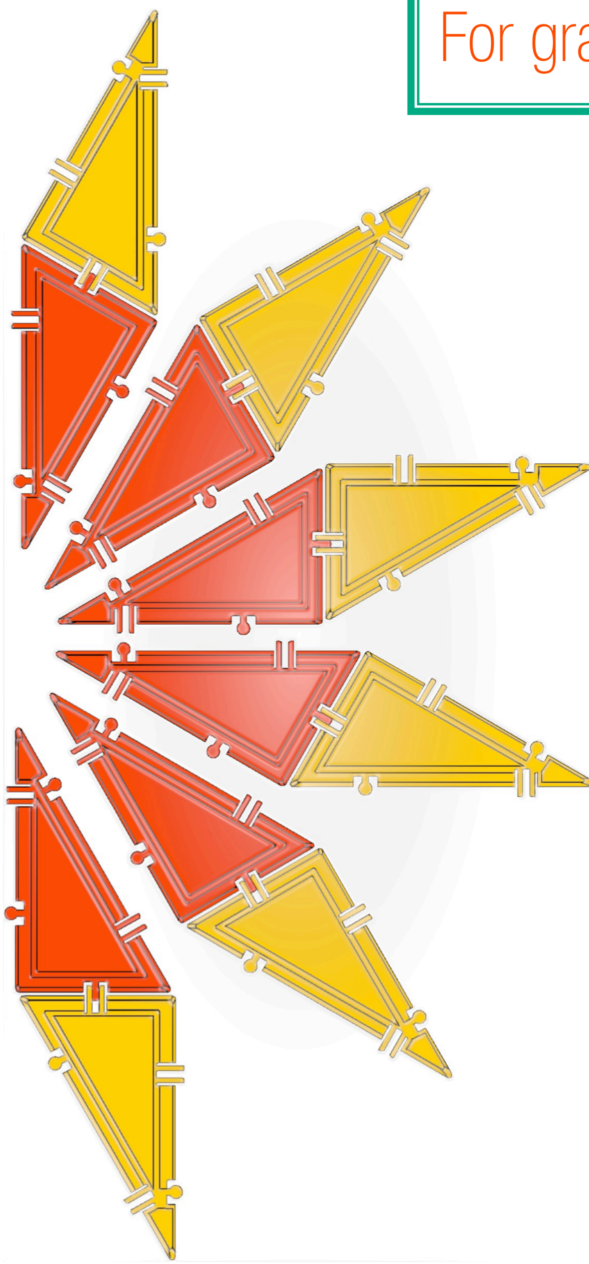
SHAPE CHALLENGE

GEOMETILES ™

Shape Challenge

For grade 4 and up



GEOMETILES ™

Patent Pending

www.geometiles.com

Introduction

The following exercises are intended to accommodate a wide range of students from grade 4 onwards. This is the second workbook in the "Shape Challenge" series. The first one is intended for grades 2 and up. It is not a prerequisite for this one, but it is beneficial for students do the 2nd grade workbook as a "warm up" for this one. The exercises in this workbook are more involved than the ones in the 2nd grade one.

It is recommended that students do the exercises in this workbook before they go on to "3D Solid Understanding". That way, students will get familiar with the various 2-dimensional shapes they can build before assembling them into 3-dimensional solids.

You can do these exercises at whatever depth is appropriate for your students. As you well know, it can be difficult to anticipate what questions students will ask. Some of the material provided in this booklet is intended to help you address questions that your students come up with—or inspire you to ask your own! Material that is provided as an optional resource is appears in smaller font. If it seems too difficult, feel free to skip it.

Activities in this workbook and the Common Core State Standards (CCSS)

How this workbook relates to the goals of the Common Core Progressions

According to the Common Core Geometric Progressions:

> *By the end of Grade 5, competencies in shape composition and de-composition… should be highly developed … Students need to develop these competencies because they form a foundation for understanding multiplication, area, volume, and the coordinate plane.*

With this in mind, it would benefit students to work through these exercises, and then follow them with the ones in the Geometiles™ "3-D Solid Understanding".

CCSS for Mathematical Content supported by activities in this workbook

> 2.G.A.1 *Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.*

Version: 10/27/15

3.G.A.1 *Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.*

4.G.A.1 *Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.*

4.G.A.2 *Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.*

CCSS for Mathematical Practice supported by activities in this workbook

MP.1 *Make sense of problems and persevere in solving them.*

Each problem consists of a goal (make a certain shape) and a set of constraints (the number of tiles students can use). Students are encouraged to try different combinations of shapes until they find one that solves the problem.

MP.3 *Construct viable arguments and critique the reasoning of others.*

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

MP.6 Attend to precision

Solving these problems makes it necessary for students to use clear definitions of various polygons in discussion with each other and their teacher.

MP.8 *Look for and express regularity in repeated reasoning.*

Students will create the more complex shapes by building upon simpler shapes they had built earlier.

Learning objectives of this workbook

a. Reinforce the idea of what a given geometrical shape is through tactile means and through constructing that shape in various sizes.
b. Recognize ways to decompose a polygon into smaller polygons.

c. Help students learn how certain polygons can be made with Geometiles™ so that they can use them later to build 3-dimensional solids.
d. Lay the foundation for the concept of similar polygons that students will encounter in later grades.
e. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
f. Learn how to collaborate with one another.

Arguably, point e. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

If students are frustrated to the point of disengagement, you can modify the problem so they construct, say, 2 or 3 triangles instead of all 6. They will still get a meaningful learning experience from doing the partial solutions.

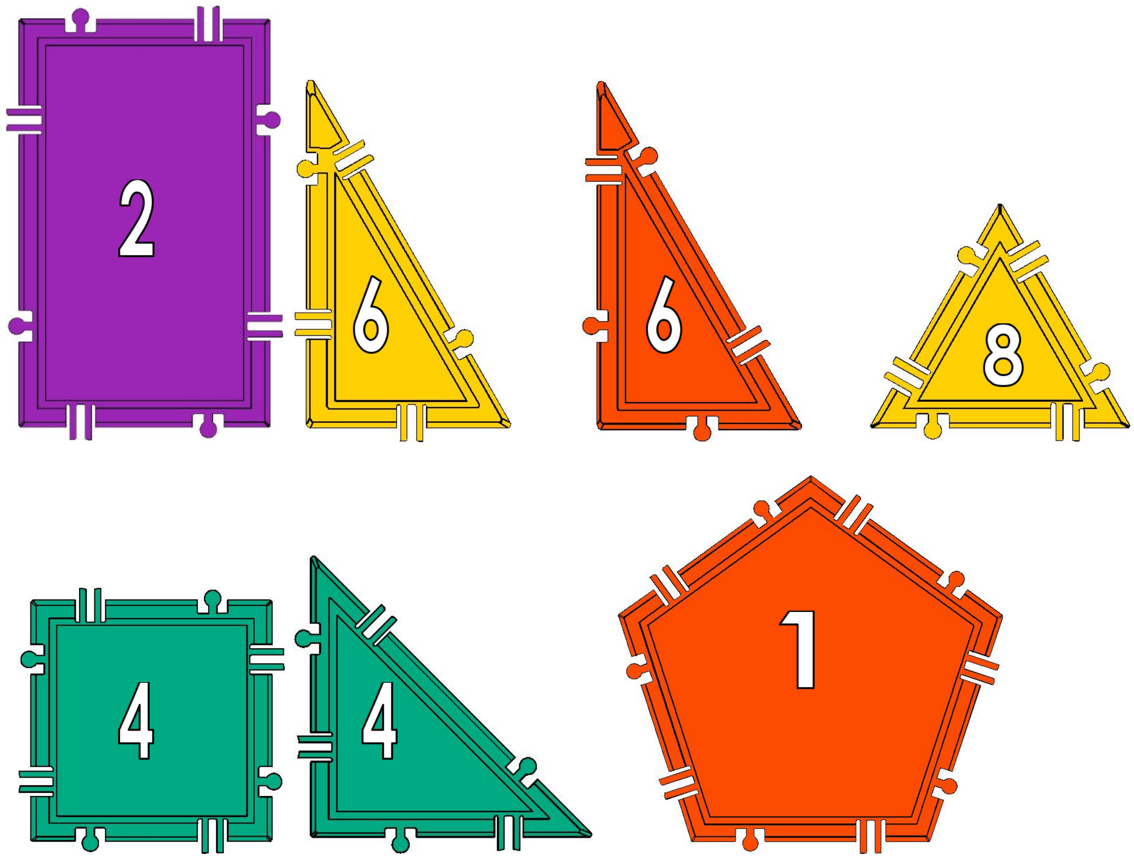
Hints are provided as necessary to help the students who are having trouble gaining momentum with solving a problem.

Recommended Classroom Plan

The exercises are designed to work with 96 tiles split evenly into two sets of 48 tiles. Each 48 set is intended for a group of 2-3 students. Therefore, the 96 piece set is meant for a total of 4 to 6 students.

Each of the two groups of students gets the following:

(colors will vary)



Note that you will have pieces in your set left over. We have purposely limited the number of tiles to which students have access to help them focus on the exercises at hand.

Problems

1. Make a **square** in 4 different sizes.
2. Make an **equilateral triangle** in 6 different sizes.
3. How many different **rectangles** (not squares) can you make with 2 tiles?
4. Make **right isosceles** triangles in 3 different sizes.
5. How many **trapezoids** can you make with 3 tiles?
6. Use 4 **right scalene** triangles to build a larger right scalene triangle.
7. Make an **obtuse** triangle.
8. Make a **rhombus** in 4 different sizes.
9. Make 3 different **parallelograms** (not rhombuses or rectangles). They need NOT have different areas, just different shapes.
10. Make 2 different size **regular hexagons** (hexagons with all equal sides and equal angles).
11. Make a **pentagon** with 2 tiles.
12. Make a **hexagon** with 2 tiles.
13. Make a **quadrilateral** that has a line of symmetry and none of whose sides are parallel to each other.
14. Make a **quadrilateral** that has no lines of symmetry and none of whose sides are parallel to each other.

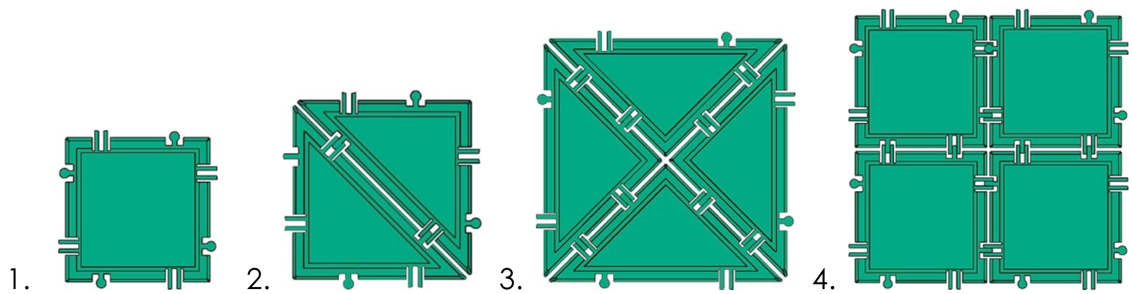
15. Try to make the shape of a STOP sign (this shape is called an **octagon**). Is your shape exactly the same as that of a STOP sign? If not, how is it different?

Answers

Please note that the colors of the tiles in the answers may be different from the colors in your set.

Problem 1

Answer



Hints

Solutions 2 and 3 are the less obvious ones. If students are not seeing them, guide them to solution 2 first. For solution 3, you can start by making a larger isosceles triangle out of two of the smaller isosceles triangles. Then you can put two of the larger isosceles triangles together. Another hint is that the side of the square is the hypotenuse of the isosceles triangle.

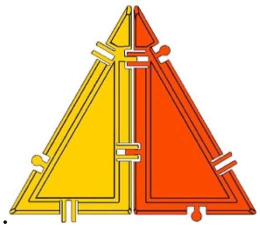
Problem 2

Answer

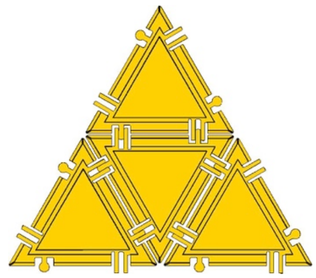
Solutions are ordered in increasing order of areas.

1.

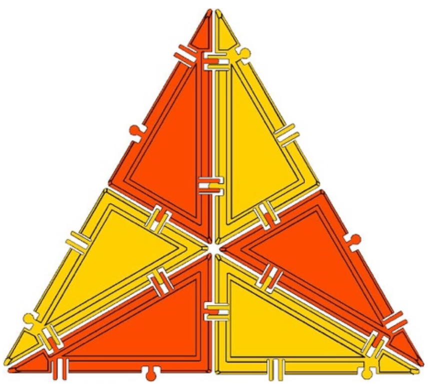
2.



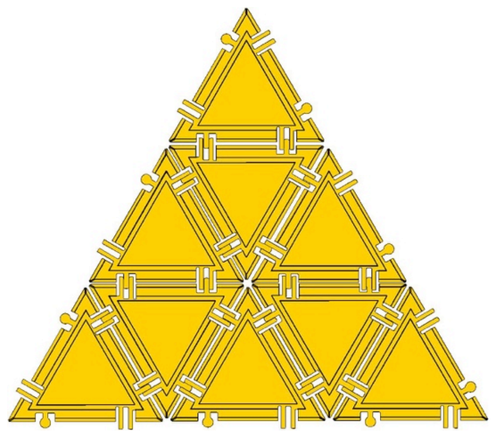
3.



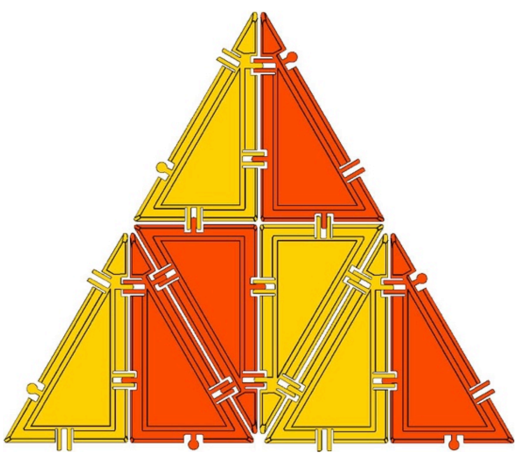
4.



5.



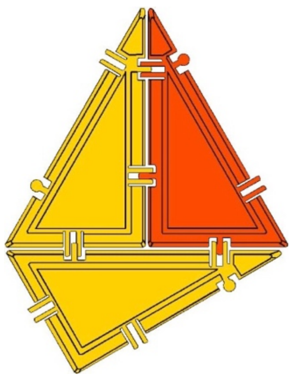
6.



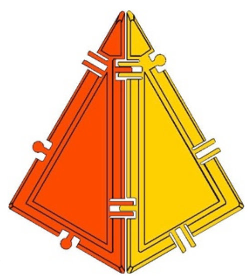
Hints

Solution 2 leads to solution 6 just as solution 1 leads to solution 3. If students are having trouble coming up with solution 2, ask them "Can you make an equilateral triangle out of these?" [hand them the two needed parts and ask them to connect them. They can be connected to form three different figures, and only one of them is an equilateral triangle].

Now ask the students, "How do you know this triangle is equilateral?" The answer is to connect as shown below to show that the sum of lengths of the two short sides is exactly equals the length of the other side:



For solution 4, have students make part of the triangle, like for example,

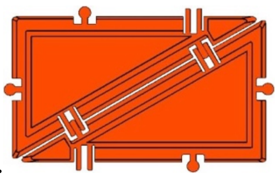
 or  ,

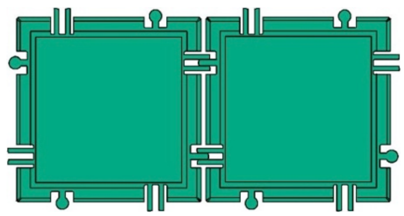
and hint that they can combine several of these parts to make a whole equilateral triangle.

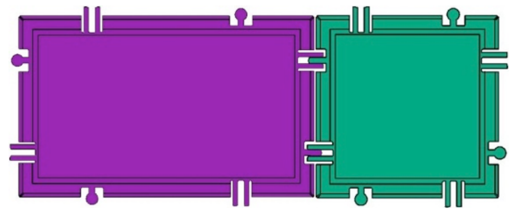
Solution 5 is part of the pattern started by solution 1 and solution 3.

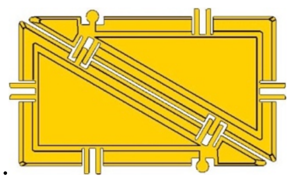
Problem 3

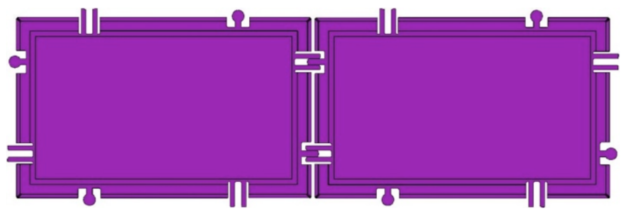
Answer

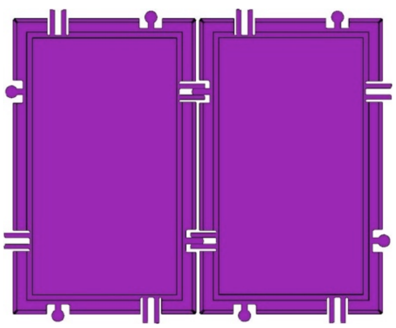
1. 

2. 

3. 

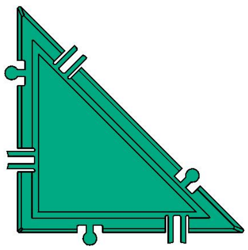
4. 

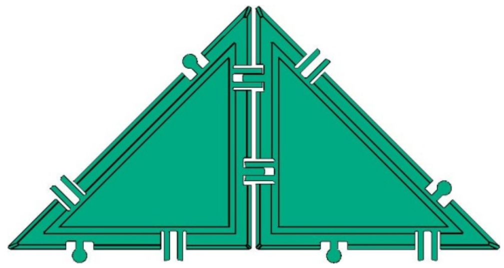
5. 

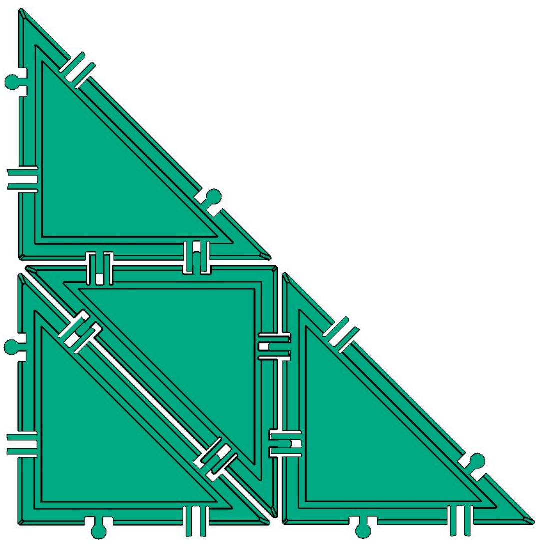
6. 

Problem 4

Answer

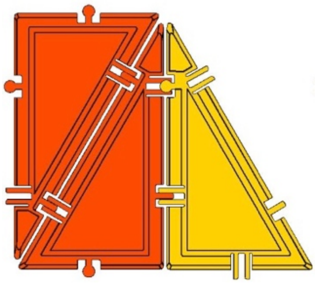
1. 

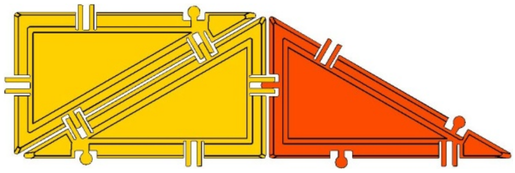
2. 

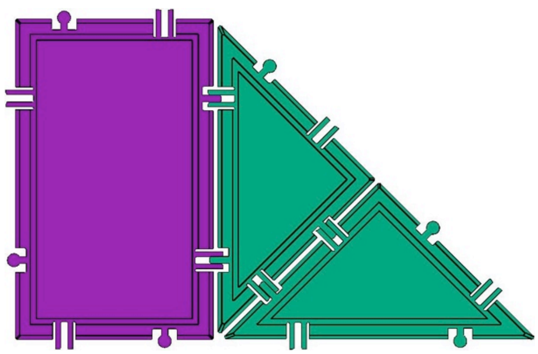
3. 

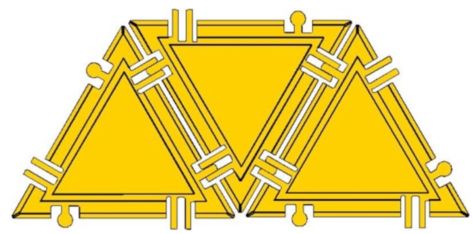
Problem 5

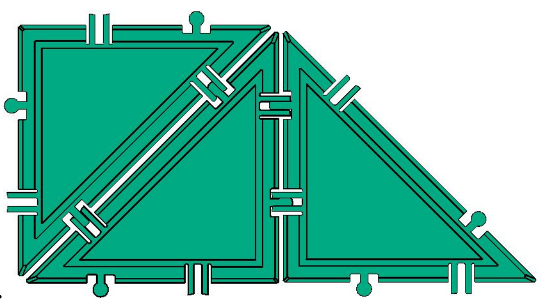
Answer

1. 

2. 

3. 

4. 

5. 

Hints

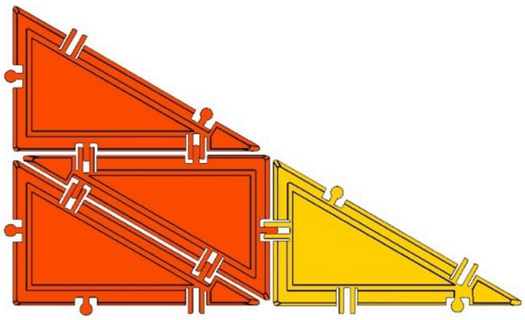
A trapezoid is a quadrilateral with two parallel sides. It need not be symmetric.

You can hint to the students that they can use the rectangles which they constructed in a previous problem and add triangles to them.

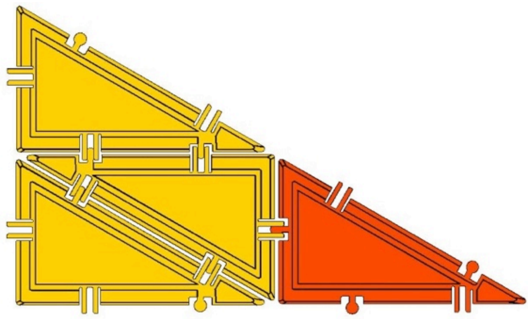
Problem 6

Answer

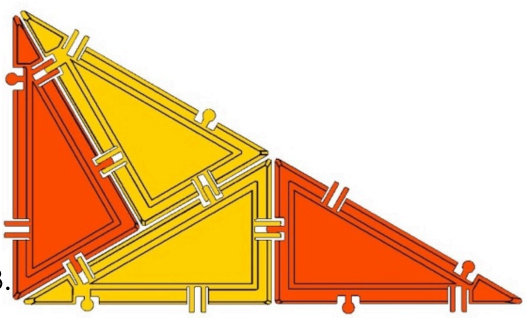
1.



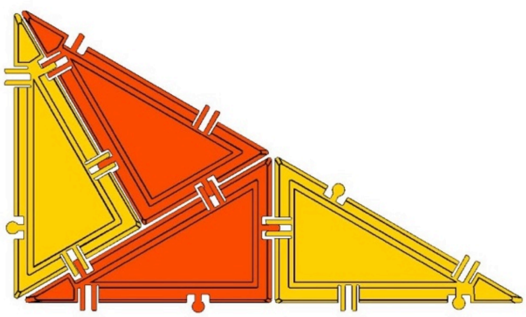
2.



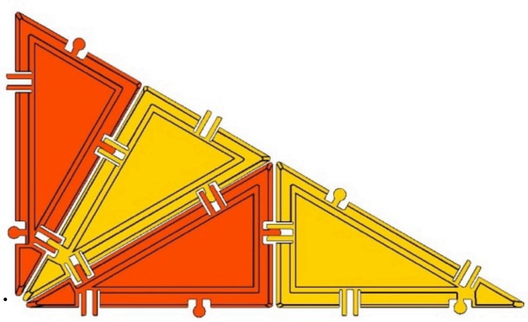
3.



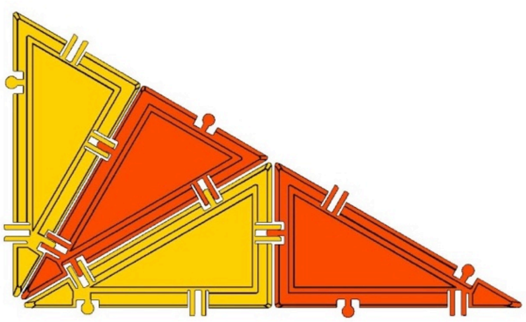
4.



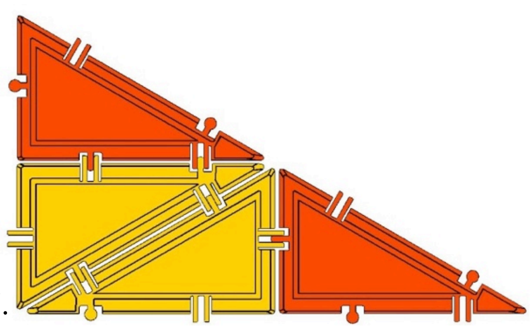
5.



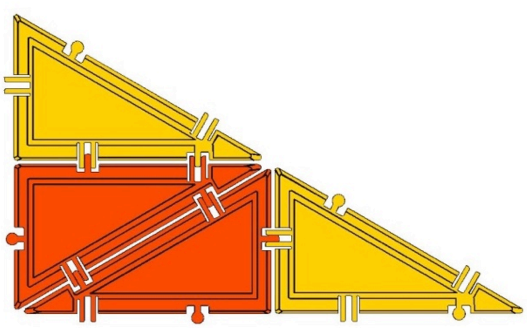
6.



7.



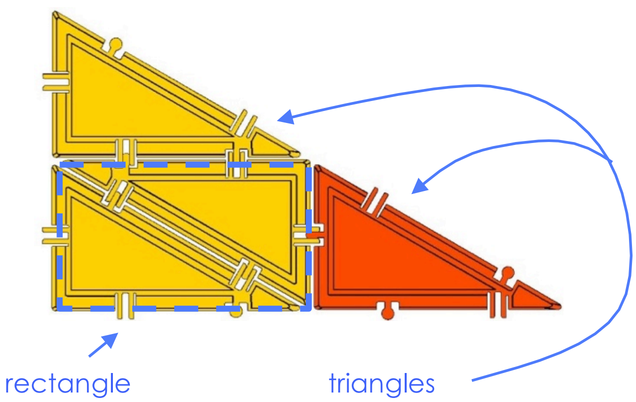
8.



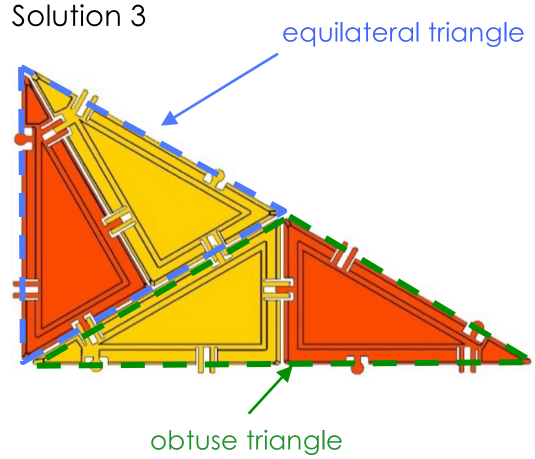
Hints

Note that all these solutions fall into 2 categories. Solutions 1, 2, 7, and 8 consist of a rectangle with two triangles attached, while solutions 3, 4, 5, and 6 consist of an equilateral triangle and an obtuse triangle connected to each other. For example,

Solution 2



Solution 3

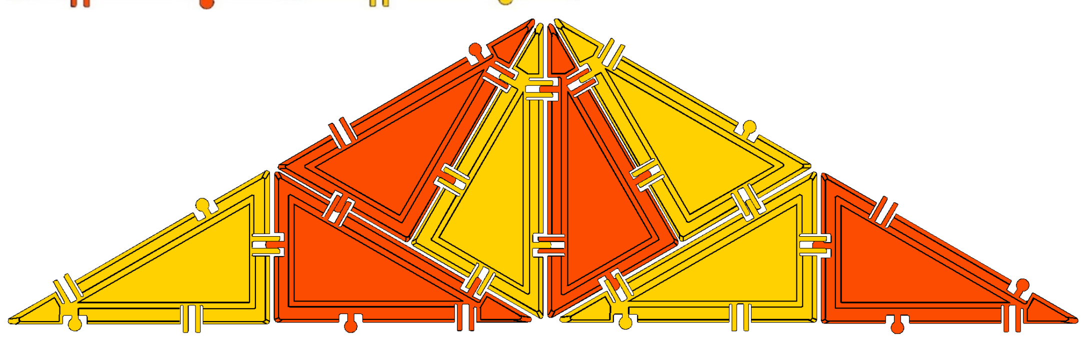


These two solutions lay the groundwork for the rest. For instance, once students see the rectangle and the two triangles attached to in in Solution 2, they can think about other ways of making the rectangle and flipping it over to arrive at the other solutions

Problem 7

Answer

1. 

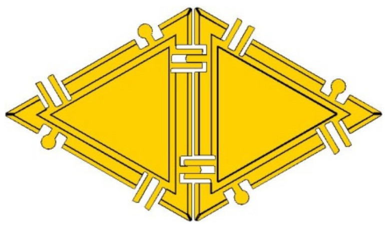
2. 

Hints

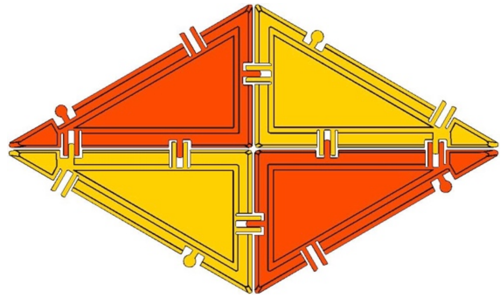
It's quite a challenge to come up with that second triangle. A key observation is that it is made of two triangles from the solution to Problem 6.

Problem 8

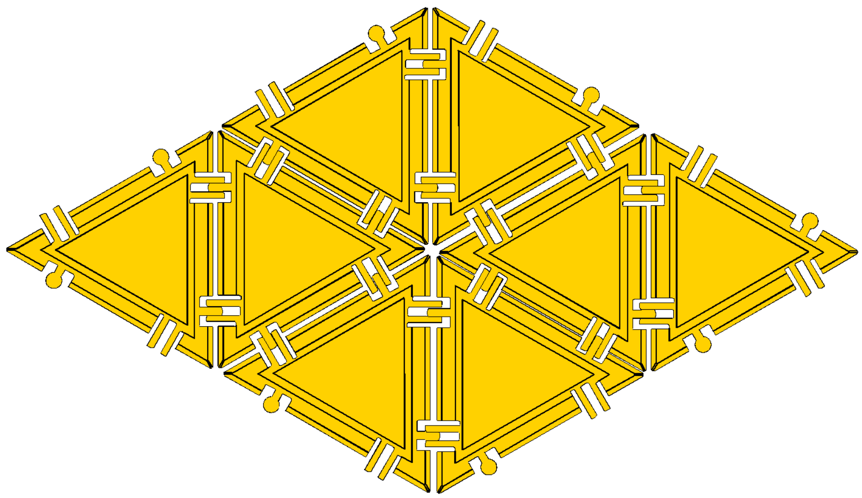
Answer



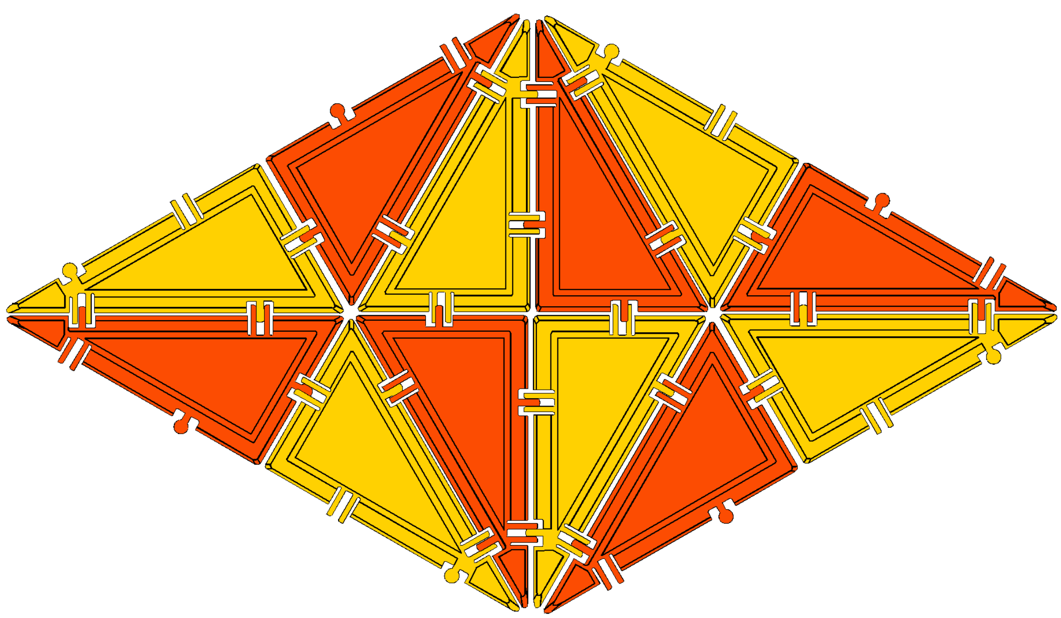
1.



2.



3.



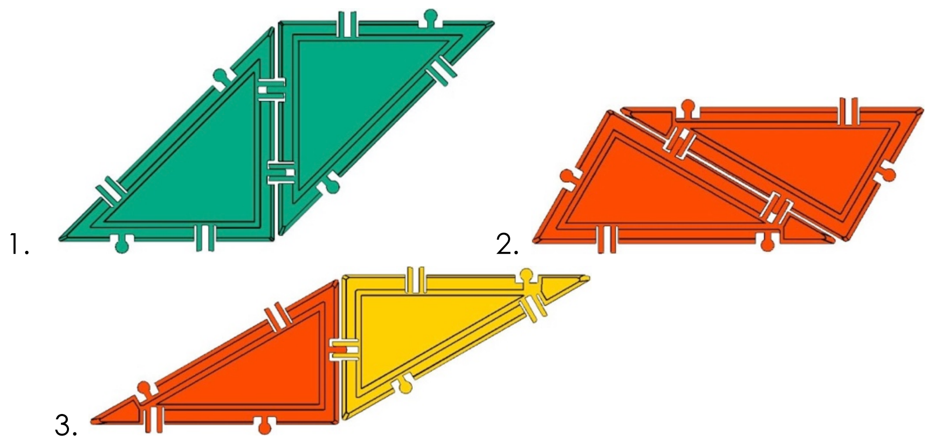
4.

Hints

Solutions 3, 4 are just two copies of Solutions 3,4 from Problem 2, respectively.

Problem 9

Answer



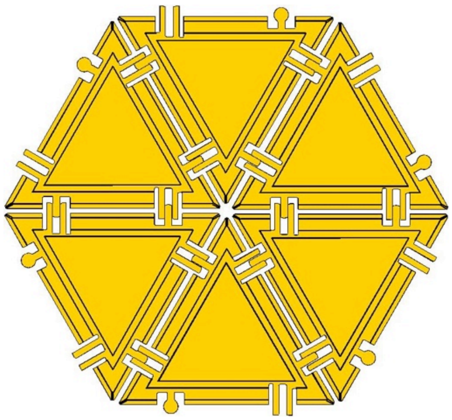
Hints

The hint here is that each parallelogram is made of two congruent triangles.

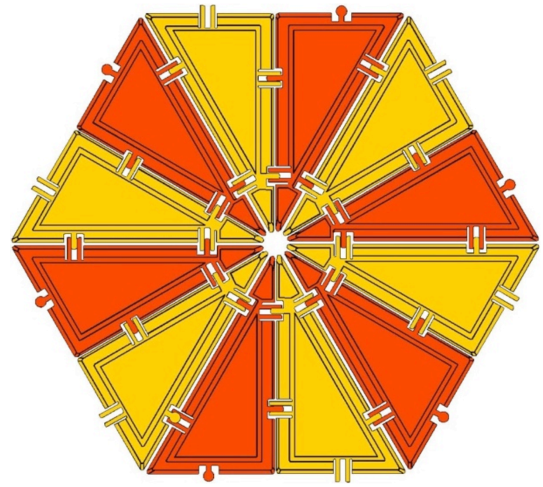
Note that the parallelgorams in solutions 2 and 3 have the same areas but different shapes.

Problem 10

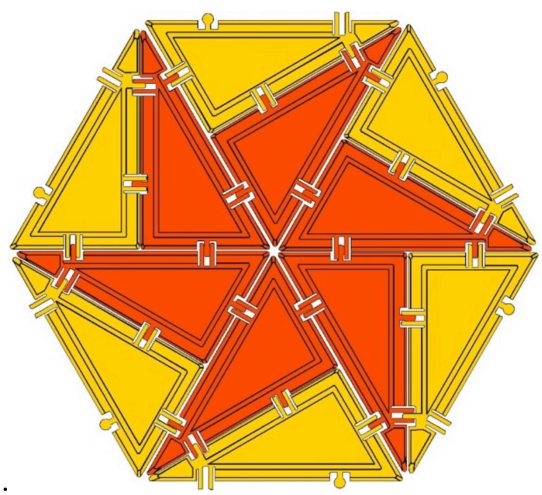
Answer



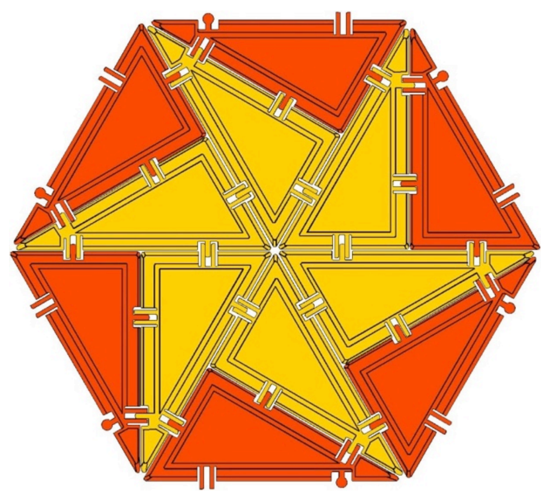
1.



2.



3.



4.

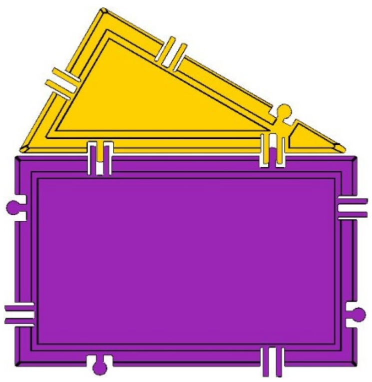
Note that there are many other ways to arrange the twelve triangles in solutions 2, 3, and 4 into into a hexagon.

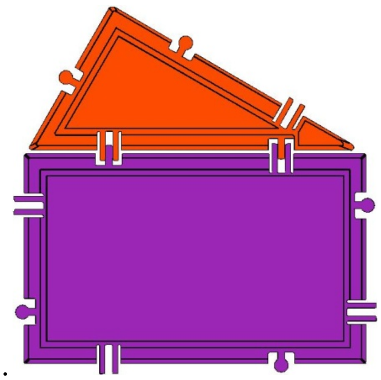
Hints

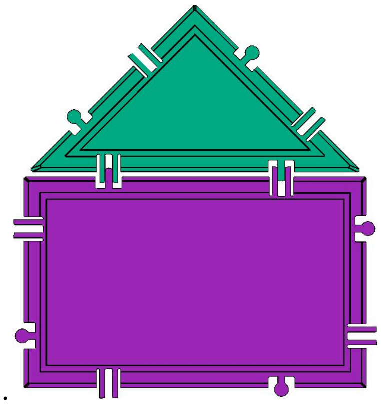
The most obvious answer is solution 1. Once students get this answer, you can remind them that there is another way to make an equilateral triangle that they learned in Problem 2, solution 2. They can then assemble six of these larger equilateral triangles in a variety of ways to make a hexagon.

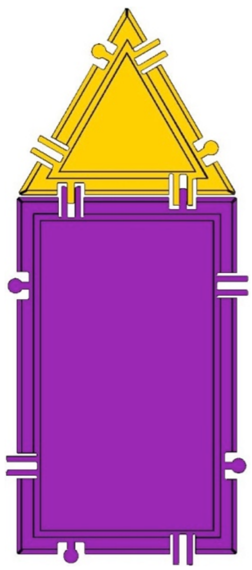
Problem 11

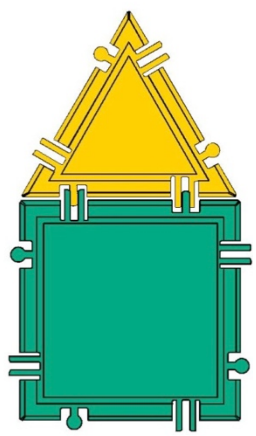
Answer

1. 

2. 

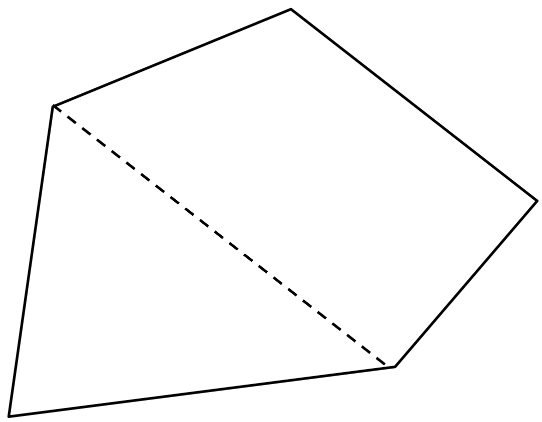
3. 

4. 

5. 

Hints

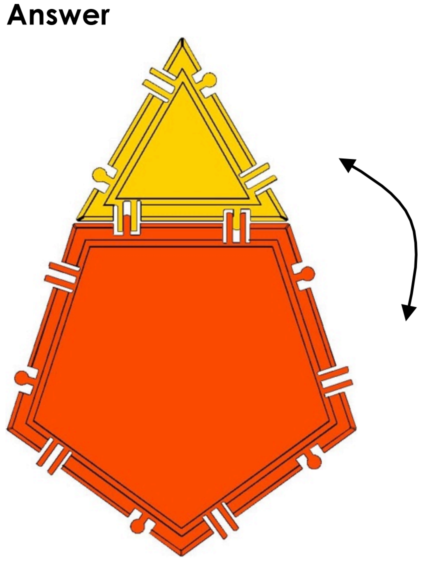
A pentagon has 5 sides, not necessarily equal in length. If students are having trouble, you can ask them to draw a random pentagon and have them split it up into 2 pieces with a line that goes through the corners:



Now ask the students into what shape the pentagon is divided. No matter how the pentagon is divided, the answers are triangle and quadrilateral. The only quadrilaterals in the set of tiles are a square and a rectangle. **Therefore, the only way to make a pentagon out of two tiles is to join a triangle to a square or to a rectangle.**

Problem 12

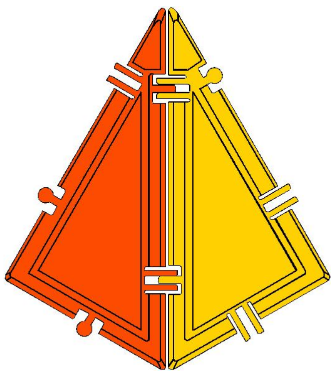
Answer



This may look like a straight line, but it is not. If students have a hard time believing this, you can align a ruler with the edge of one of the triangle or pentagon. Later on students will learn that the angles of the triangle and pentagon do not add up to 180°.

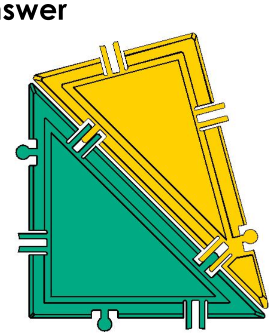
Problem 13

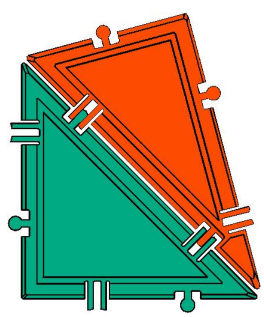
Answer



Problem 14

Answer

1. 

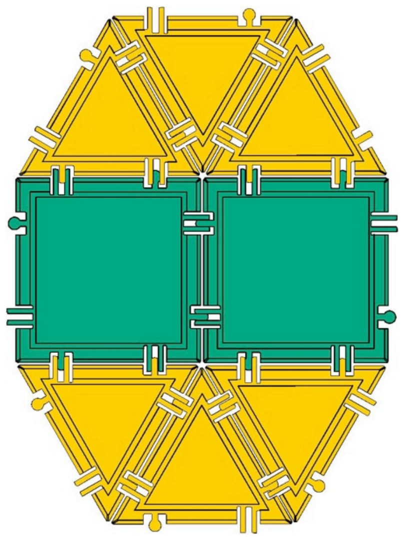
2. 

Hints

This is quite a challenging problem, so it's fine to give students detailed hints such as which parts to use to make the quadrilateral. They may attempt to make quadrilaterals that are parallelograms. If that happens, have the other students in their group comment on the correctness of their answer. The goal here is to make sure students really understand the various categories of quadrilaterals.

Problem 15

Answer



This is not exactly the same as a STOP sign, because in a STOP sign all the angles are the same. This is why it looks like a "skinny" STOP sign. It is similar to a STOP sign because it has all sides of the same length.

It would be a good follow up exercise to find the angles in a STOP sign and compare it to the angles in the octagon on the left.

Hints

You can begin by asking students to describe the STOP sign qualitatively. How many sides does it have? How many vertices (corners)? How do the lengths of the 8 sides compare to one another? What about the angles?

By the time students have reached this exercise, they will have made the green rectangle in the picture above (Problem 3) as well as the yellow trapezoids (Problem 5). If students are stuck, you can suggest that they look back on those shapes and try to connect them.