SHAPE CHALLENGE

# GEOMETILES ™

## Shape Challenge

For grades 2 and up

TM

Patent Pending



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# Introduction

The following exercises are intended to accommodate a wide range of students from grade 2 onwards. It is recommended that students do the exercises in this booklet before they go on to "3D Solid Understanding". That way, students will get familiar with the various 2-dimensional shapes they can build before assembling them into 3-dimensional solids.

You can do these exercises at whatever depth is appropriate for your students. As you well know, it can be difficult to anticipate what questions students will ask. Some of the material provided in this booklet is intended to help you address questions that your students come up with—or inspire you to ask your own! Material that is provided as an optional resource is appears in smaller font. If it seems too difficult, feel free to skip it.

# Activities in this booklet and the Common Core State Standards (CCSS)

## Common Core Progressions

The Progressions for the Common Core Math Standards are a series of documents each of which follows the development of a math topic throughout several grade levels. The Progressions allow a teacher to follow the development of, say, geometry, from grades K to 6. This gives the teacher a bird's eye view of where his/her students stand with respect to geometry, and what the geometry goals are in future grades.

At the time of this writing, the Progressions are in draft form. They are being written in part by the original authors of the CCSS, as well as other educators and mathematicians.

### How this workbook relates to the goals of the Common Core Progressions

According to the Common Core Geometric Progressions:

> *By the end of Grade 5, competencies in shape composition and de-composition… should be highly developed … Students need to develop these competencies because they form a foundation for understanding multiplication, area, volume, and the coordinate plane.*

With this in mind, it would benefit students to work through these exercises, and then follow them with the ones in the Geometiles™ "3-D Solid Understanding".

### CCSS for Mathematical Content supported by activities in this workbook

2.G.A.1 *Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.*

### CCSS for Mathematical Practice supported by activities in this workbook

**MP.1 *Make sense of problems and persevere in solving them.***

Each problem consists of a goal (make a certain shape) and a set of constraints (the number of tiles students can use). Students are encouraged to try different combinations of shapes until they find one that solves the problem.

**MP.3 *Construct viable arguments and critique the reasoning of others.***

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

**MP.6 *Attend to precision***

Solving these problems makes it necessary for students to use clear definitions of various polygons in discussion with each other and their teacher.

# Learning objectives of this workbook

a. Reinforce the idea of what a given geometrical shape is through tactile means and through constructing that shape in various sizes.
b. Recognize ways to decompose a polygon into smaller polygons.
c. Help students learn how certain polygons can be made with Geometiles™ so that they can use them later to build 3-dimensional solids.
d. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
e. Learn how to collaborate with one another.

Arguably, point d. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

If students are frustrated to the point of disengagement, you can modify the problem so they don't need to construct, say, every single parallelogram possible. They will still get a meaningful learning experience from doing the partial solutions.

Hints are provided as necessary to help the students who are having trouble gaining momentum with solving a problem.

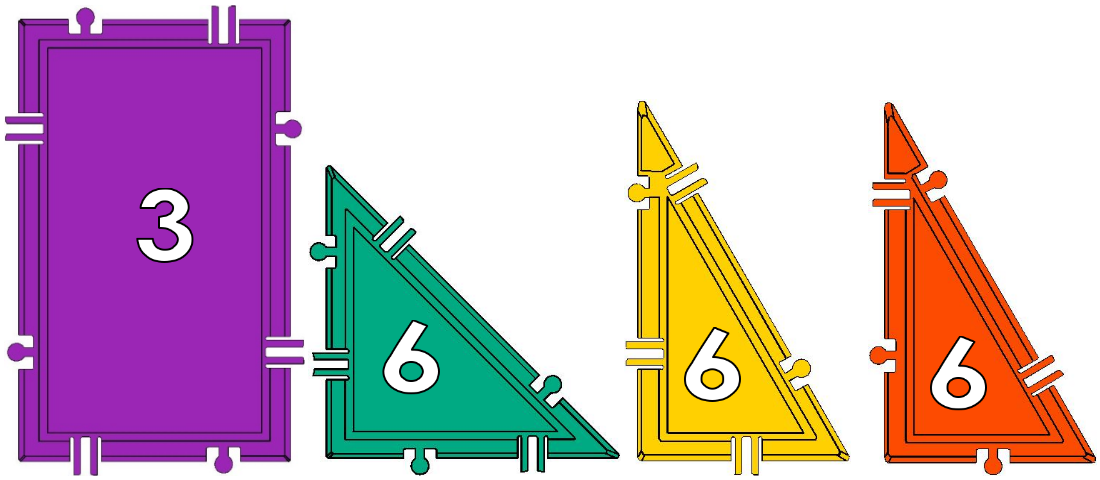
## Recommended classroom plan

The exercises are designed to work with 48 tiles (half of a 96-piece set), but not all at the same time. The 48 tiles will be split up into two sets: Set A (21 pieces) and Set B (27 pieces). These sets are defined in the **Problems** section. Have one group of students work with Set A and another work with Set B. Then switch the sets.

Each group (Set A and Set B) can be made of 2-3 students.  So the 96-piece set allows two groups to be working on Set A, and two groups to be working on Set B simultaneously. This makes a total 8 to 12 students per 96-piece set.

**SET A**

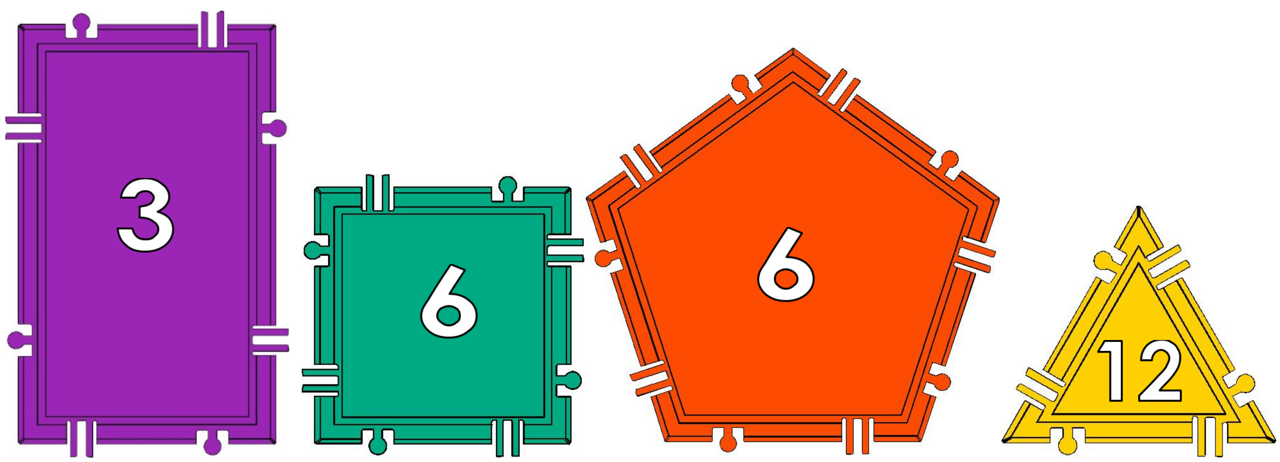
Shapes and quantities needed for 1 group of 2 to 4 students:




Note that yellow and orange triangles are <u>not</u> the same. You may want to point out the difference to the students.

**SET B**

Shapes and quantities needed for 1 group of 2 to 4 students:



## Problems

### SET A

1. Make a **square** with 2 tiles.
2. How many different **rectangles** (that are not squares) can you make with 2 tiles?
3. How many different **triangles** can you make with 2 tiles? Do any of them have all sides of the same length?
4. How many different **parallelograms** (not rhombuses) can you make with 2 tiles?
5. Make a **rhombus** with 4 tiles.
6. How many different **trapezoids** can you make with 3 tiles?
7. How many different **pentagons** can you make with 2 tiles?
8. How many different **hexagons** can you make with 3 tiles? Do the sides of your hexagons all have the same length?
9. Now make a **hexagon** all of whose sides have the same length.
10. Make a **pentagon** with 3 tiles.
11. Use 2 tiles to make a **quadrilateral** (not a parallelogram or trapezoid).
12. Make an **octagon**. Do all of its edges have the same length? How is it similar to a STOP sign? How is it different from a STOP sign?

# Problems

### SET B

1. Make a **square** with 4 tiles.
2. Make a **rhombus** with 2 tiles.
3. How many different **rectangles** can you make with 2 tiles?
4. Name the shape you get when you join the triangle to the pentagon.
5. Make a **triangle** out of 4 triangles.
6. Now make an **even larger triangle** out of the smaller triangles. How many triangles did you use?
7. Make a **trapezoid** out of 3 triangles.
8. Make a **parallelogram** out of 4 triangles.
9. How many different **pentagons** can you make with 2 tiles?
10. How many different **hexagons** can you make with 3 tiles? Do the sides of your hexagon all have the same length?
11. Now make a **hexagon** all of whose sides have the same length.
12. Make as many **different shapes** as you can using 4 triangles. You need to use all 4 triangles in every shape.

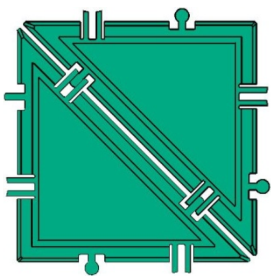
# Answers

**Please note that the colors of the tiles in the answers may be different from the colors in your set.**

## Set A

**Problem 1**

**Answer**



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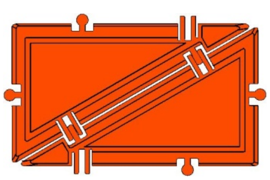
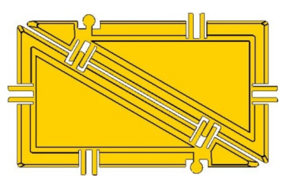
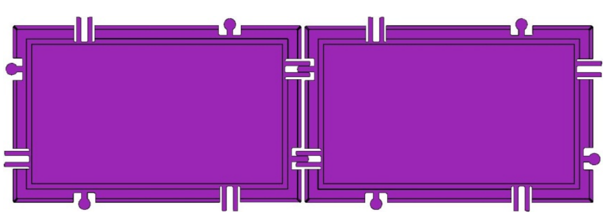
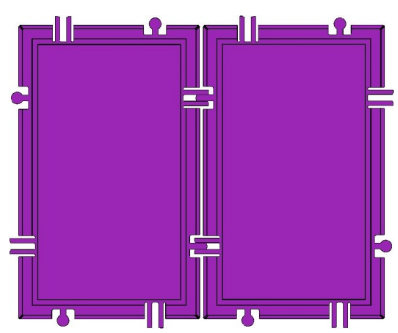
**Problem 2**

**Answer**

1.  or 

same rectangle made with different pieces

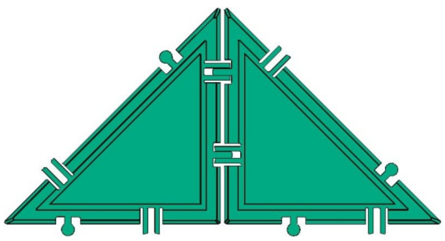
3. 

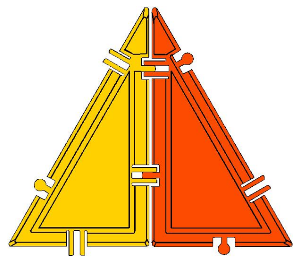
4. 

**Problem 3**

**Answer**

1. 

2. 

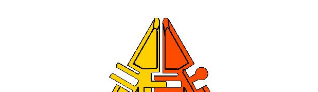
3. 

**Hints**

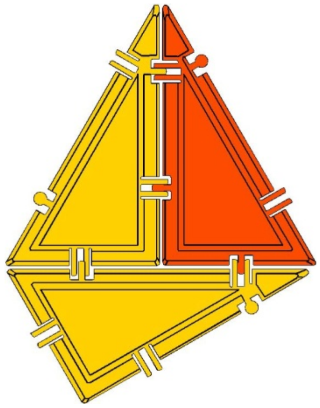
If students are having trouble with this problem, you may want to note that the only way they can make a triangle with two tiles is if both tiles are triangles.

Now, how do we prove that one of the triangles is equilateral? In other words, how do we show that the lengths of all 3 of its sides are equal to one another?

We know the left and right side lengths are equal to each other because they match up when we fold the triangle in 2:



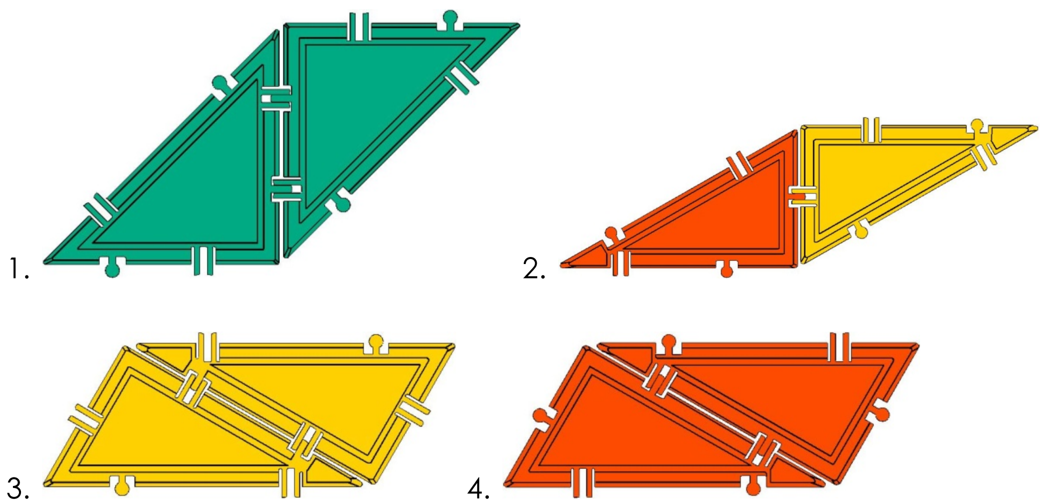
Now we just need to show that the bottom side length equals the other side lengths. We do this easily by taking a copy of either the red or yellow triangle and attaching it to the bottom:



The sides match exactly. Therefore, they have the same length.

**Problem 4**

**Answer**

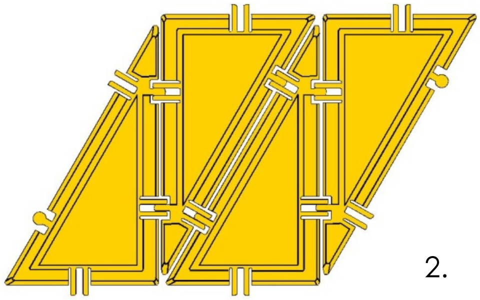


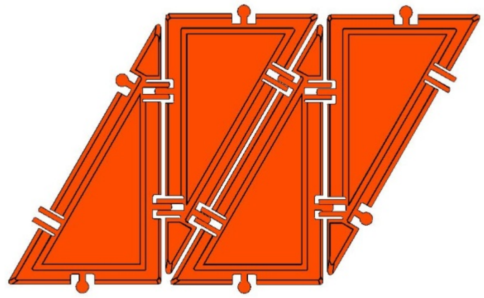

**Hints**

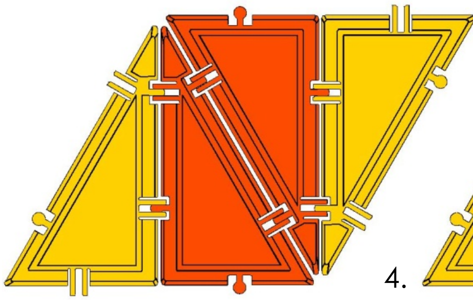
This is a good time to point out to the students that any parallelogram can be made of two identical triangles.

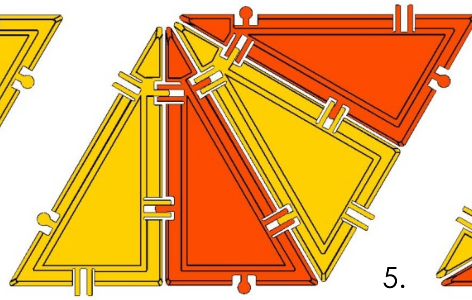
**Problem 5**

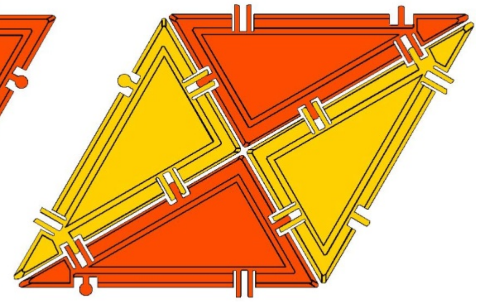
**Answer**

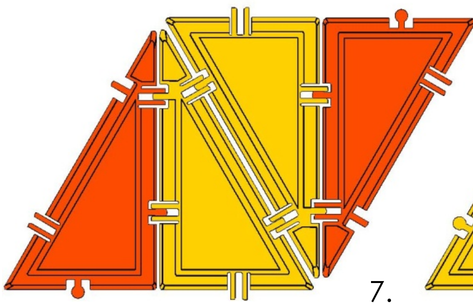
1. 

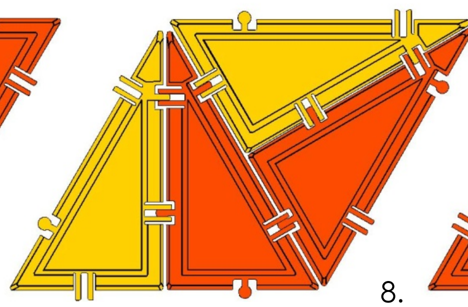
2. 

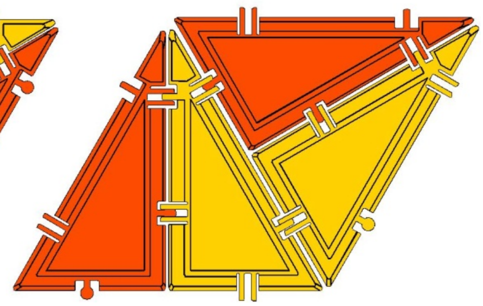
3. 

4. 

5. 

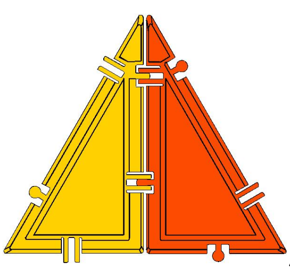
6. 

7. 

8. 

**Hints**

It would be nice to see how many of these students can make. If you get into this exercise, it would be great to organize the 8 possibilities. One way to do this would be to consider first the rhombuses made of a single type of tile. That would be solutions 1 and 2. Notice that the remaining 6 solutions are each made of two of these equilateral triangles,

,

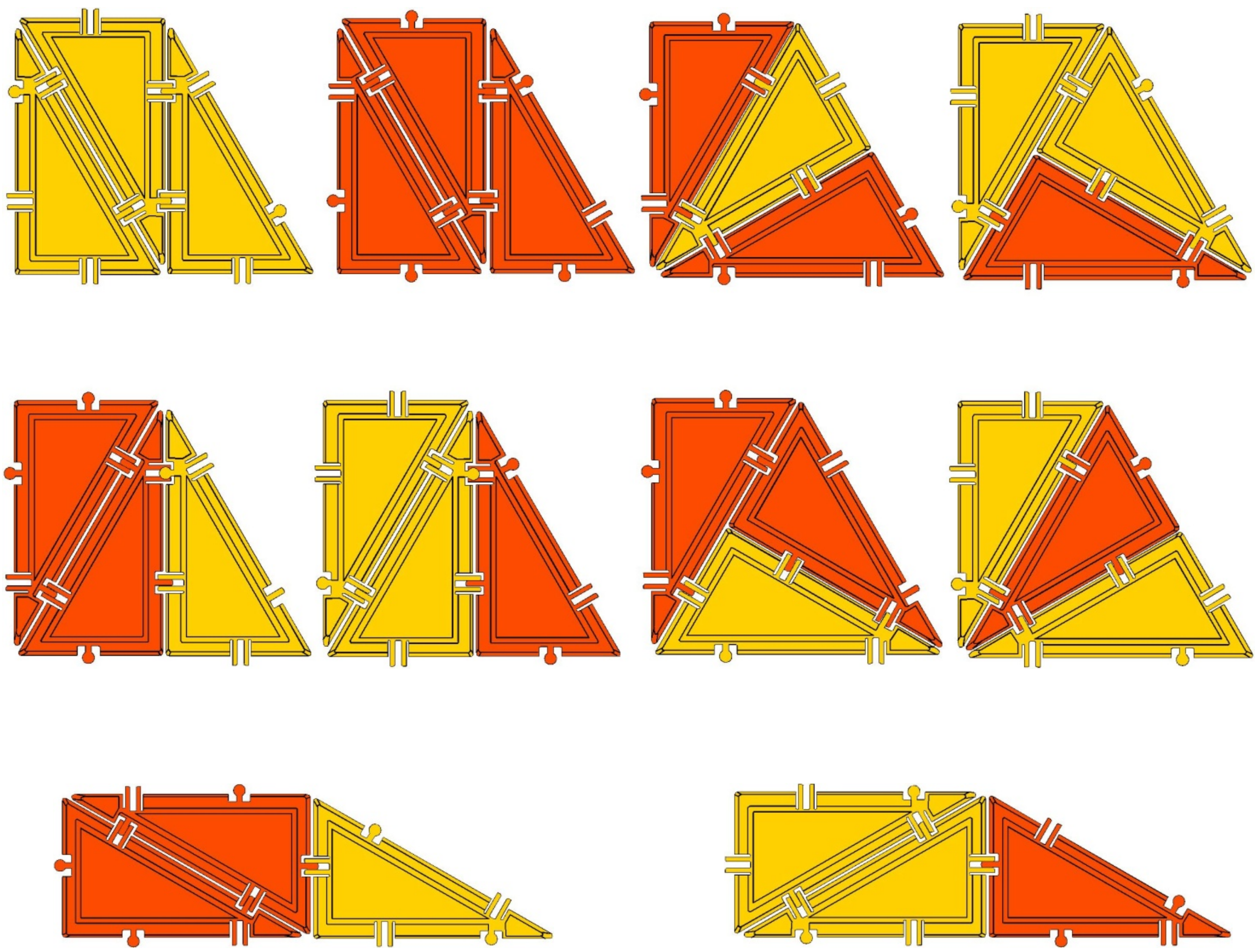
put together in a variety of ways.

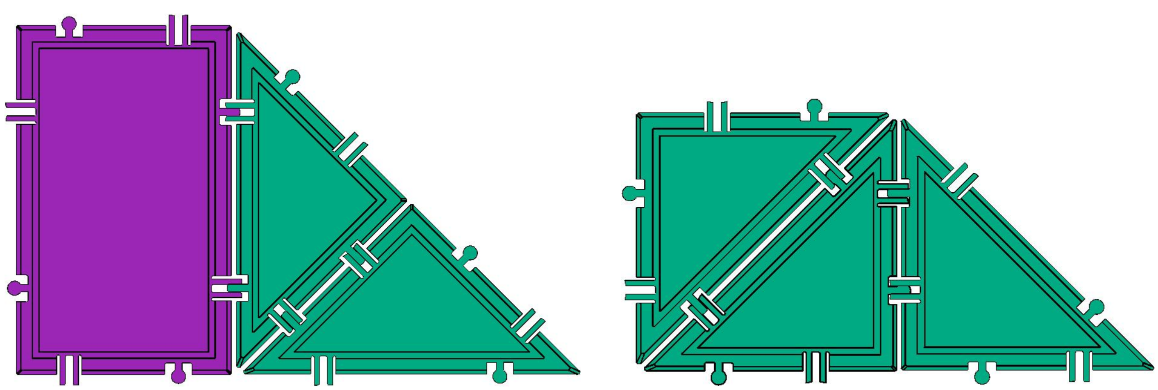
**Problem 6**

**Answer**

There are 10 possible ways to make a trapezoid with just the 30-60-90 triangle tiles:



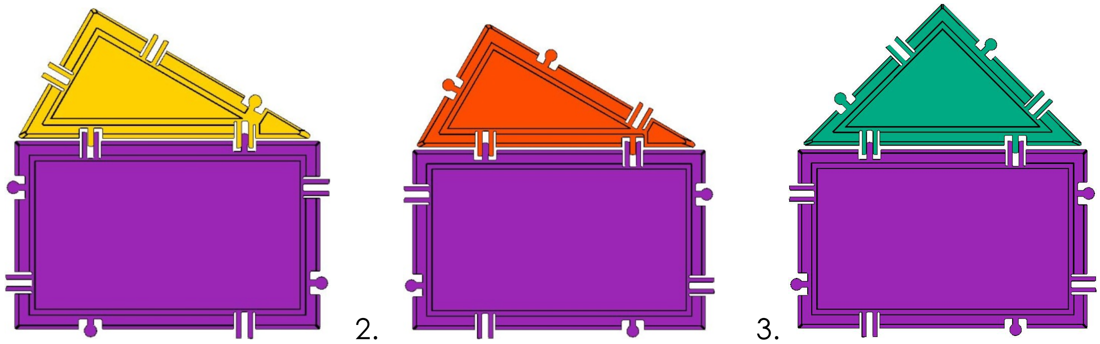
And two additional ways:



**Problem 7**

**Answer**



1. 2. 3.

**Hints:**

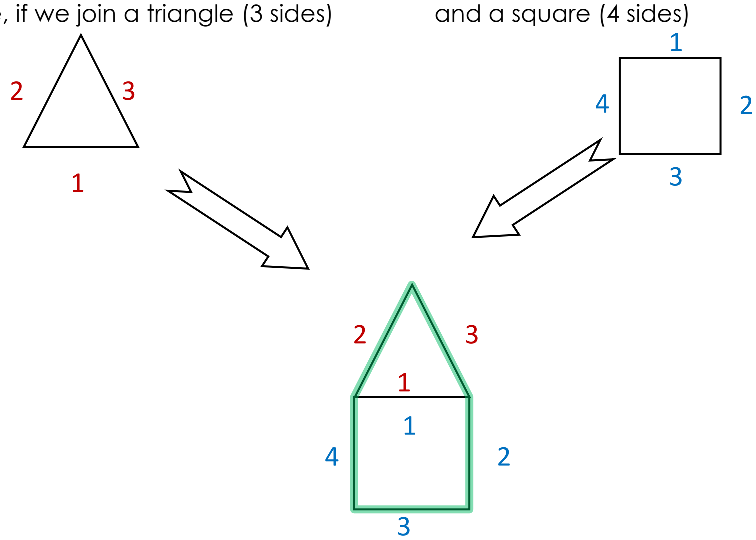
Students may be surprised that these are not shaped like the pentagonal tile in this set. In other words, they are not *regular* pentagons. A *regular* pentagon has all equal sides and equal angles. For example, the Pentagon building in Washington D.C. is a regular pentagon. They were simply asked to make a pentagon, which means a figure with 5 sides. If your students are ready for a challenge, you can engage them in the following discussion.

How do we know that these three pentagons are the only ones we can make with 2 tiles? This is a very typical type of question that mathematicians and scientists often ask themselves, and, if your students are up for it, a worthwhile one to discuss with them.

When we join together two polygons and we count the number of sides in the resulting figure, we get

# of sides in polygon A + # of sides in polygon B - 2.

For example, if we join a triangle (3 sides) and a square (4 sides)



we get a figure with 5 sides (shown with green outline). That comes from

3+ 4 – 2 = 5.

The 3 comes from sides the triangle, the 4 from the sides of the square, and the 2 comes from the fact that we "lost" side 1 of the triangle and side 1 of the square when we joined them together.

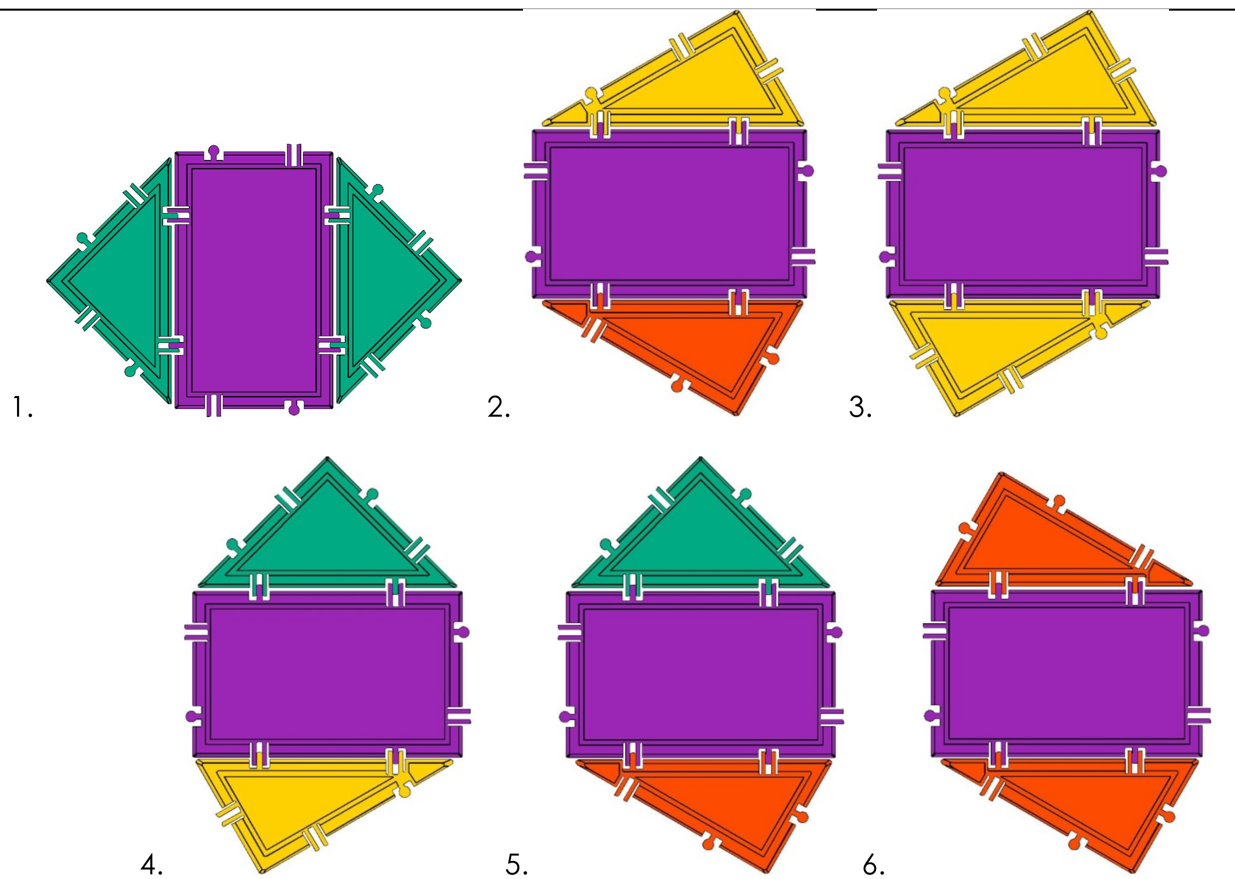
More generally, when we search for two tiles that join to form a 5-sided figure, we want tiles such that the sum of their sides is 7 (that way 7-2=5 and we get a pentagon). You can brainstorm with the students about what numbers add up to get 7:

1+6
2+5
3+4

The only possibility worthy of our consideration is 3 +4, because there's no such thing as a 1-gon or a 2-gon. So we know we can only get a pentagon by joining a triangle tile with a quadrilateral tile. The only quadrilateral tile we have in this set is the rectangle. So the only pentagons possible made from 2 tiles are formed by connecting triangles to rectangles.

**Problem 8**

**Answer**



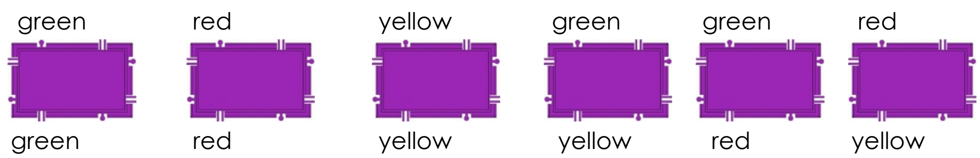

Clearly, none of these hexagons are equilateral.

**Hints**

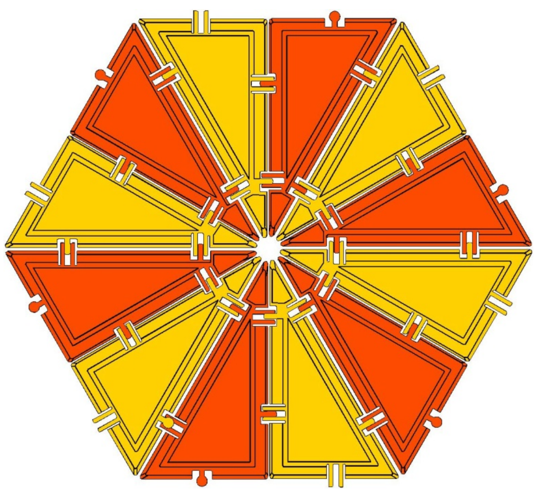
We can start off with the pentagon from the previous question, and to see what other triangle can be added to the rectangle from the other side to make a hexagon.

Here is an optional way to extend this discussion.
Let us enumerate all the possibilities if we have 3 different triangles that can be connected to the rectangle. This step is an early introduction to outcomes in probability that will be extremely useful to students in middle school and onward. The rectangle connects to any of the triangles on two of its long sides. The triangles will be referred to by their colors



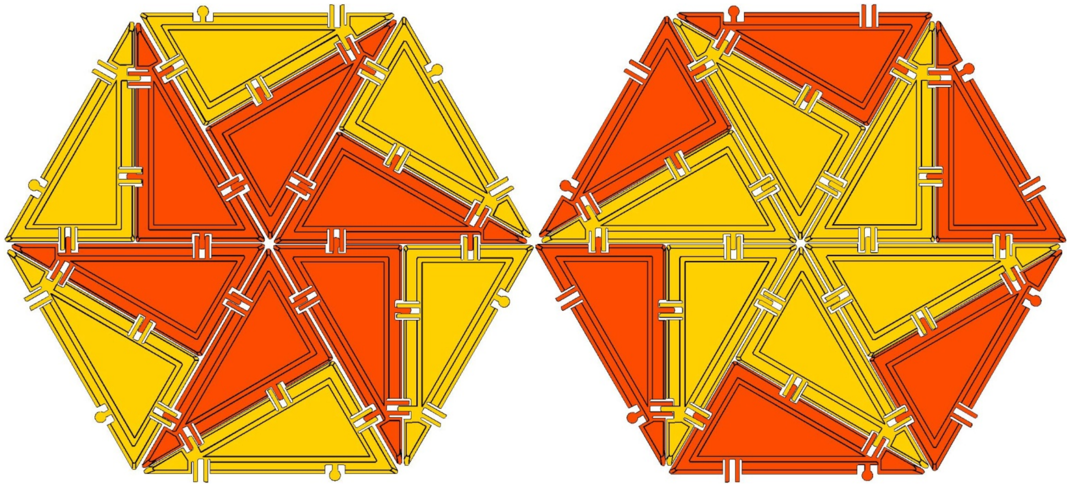

In coming up with the 6 possibilities above, it is crucial to keep your thinking organized. In the order used above, we first considered pairs of the same color (green/green, red/red, yellow/yellow) and then all the possible pairs of 2 different colors (green/yellow, green/red, red/yellow). You can come up with your own method of organization if you want.

1. We're obviously going to need more than 3 tiles for this! You may need to remind students of the equilateral triangle they made as part of the answer to Problem 3. If they join 6 of these triangles together they will get this:

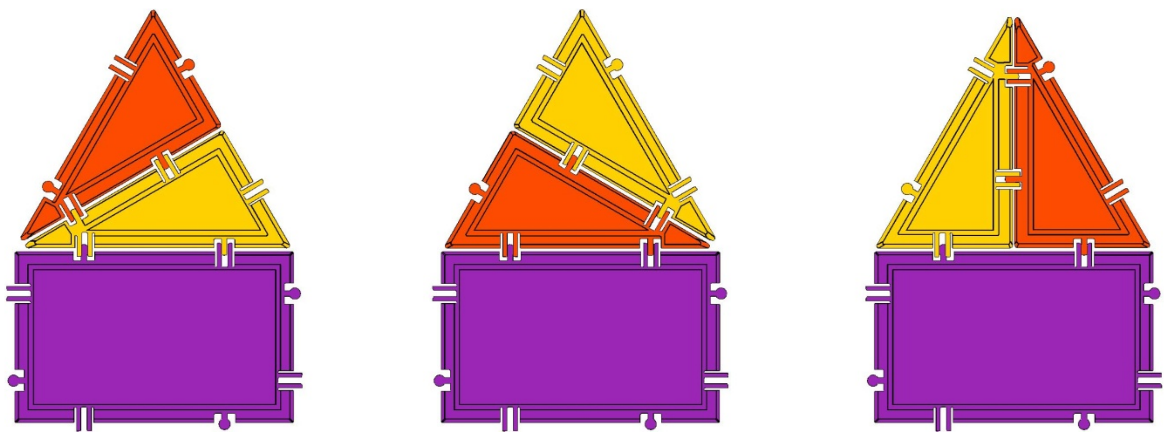


The are many other ways to arrange the above 12 triangles into a hexagon. Here are some of the more aesthetically pleasing examples:

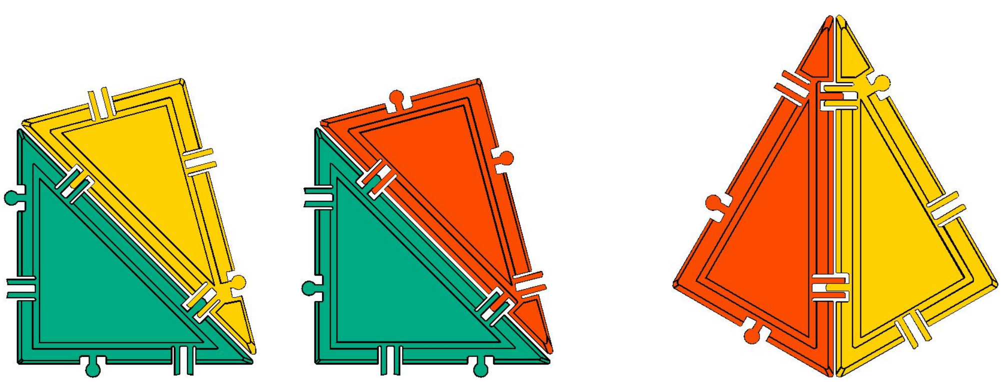
 



2. These are the only 3 possibilities:



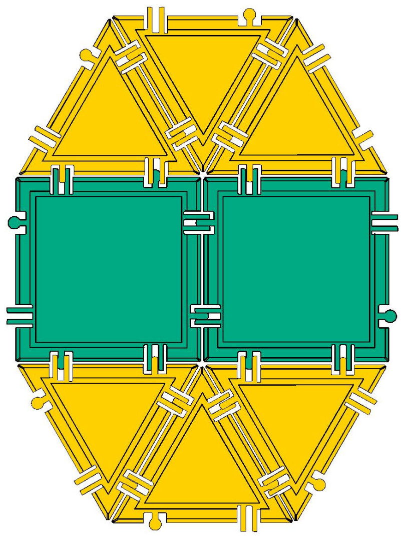
3.



12.

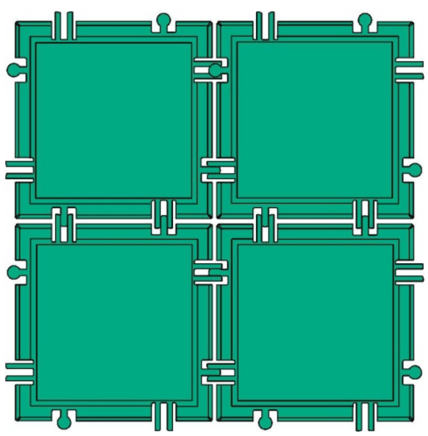
**Answer**



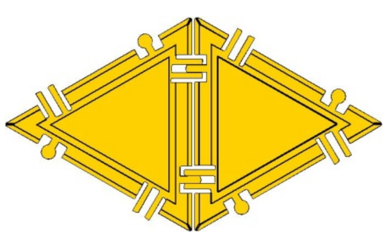
This is not exactly the same as a STOP sign, because in a STOP sign all the angles are the same. This is why it looks like a "skinny" STOP sign. It is similar to a STOP sign because it has all sides of the same length.

## SET B

1.

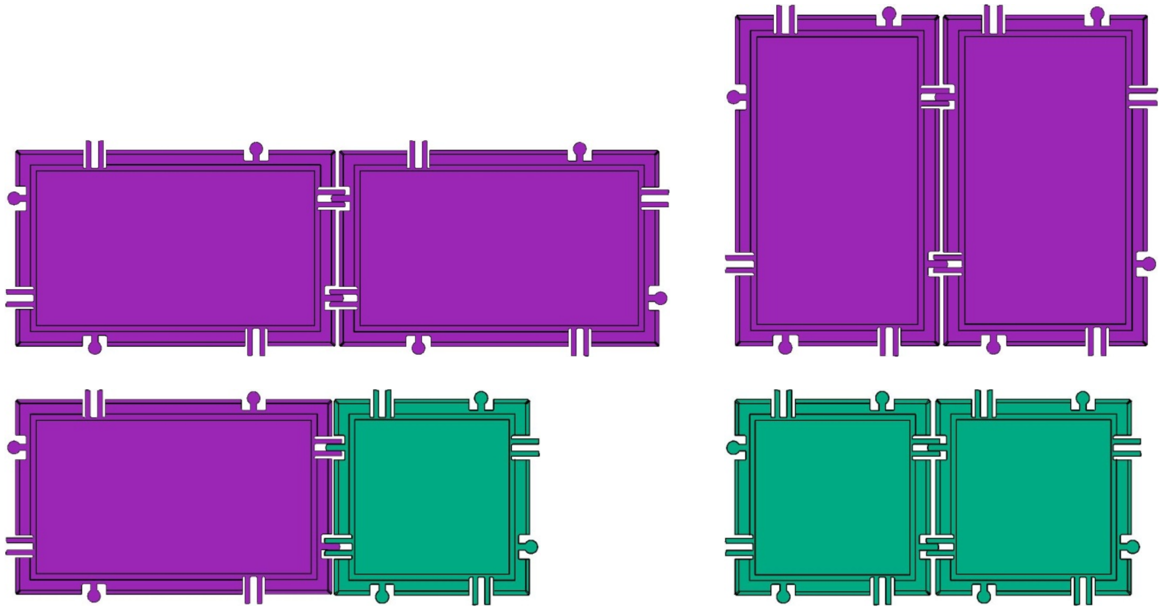


2.

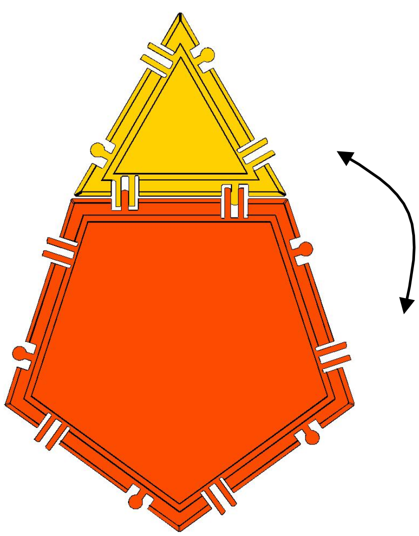


3.



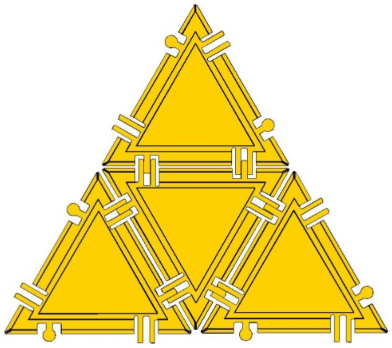
4. This is a hexagon.



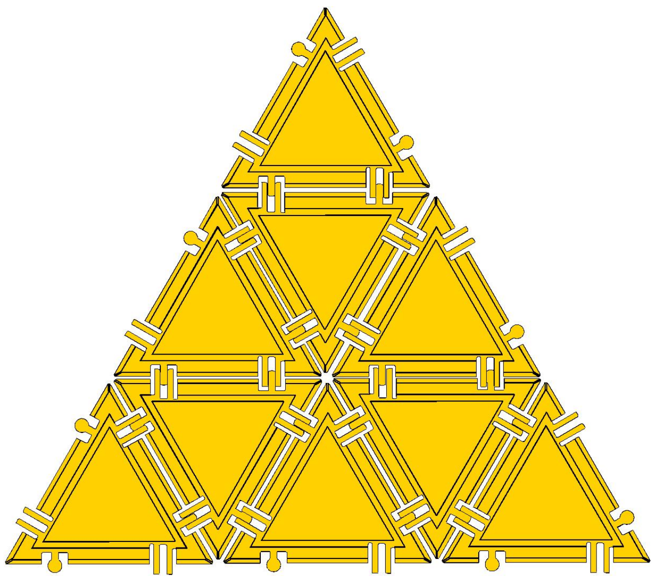
This may look like a straight line but it is not. If students have a hard time believing this, you can align a ruler with the edge of one of the triangle or pentagon. Later on students will learn that the angles of the triangle and pentagon do not add up to $180^{o}$.

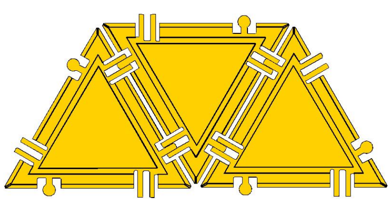
5.



6.



7.



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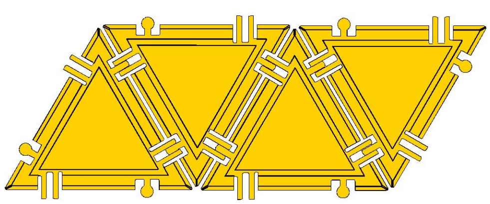

9. Students may be surprised that these are not shaped like the pentagonal tile in this set. In other words, they are not *regular* pentagons. A *regular* pentagon has all equal sides and equal angles. For example, the Pentagon building in Washington D.C. is a

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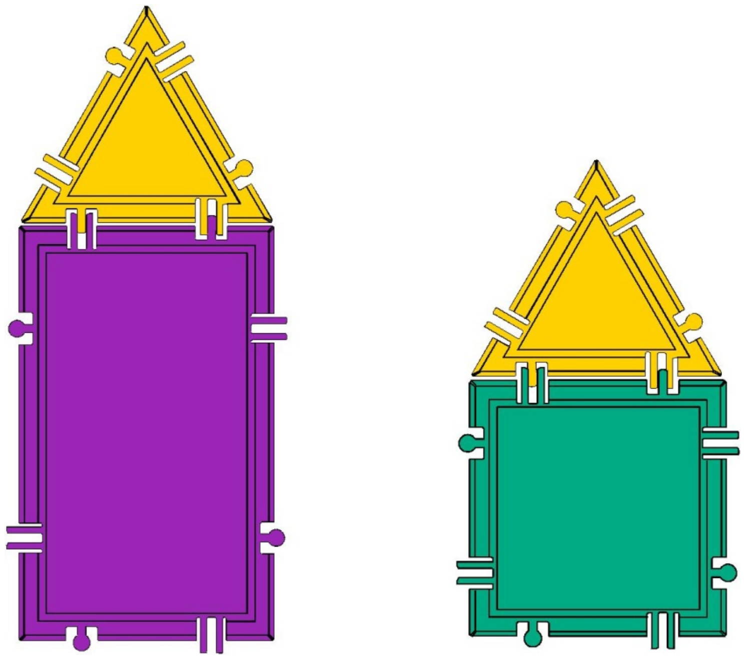

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When we join together two polygons and we count the number of sides in the resulting figure, we get

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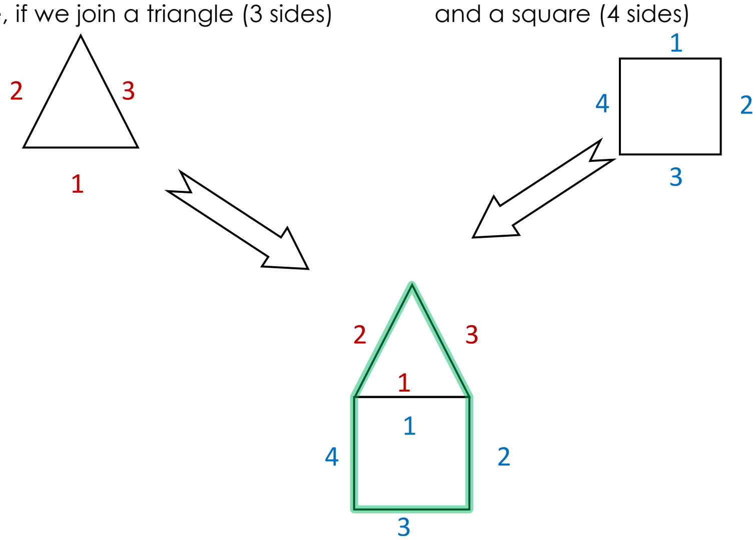
For example, if we join a triangle (3 sides) and a square (4 sides)




we get a figure with 5 sides (shown with green outline). That comes from

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The 3 comes from sides the triangle, the 4 from the sides of the square, and the 2 comes from the fact that we "lost" side 1 of the triangle and side 1 of the square when we joined them together.

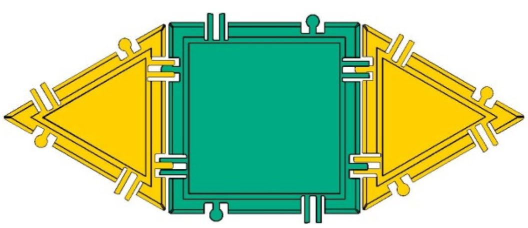
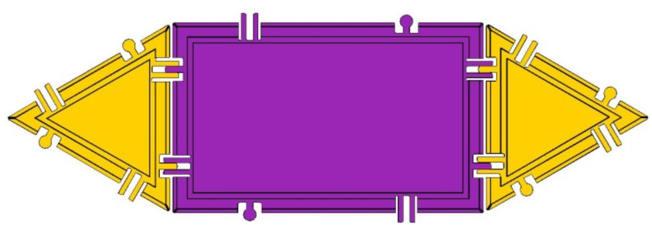
More generally, when we search for two tiles that join to form a 5-sided figure, we want tiles such that the sum of their sides is 7 (that way 7-2=5 and we get a pentagon). You can brainstorm with the students about what numbers add up to get 7:

1+6
2+5
3+4

The only possibility worthy of our consideration is 3 +4, because there's no such thing as a 1-gon or a 2-gon. So we know we can only get a pentagon by joining a triangle tile with a quadrilateral tile. The only quadrilateral tiles we have in this set are the square and the rectangle. So the only pentagons possible made from 2 tiles are formed by connecting the triangle to a square or a rectangle.

If you or your students are interested in terminology, the pentagon made from a triangle and a square is called an *equilateral* pentagon, because all of its sides are equal. However, it is not a regular pentagon, since its angles are not equal to each other. A triangle that is equilateral is also regular, but with all the other polygons equilateral need not be regular.  For an extra challenge, you can ask your student to name a quadrilateral that is equilateral but not regular [a rhombus].

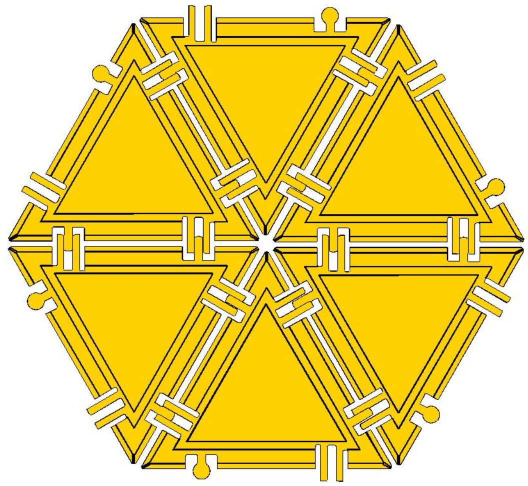
10. We can start off with the pentagon from the previous question, and to see what other triangle can be added to the rectangle from the other side to make a hexagon.

Clearly neither of the above hexagons are equilateral.

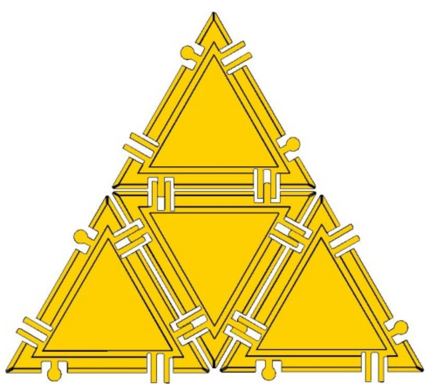
 

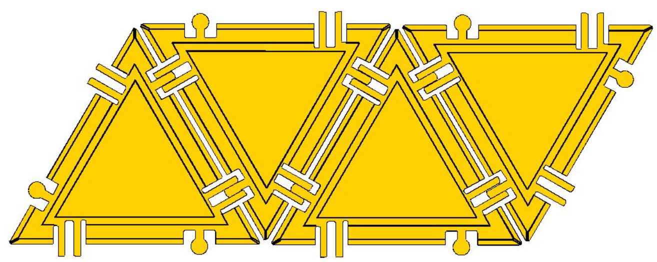
11.



12. You may want to split up the 12 triangular tiles into 3 groups of 4, and give each pair of students 4 tiles to work with.

We have already seen the equilateral triangle and the parallelogram.



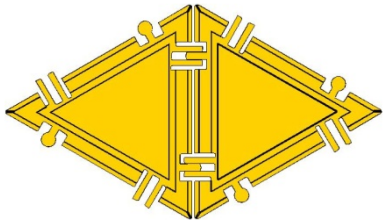


Is this it? How do we go about finding all the possibilities systematically?

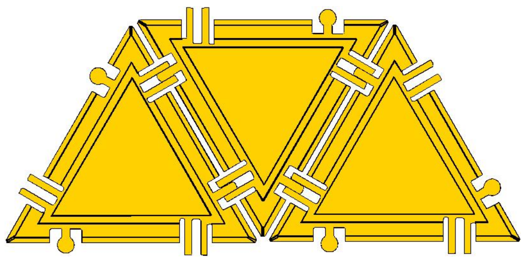
Let us first ask a simpler problem: how many shapes can we get from connecting just 2 triangular tiles? This is a very important general strategy for problem solving that would be of great long term benefit to your students. As stated in the Standards Mathematical Practice, mathematically proficient students

*try special cases and simpler forms of the original problem in order to gain insight into its solution.*

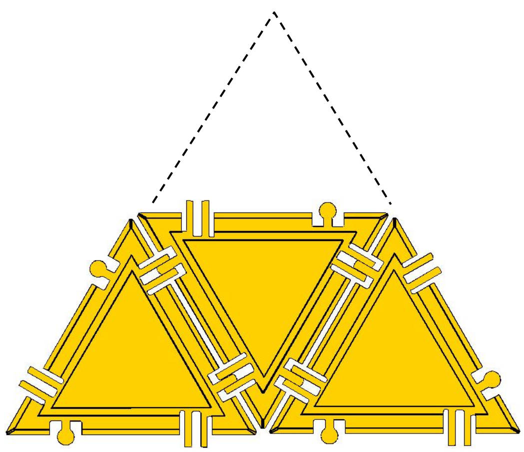
We have already worked with 2 triangular tiles in Problem 2 of this section, and they will only make one shape: the rhombus.



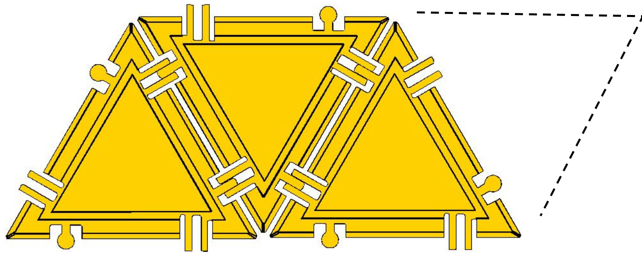
What are our possibilities if we connect the 3rd tile? It looks like we can connect the 3rd tile to any of the 4 sides of the above rhombus. However, no matter which side we choose, the result will always be the same: the trapezoid of Problem 7.



We still need to investigate what happens when we attach that last 4th tile to the trapezoid. Now things are getting a bit more interesting. If we put the tile on top, we get the equilateral triangle that we have already seen.



If we put the triangle on either of the sides, we will get a parallelogram:



Note that since this trapezoid is symmetric about its middle, we get the same figure (rotated ) whether we attach the triangle to the left or right side.

Finally, we can try attaching the 4th tile to the bottom of the trapezoid. We will get something new: