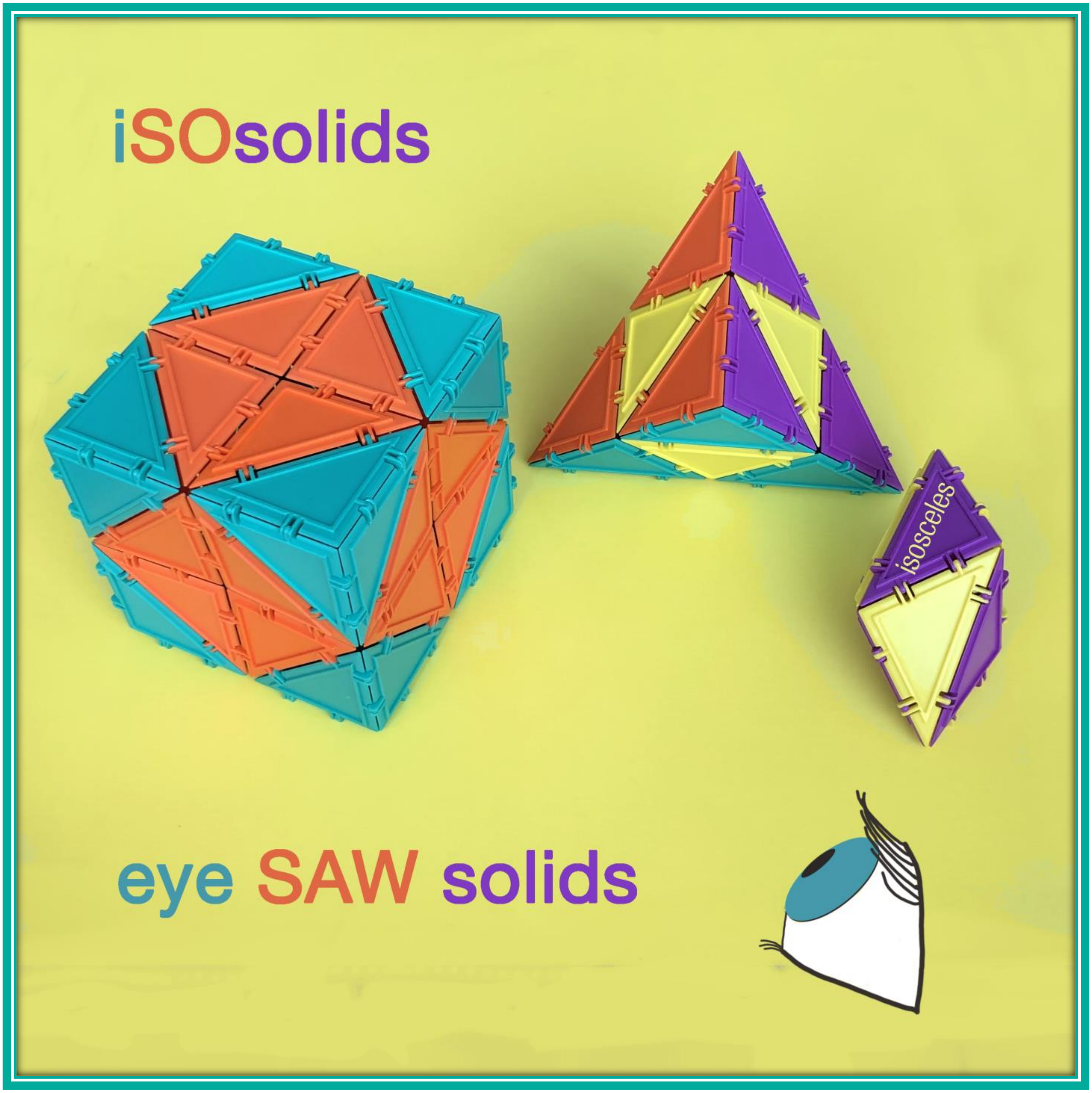
iSOsolids



eye SAW solids

TM

Activities with 48 Geometiles® right isosceles triangles.

Introduction

These games are based on the fact that one can make surprisingly many closed solids out of these right isosceles triangles. Constructing these solids, and doing it quickly, will hone the students' spatial reasoning abilities as well their fine motor skills.

Some of the solids, like cubes, are fairly straightforward to construct. The larger cubes are great to do in teams, with a couple of students working on a single face. Other solids are much less obvious, especially the non-convex ones. The games presented here vary in their level of complexity – there is something for everyone!

How to get started

In the beginning it may not be obvious to the students that so many different size cubes are possible with these isosceles triangles. In fact, some of these shapes are downright difficult to construct. They will need to explore the various shapes they can create. Here are some possible approaches to try.

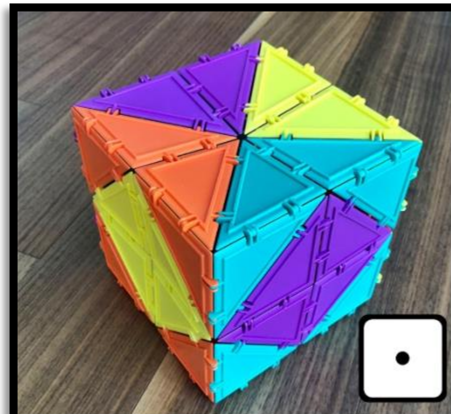
- **Start off by taking two triangles and naming the shapes that are created when you connect them. What are all the possible shapes you can make with two right isosceles triangles (square, parallelogram, another right isosceles triangle) This demonstrated in this video. https://youtu.be/QCln1hev7Ps**
- **Can you make squares in different sizes? See how many you can make? These squares will be the faces of your cube(s). How many faces does a cube have? What is the largest cube you can make with 48 isosceles triangles?**
- **Can you make a cube? What about a larger cube?**
- **Notice that once you make one shape, you can make a larger version of that shape by substituting two isosceles triangles where there used to be one. For example, you can see this in the shapes using 8 and 16 isosceles triangles in the second picture of this document.**
- **It's good to give students ample time to experiment with all the shapes before they start competing with each other.**
- **We welcome any suggestions on how to get started based on your classroom experience!**

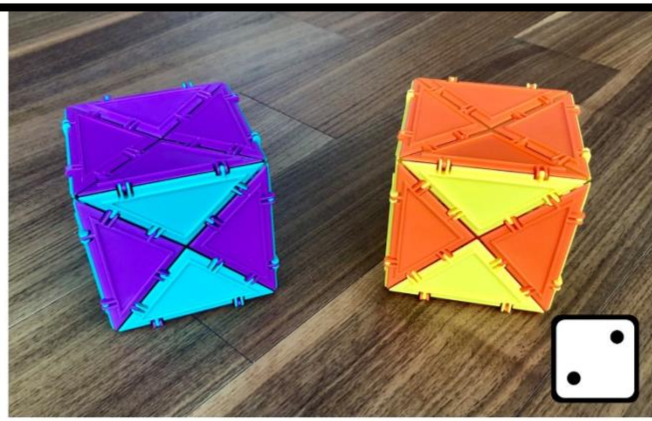
Game Ideas

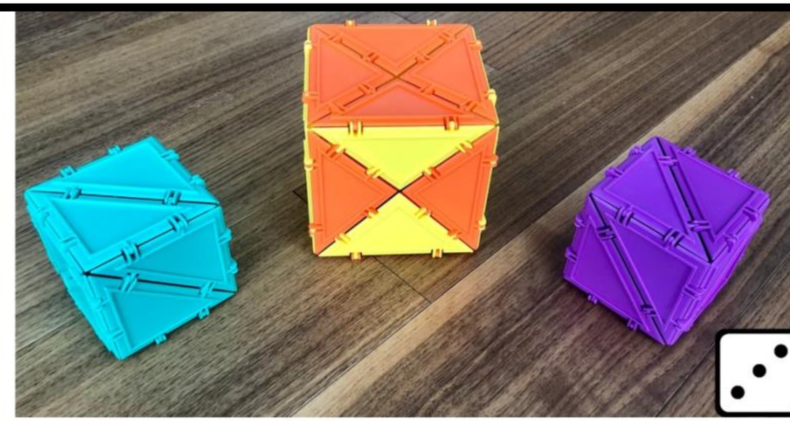
Here are two potential "games in development" for students with this set. We call something a "game in development" if we offer rules for a game as a starting point for students—they should feel free to change them, or create their own rules if it makes the game more engaging. They can even change the names of these games.

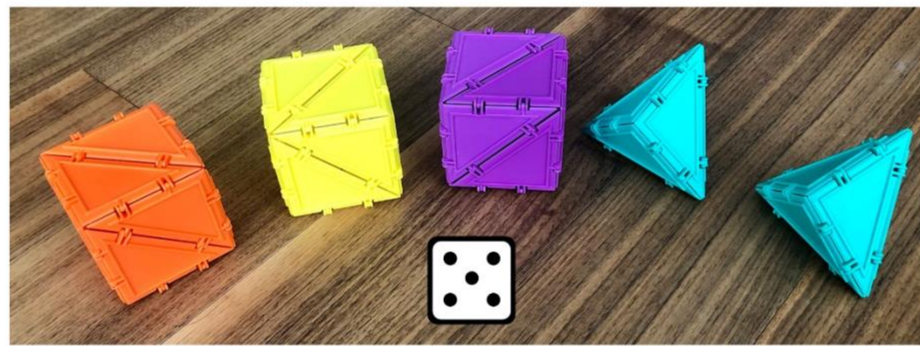
Game 1: Isoscecubes

Toss the die. Whatever number comes up on top is how many (mostly) cubes you need to make out of all 48 tiles. This game is intended for 4-5 students. The basic form of the game is summarized in the image below.









ISOSCECUBES

How many can you make with **48** isosceles triangles?

Toss a die

and start building!

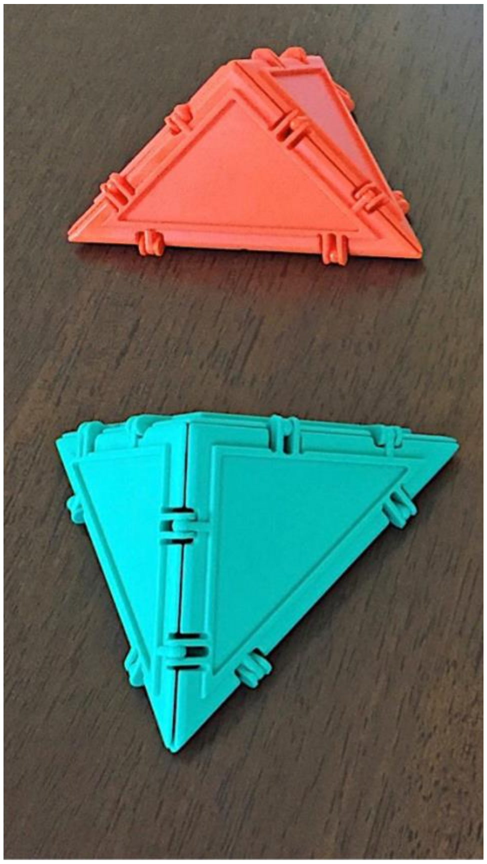
Additional notes on Isoscecubes:

Relationship of double tetrahedra and cubes

The double tetrahedra you see in last two pictures corresponding to  and  look like this when viewed from the side.

The double tetrahedron above has an interesting relationship to the cube below.

Each double tetrahedron has volume that is 1/3 of the volume of the cube. To see this, first imagine splitting the double tetrahedron in two equal halves like this:

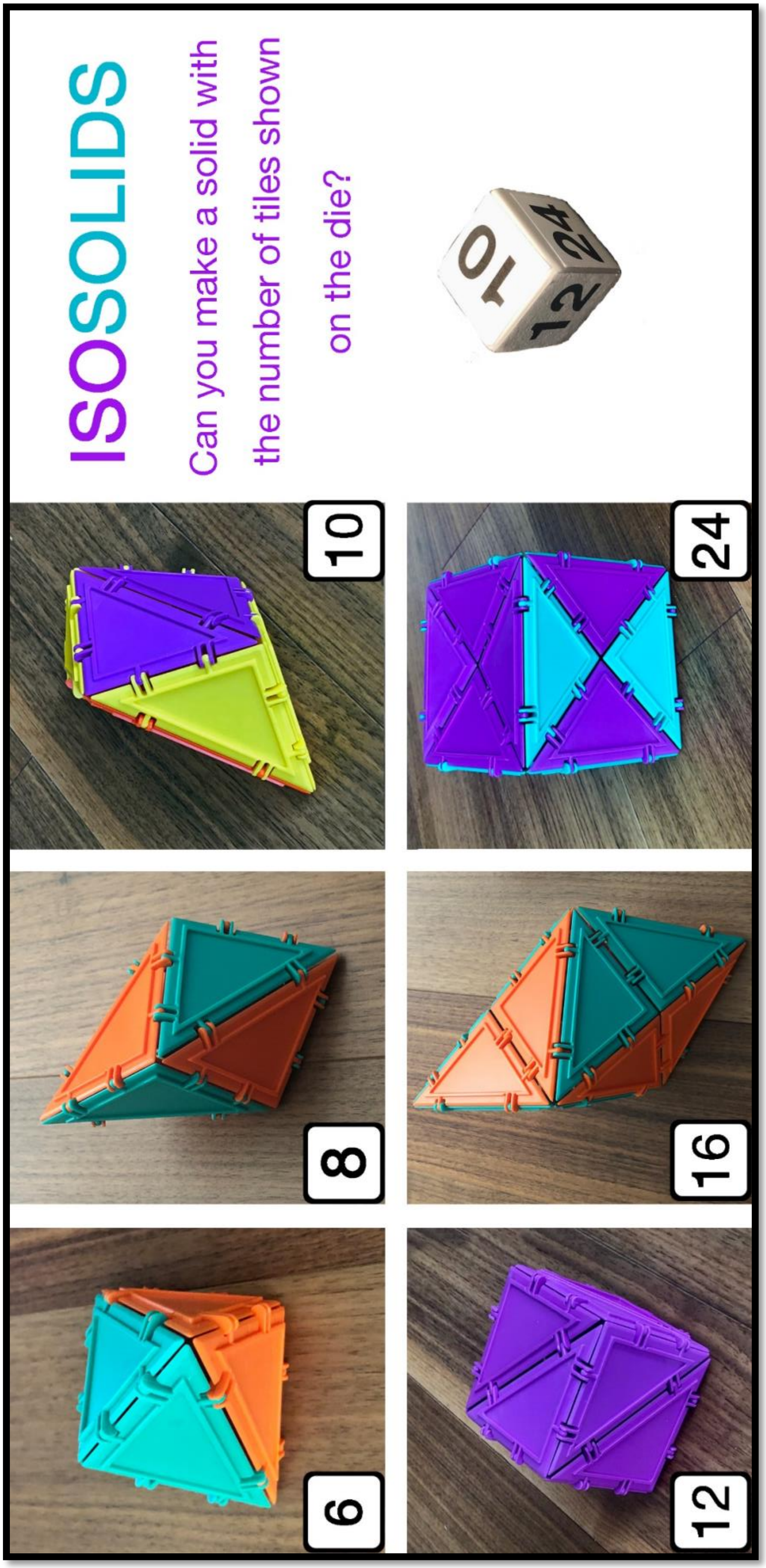


You may recognize each of the tetrahedra as being the corner part of a cube.

Further details are discussed in the booklet Volume and Calalieri's Principle.

Game 2: Isosolids (you pronounce it with the accent on the first "o" so it rhymes with Isosceles. Dorky, yes!)

Toss the custom made die. You can make your own by buying a blank die and putting your stickers on it. Whatever number comes up on top is how many pieces you need to use to make your solid. This game is best played as a race between two teams of 3-4 students, each with 24 tiles.


ISOSOLIDS
Can you make a solid with the number of tiles shown on the die?
10
12
24
10
8
6
24
16
12

Each team rushes to make either closed solid, and the team which is fastest scores 2 points.

If the other team manages to construct the correct solid, but is not the first one to do it, it scores 1 point. If they cannot construct the solid at all, they need to seek help from the other team. Once a solid is constructed, it stays together until the die is rolled again.

Note that it is quite challenging to come up with shapes that uses exactly 8, 10, and 16 triangles. Students shouldn't feel bad if they find this task formidable at first!

Further thoughts

The list of polyhedra made of a given number of triangles in the previous picture was not complete. For one, we only considered convex polyhedra. If your students are ready for an adventure, we can expand our range to cover concave polyhedra as well. Here are some interesting polyhedra, some of them concave, that you can incorporate into your game. They are listed here, along with the number of isosceles triangles used to build them, up to 24 triangles.

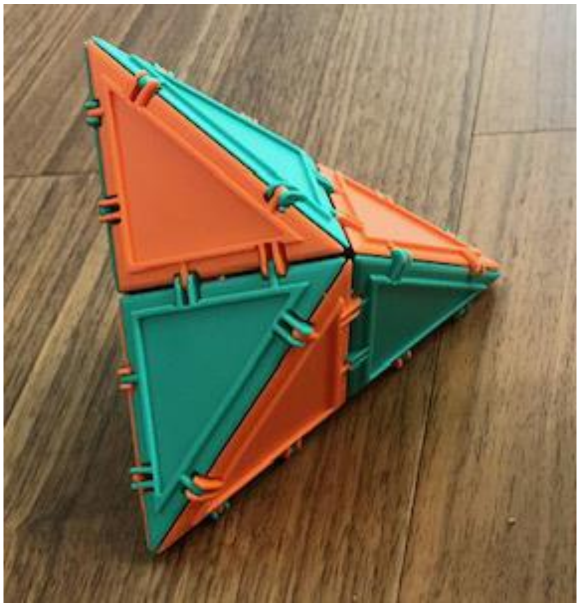
10 triangles:

Concave polyhedron made of 10 triangles

Convex polyhedron made of 10 triangles

Note the difference between this concave version and the convex version shown earlier. It is quite subtle: the pair of purple tiles was oriented differently.

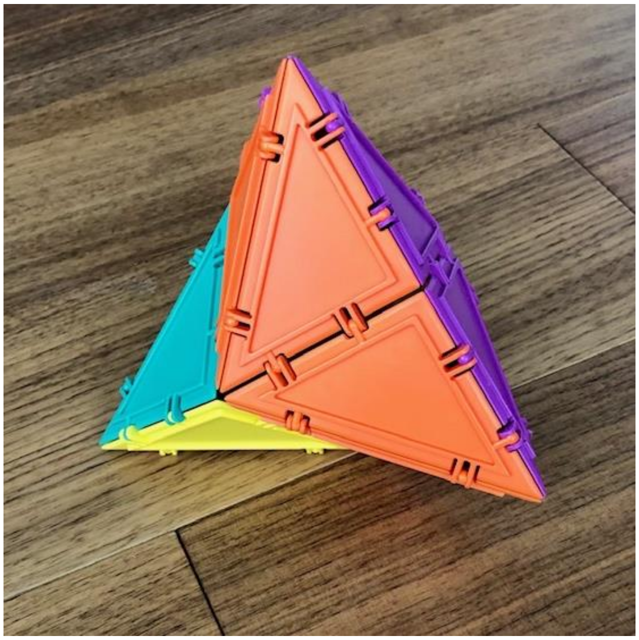
12 triangles



Another double tetrahedron

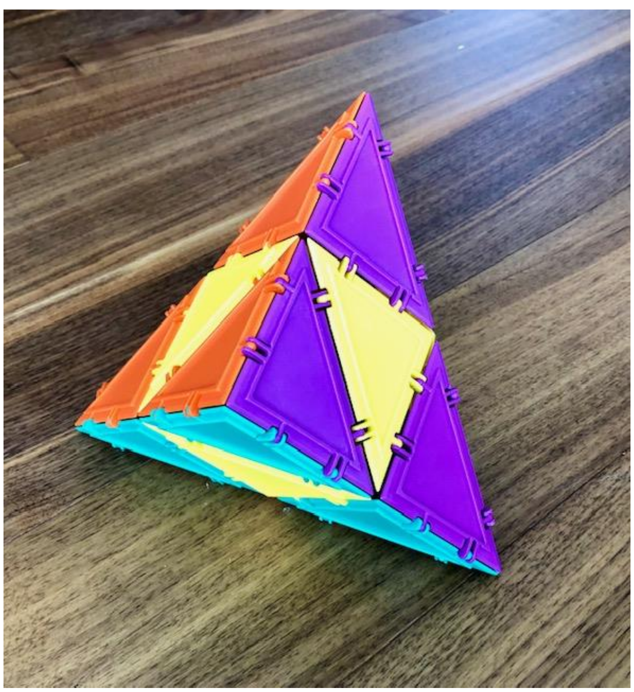
16 triangles

This last structure spins! But that's not all. It almost fits the category of monohedral isosceles bipyramids described in David Eppstein's paper arXiv:2009.00116, Figure 8. Construction of this spinner is explained in the Geometiles Spinners booklet.



The twisted pillow? I don't know the name of this polyhedron, but it sure looks like a pillow cut in half (the purple/orange being one half and the yellow/green being the other) and then twisted.

24 triangles



This is a larger double tetrahedron with each face consisting of 4 isosceles triangles.

This is a sort of stellated [regular octahedron](). Each triangular face is replaced with 3 interconnected triangles.

Those are all the shapes we could find so far. If you find more, please let post on social media and tag us!

We are @geometiles on Twitter, Instagram, and Facebook.

