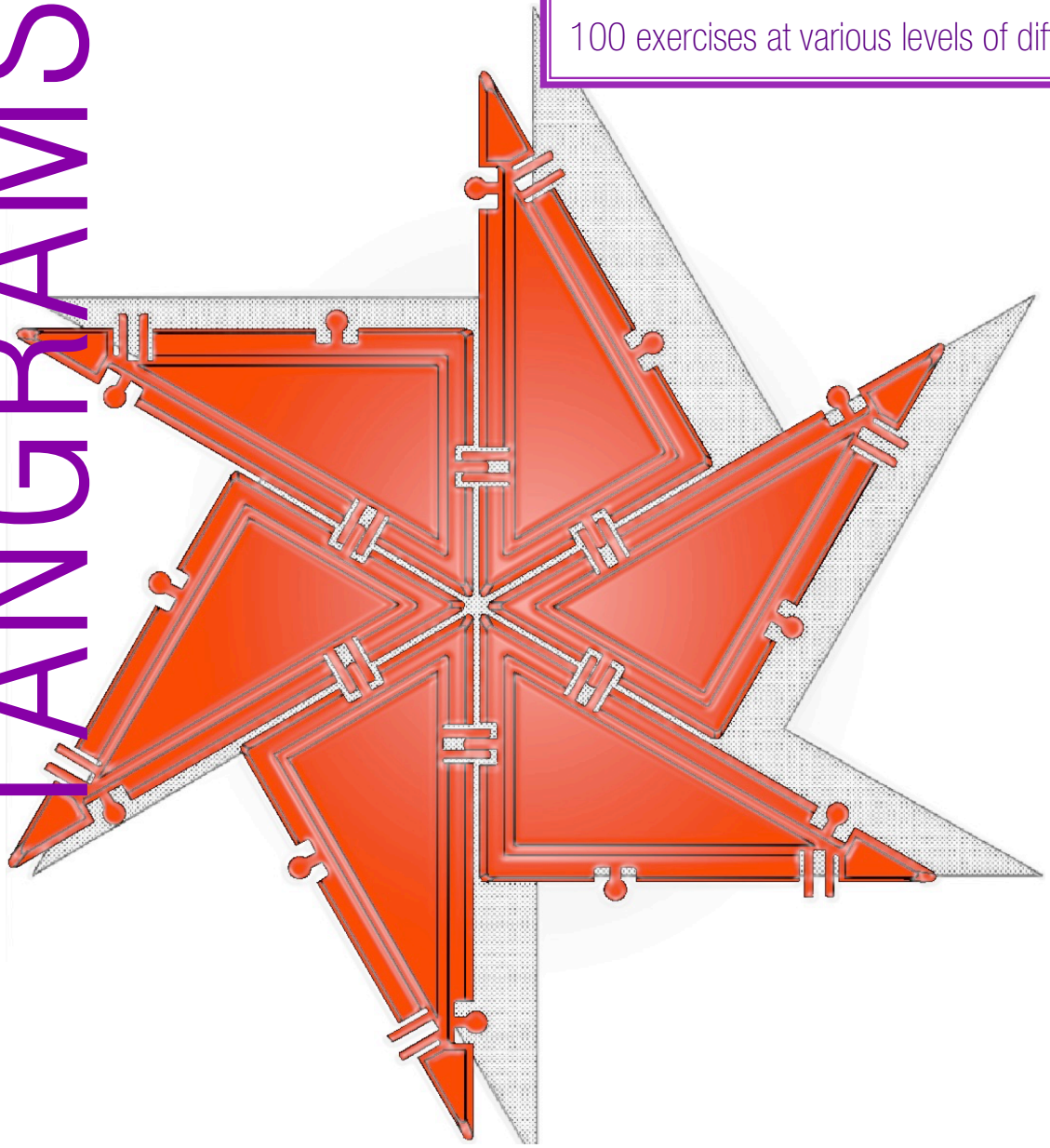
GEOMETILES ™

Tangrams

For grade K and up

100 exercises at various levels of difficulty

TANGRAMS



™

Patent Pending
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Introduction

This workbook is the perfect place to start your exploration with Geometiles™. There are no mathematical terms used, so the exercises are suited for children as young as kindergarteners.

The problems provide a great opportunity for children to hone their problem-solving skills, particularly logical and spatial reasoning. If at all possible, try to avoid solving the problem for the child, or having the child walk away from the problem—unless she wants to come back to it later. It is worthwhile to have the child struggle a bit, as the "Aha!" moment when she solves the puzzle is both rewarding and confidence-building.

Do encourage children to describe the shapes qualitatively; for instance "This one looks sharper/skinnier" or "These two sides look like they are the same length". Please be sure to read the booklet "Getting acquainted" prior to starting this workbook.

The Rules

Explain to the students that they are supposed cover the shaded figures with the tiles precisely.

The rules are:

1. Any shapes that are next to each other have to snap together.
2. The edges of the shape should be exactly on top of the edge of the shaded figure. They should not be inside the shaded figure or outside its border. See example below.

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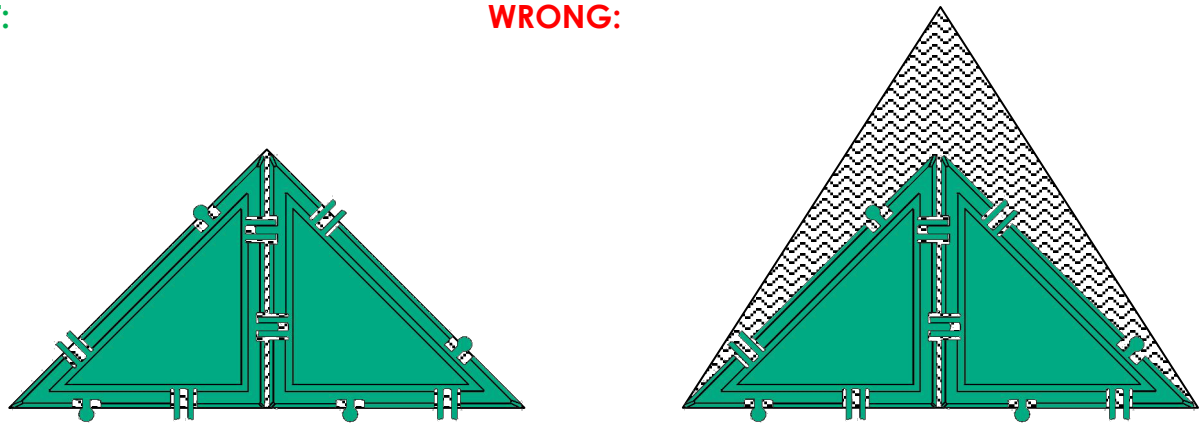
RIGHT: **WRONG:**



The tangrams have been arranged in order of increasing difficulty by groups.

Hints

Here are some hints that you may want to give the child who shows signs of frustration.

- Tell the child how many pieces are needed to solve the problem. You can get these numbers from the solutions section.
- If you feel that the child is going off on the wrong path, here are some questions you may want to ask him/her:
    1. Does the length of the piece you are holding match the length of the edge you are putting the piece on?
    2. If you put down the shape this way, is there going to be room in the shaded region to place any other shape? Is there any other shape you see that can fit into the given space?
    3. Do you think the corner of the triangle you are holding matches the shape of the corner [angle] of the shaded area?
    4. Have you tried rotating/flipping the shape?
    5. It seems that you have tried everything with this particular shape. How about trying a different shape?
- The amount of help children need with the tangrams varies vastly from child to child. Some children can solve these problems fairly independently, while others may need you to put down one or more pieces in order to complete

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the puzzle. You may need to tell the child which pieces to use. Hints, no matter how abundant, are fine as long as the child physically completes the puzzle himself.

- Some children get very emotional working through the tangrams. The emotions range from elation at having completed the puzzle, to despair, to declarations that the puzzle is impossible (!). It will be very helpful for you to work through those emotions with the child.

Activities in this workbook and the Common Core State Standards (CCSS)

Common Core Progressions

The Progressions for the Common Core Math Standards are a series of documents each of which follows the development of a math topic throughout several grade levels. The Progressions allow a teacher to follow the development of, say, geometry, from grades K to 6. This gives the teacher a bird's eye view of where his/her students stand with respect to geometry, and what the geometry goals are in future grades.

At the time of this writing, the Progressions are in draft form. They are being written in part by the original authors of the CCSS, as well as other educators and mathematicians.

How this workbook relates to the goals of the Common Core Progressions

According to the Common Core Geometric Progressions:

> *By the end of Grade 5, competencies in shape composition and de-composition… should be highly developed … Students need to develop these competencies because they form a foundation for understanding multiplication, area, volume, and the coordinate plane.*

With this in mind, it would benefit students to work through these exercises, move on to "Shape Challenge", and work their way to "3-D Solid Understanding".

CCSS for Mathematical Content supported by activities in this workbook

1.G.A.2 *Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.*

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CCSS for Mathematical Practice supported by activities in this workbook

MP.1 ***Make sense of problems and persevere in solving them.***

Each problem consists of a goal (make a certain shape) and a set of constraints (the tiles students can use). Students are encouraged to try different combinations of shapes until they find one that solves the problem.

MP.3 ***Construct viable arguments and critique the reasoning of others.***

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

MP.8 ***Look for and express regularity in repeated reasoning.***

Students will create the more complex shapes by building upon simpler shapes they had built earlier.

Learning objectives of this workbook

a. Recognize ways to decompose a polygon into smaller polygons.
b. Help students learn how certain polygons can be made with Geometiles™ so that they can use them later to build 3-dimensional solids.
c. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
d. Learn how to collaborate with one another.

Arguably, point c. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

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Recommended classroom plan

The exercises are designed so that 4 groups of students simultaneously work with a set of 96 tiles. The set is be split up into 4 smaller sets: two copies of SET A (21 pieces per single set) and two copies of SET B (27 pieces per single set). Have two groups of students work with SET A, each with a different color combinations, and another two groups work with SET B, each with a different color combinations. Then switch the sets so that the SET A students will get a chance to work with SET B and vice versa. It might be good to separate tiles into sets and put the sets in clear plastic bags before the lesson starts

This picture tells you exactly how many of each tile to give to each group. The two versions of SET A differ only in colors. So do the two types of SET B.

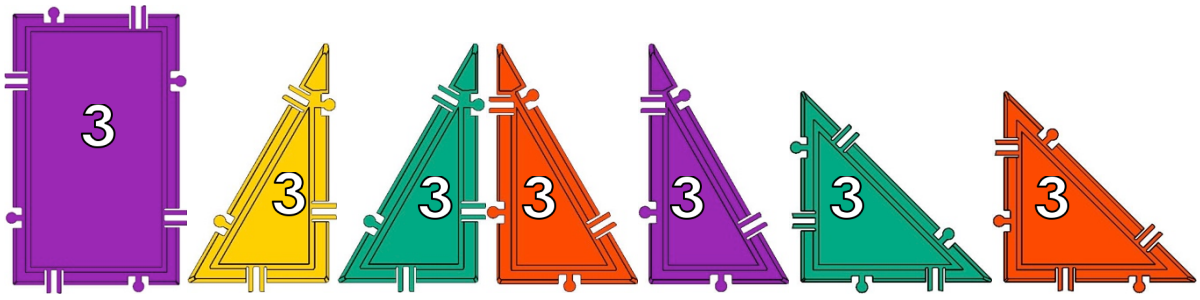
Each group can be made of 2-3 students. So the set of 96 tiles is designed to work for 8 to 12 students.

Level 1 problems are solved with 2 tiles.

Level 2 problems are solved with 3 or 4 tiles.

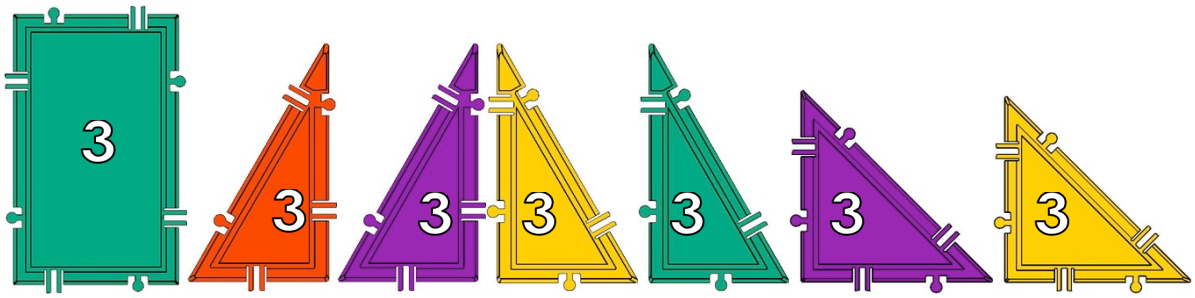
Level 3 problems are solved with 5 tiles or more tiles.

SET A, color combination 1



21 tiles

SET A, color combination 2



21 tiles

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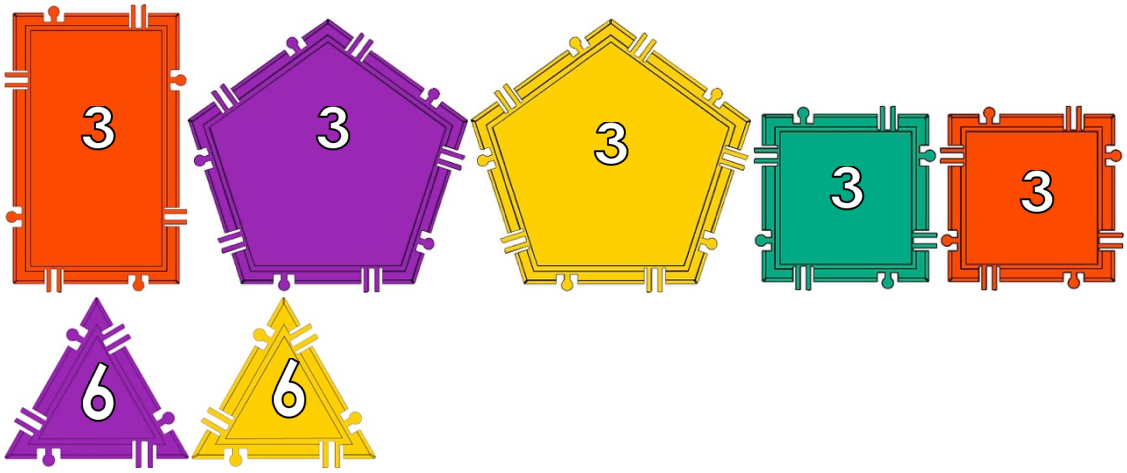
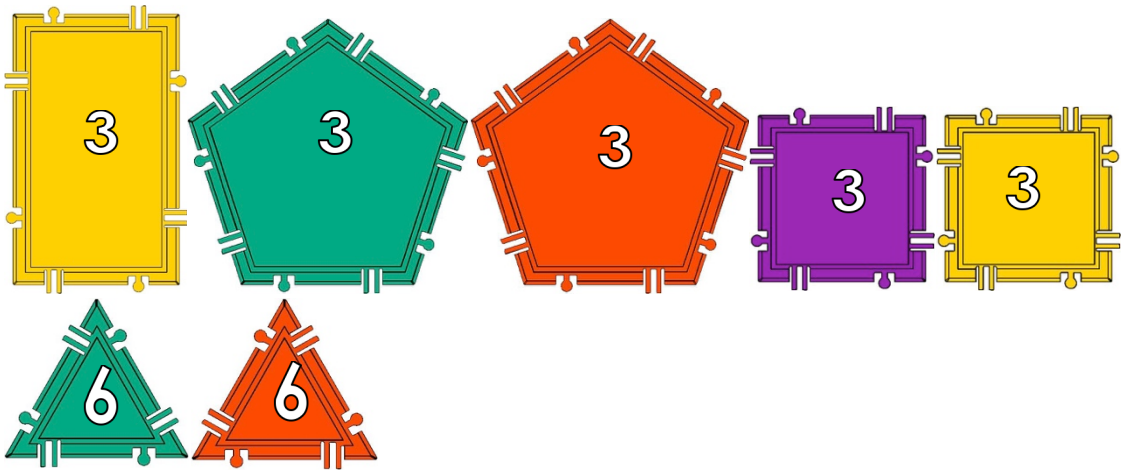
SET B, color combination 1



27 tiles

SET B, color combination 2



27 tiles

Total: 96 tiles

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Group A

Level 1

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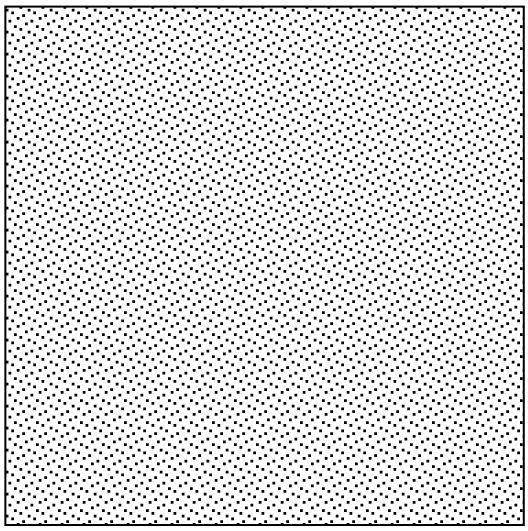

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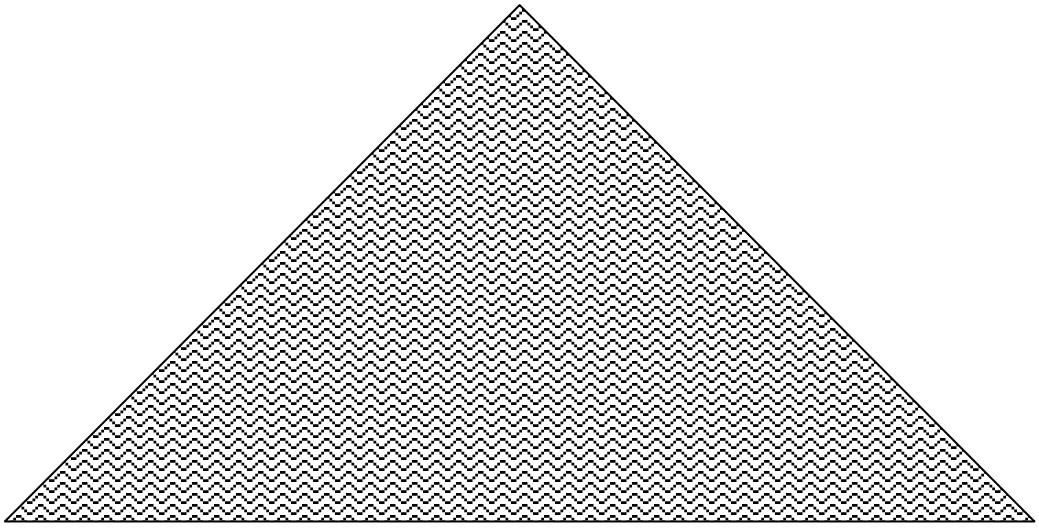

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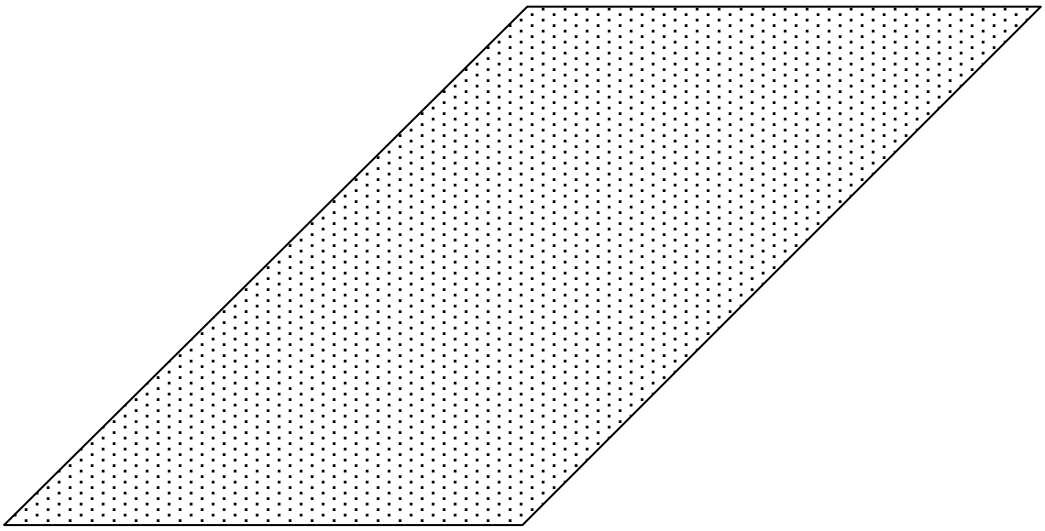

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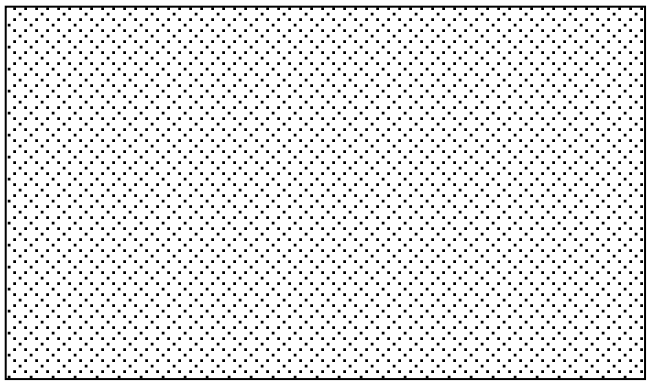

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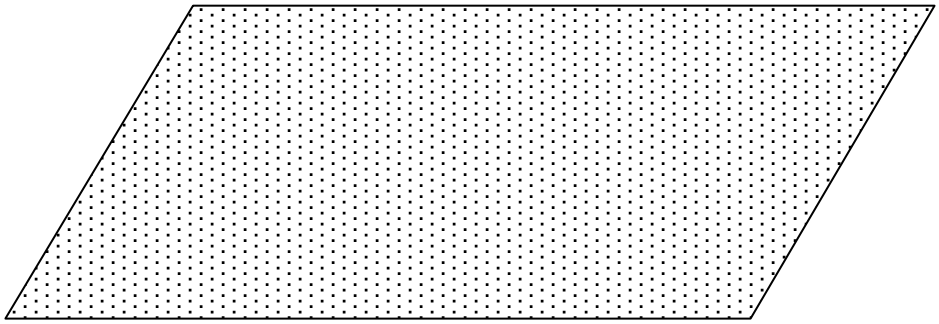

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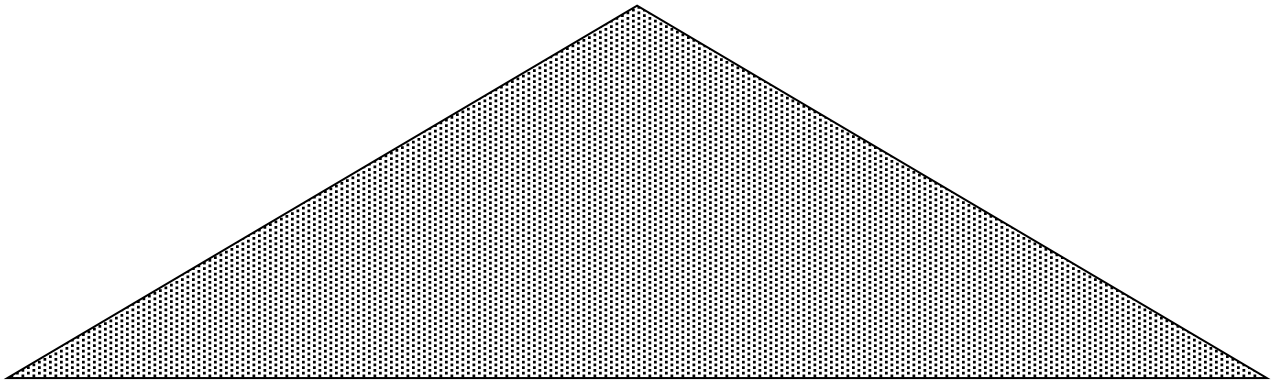

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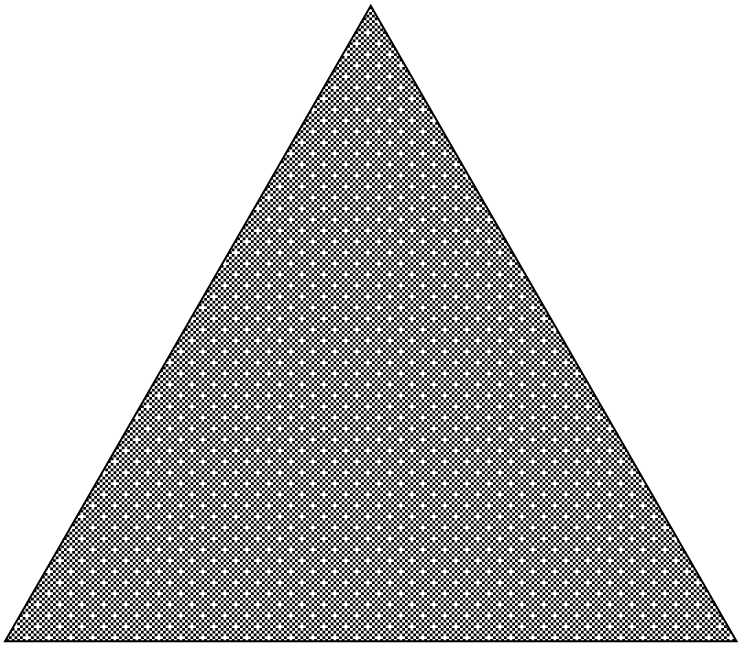

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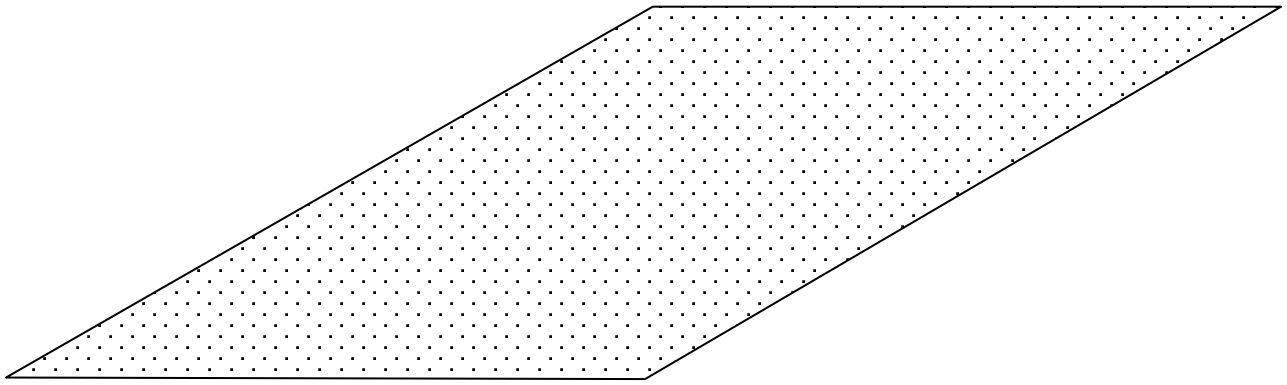

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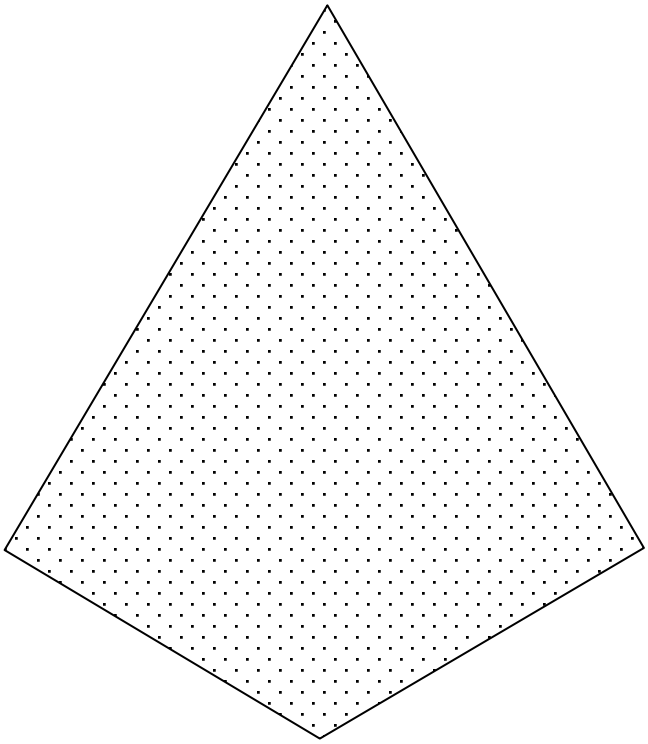

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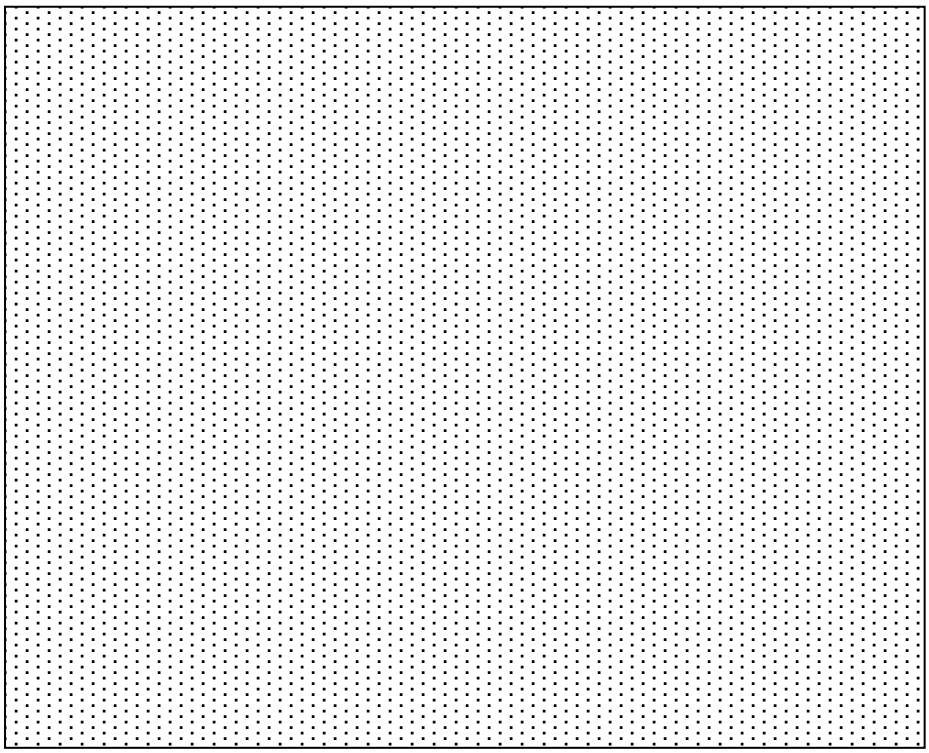

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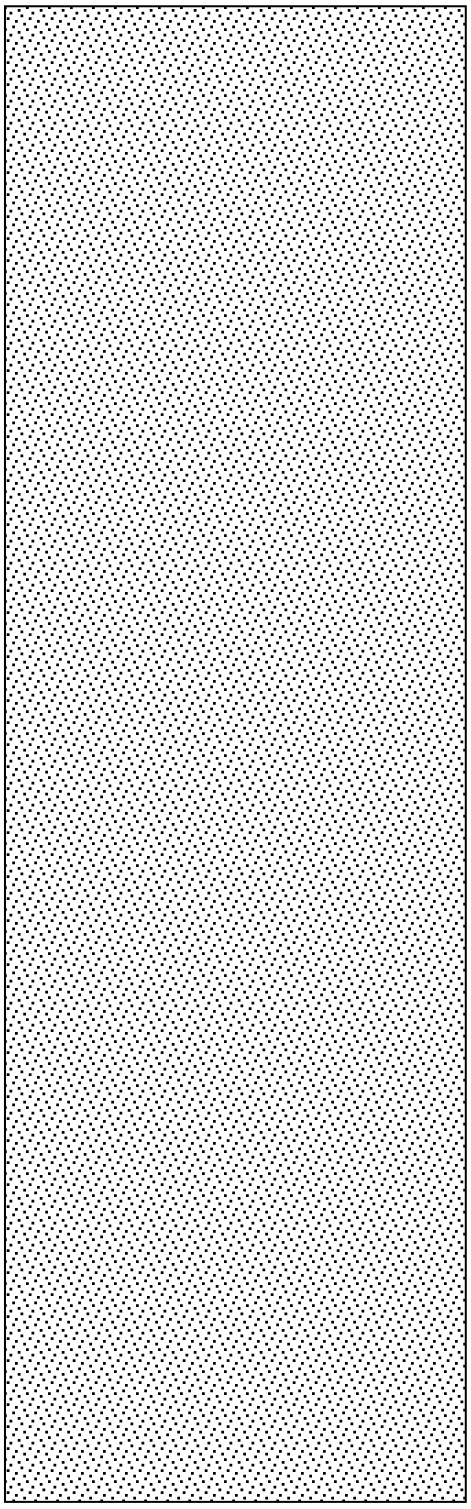

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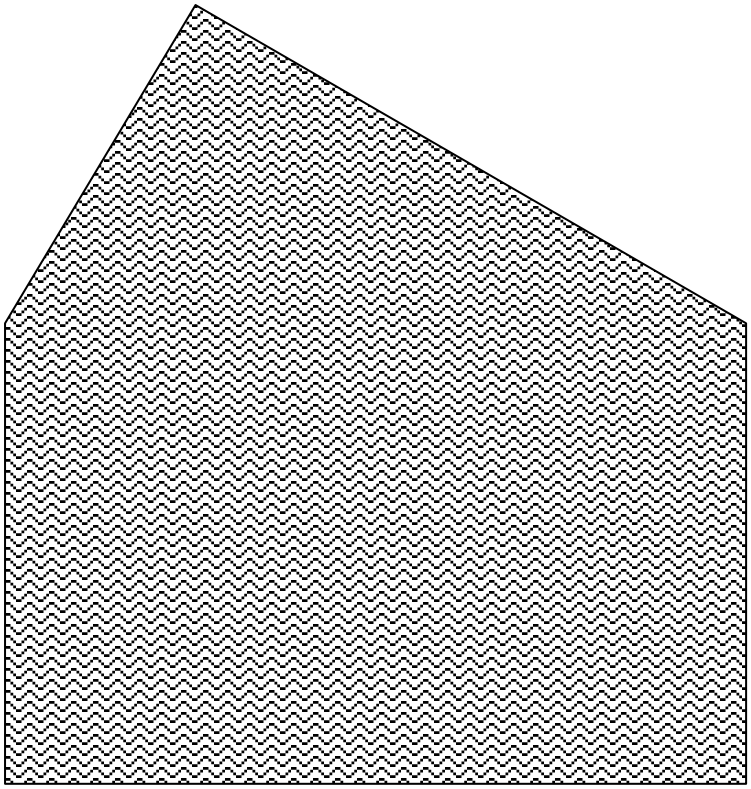

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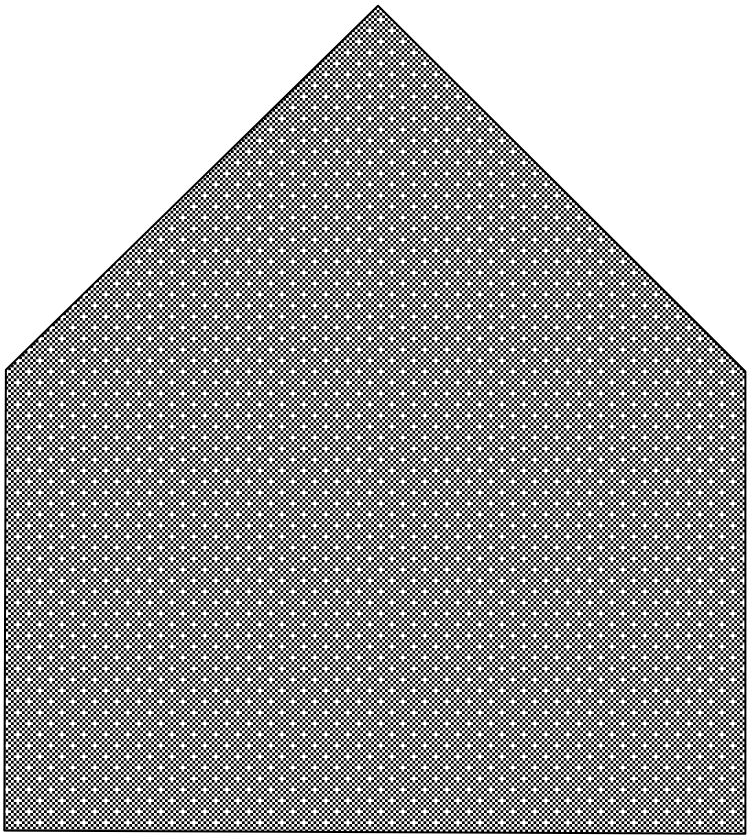

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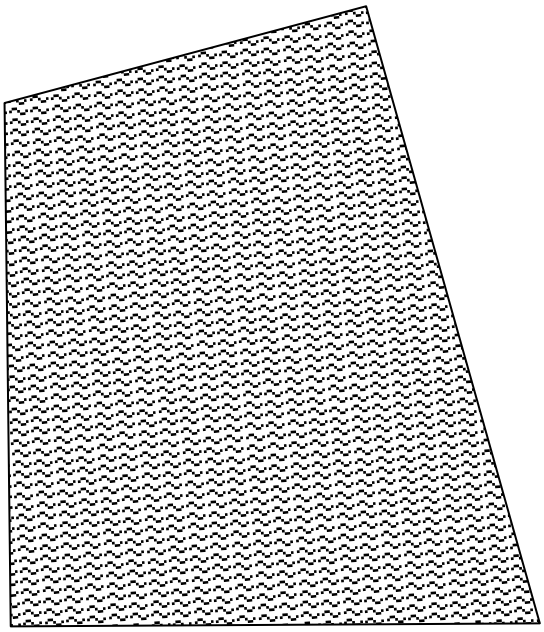

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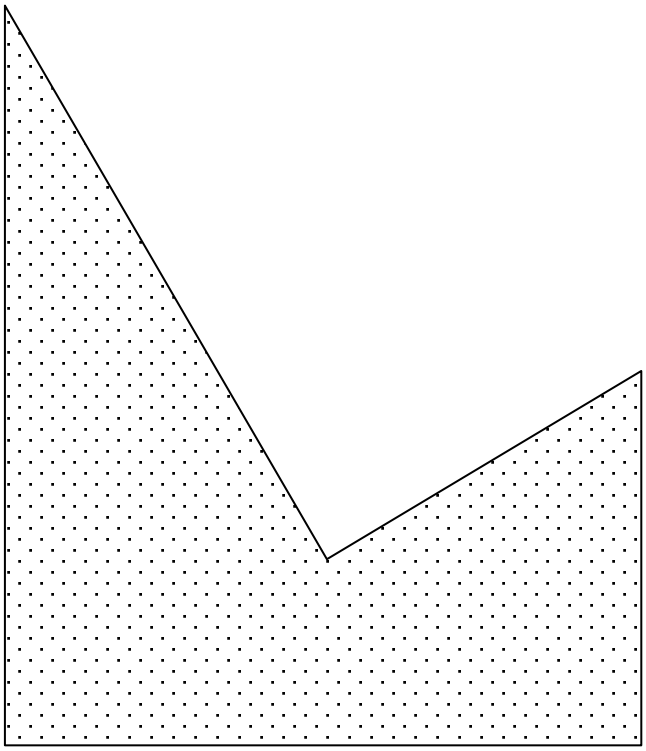

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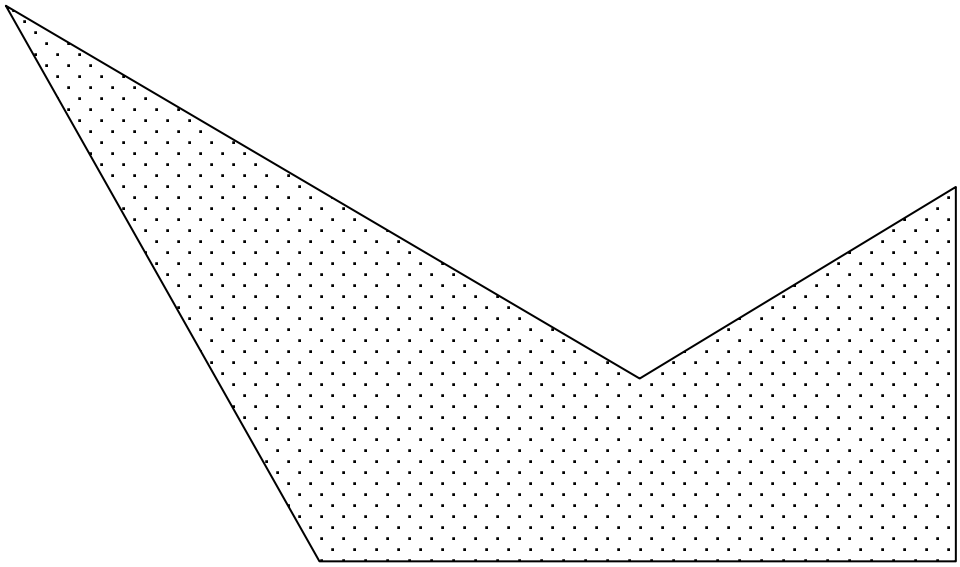

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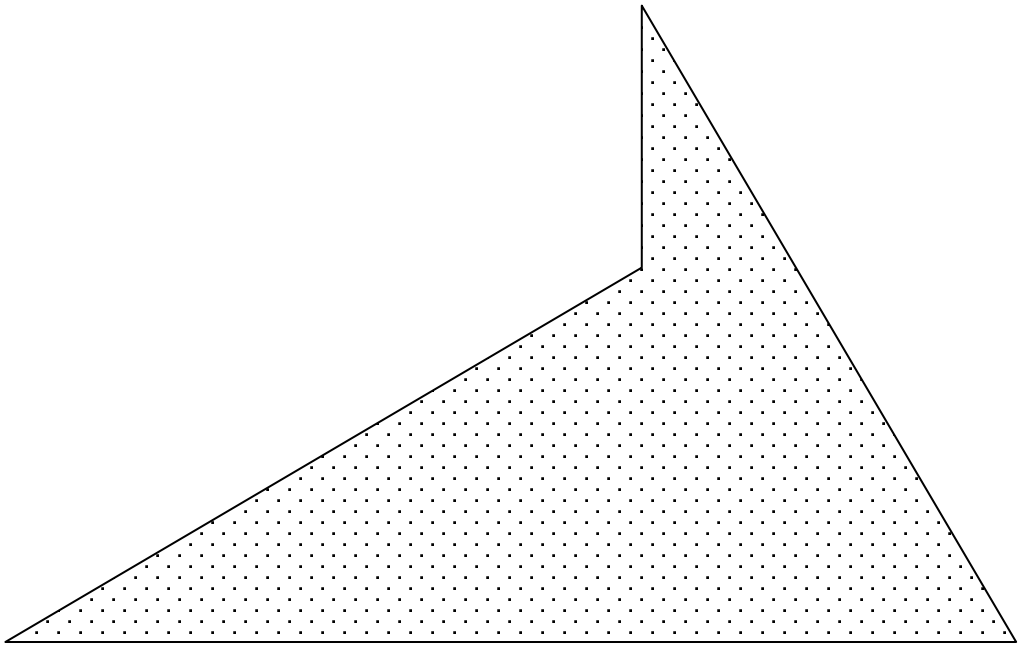

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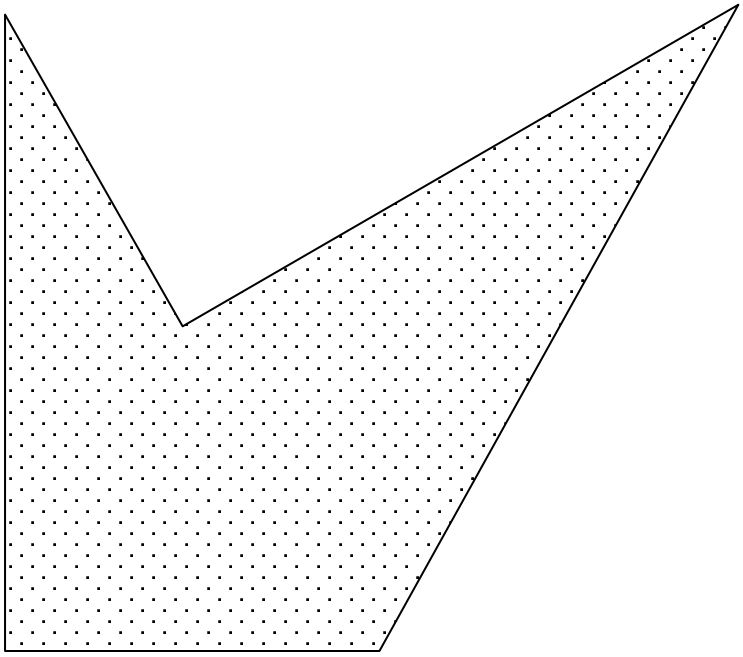

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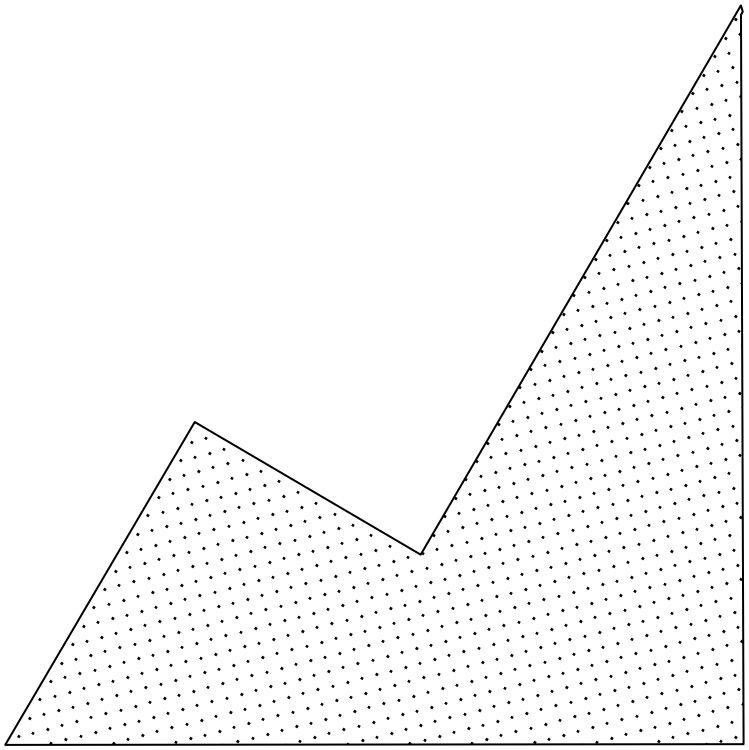

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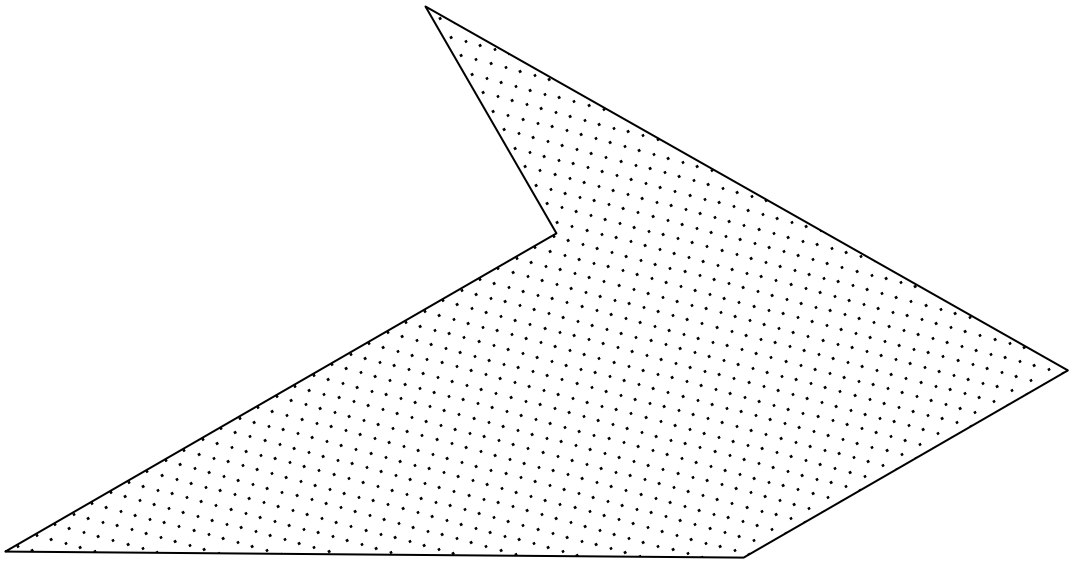

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Group A

Level 2

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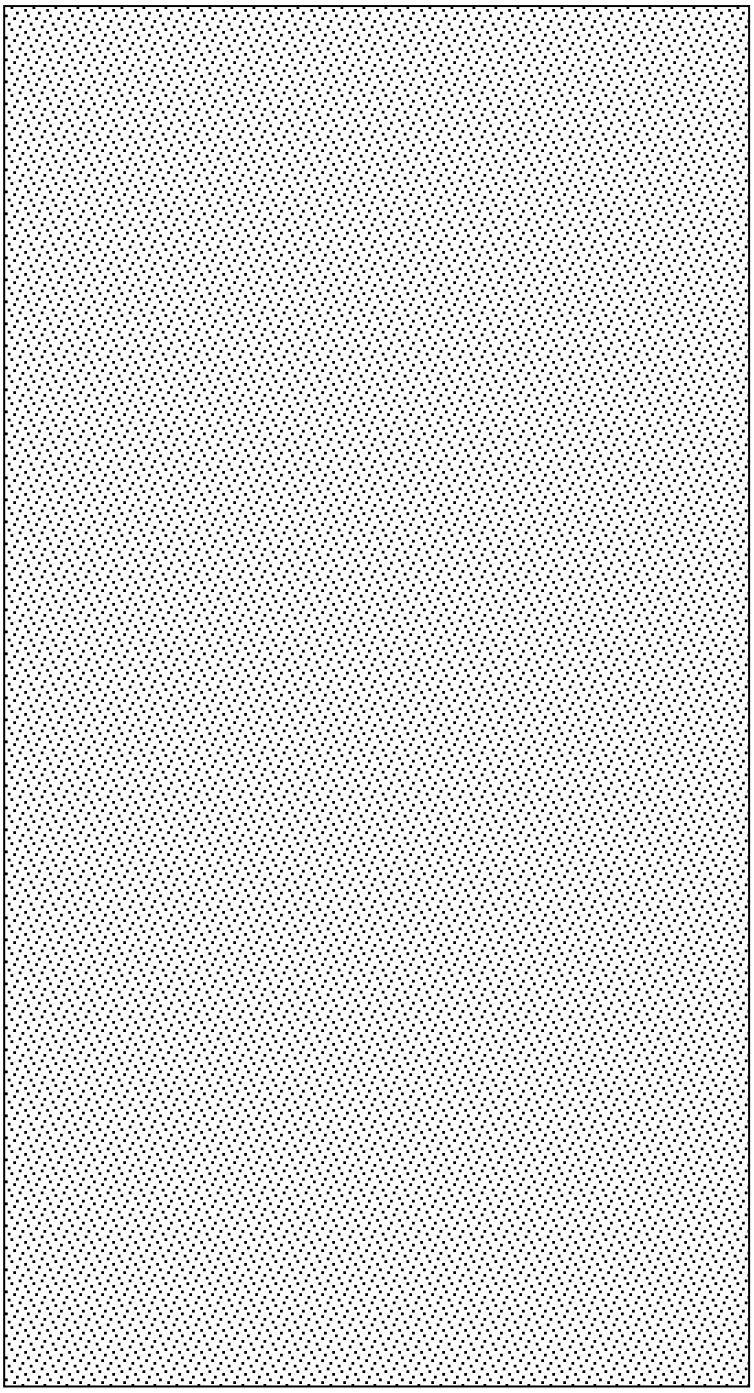

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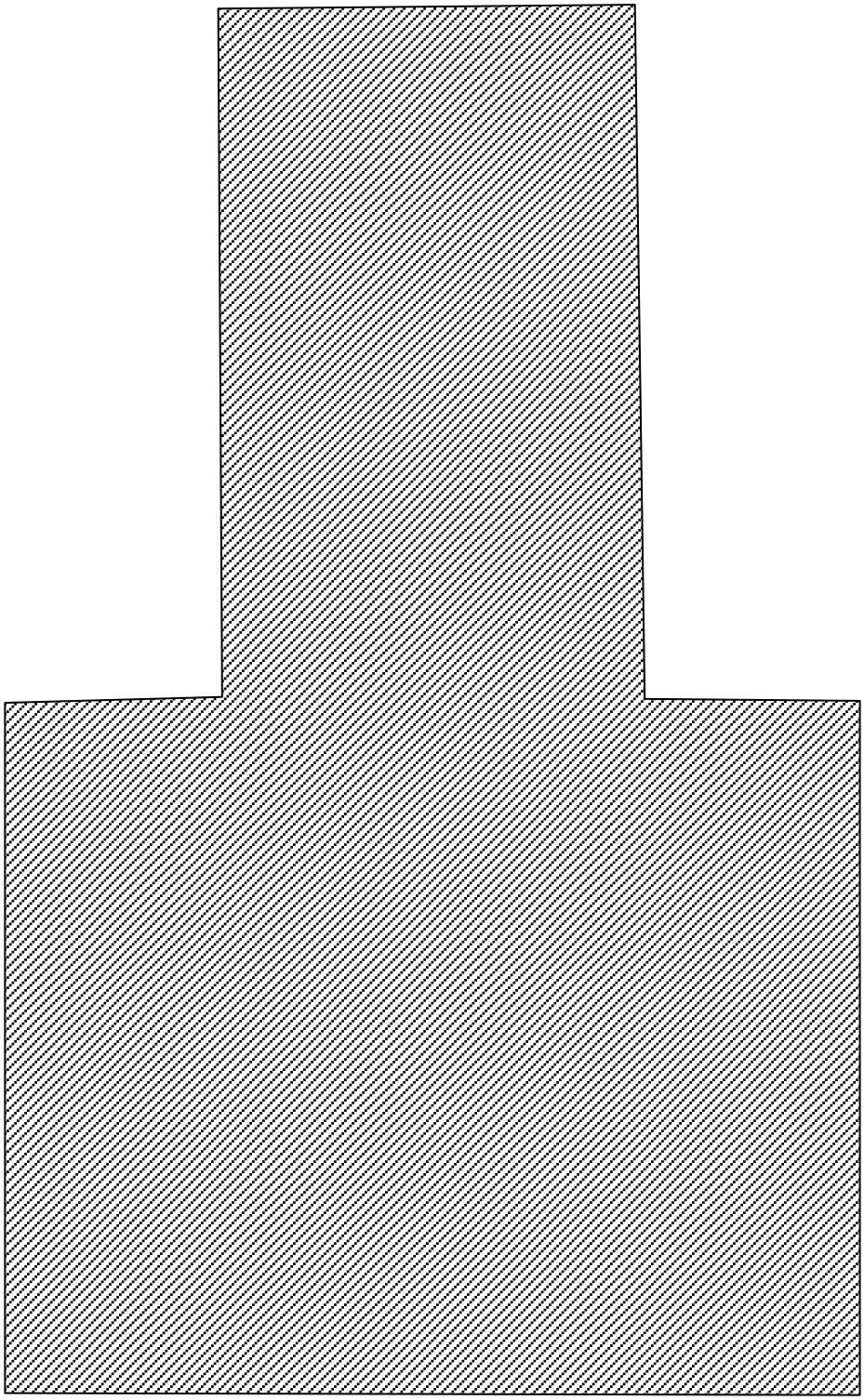

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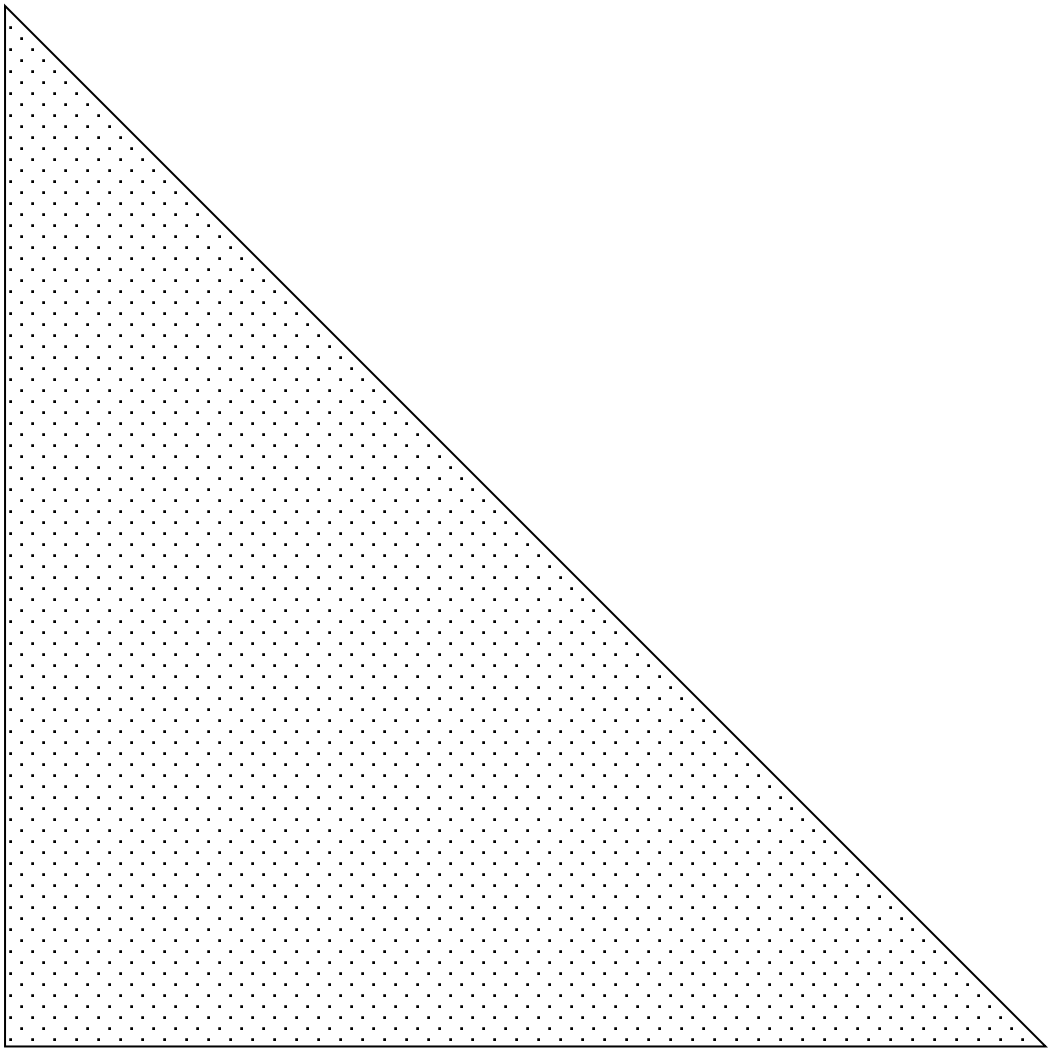

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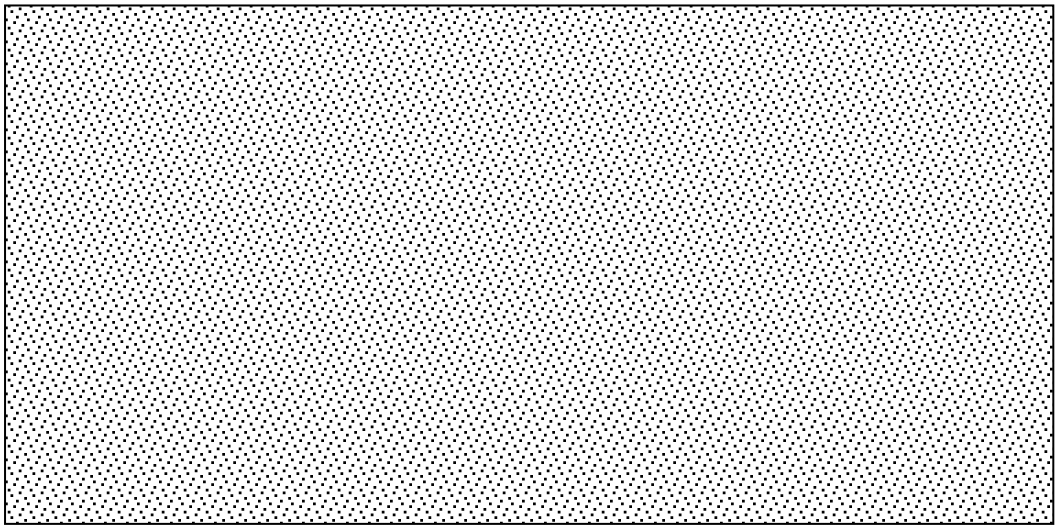

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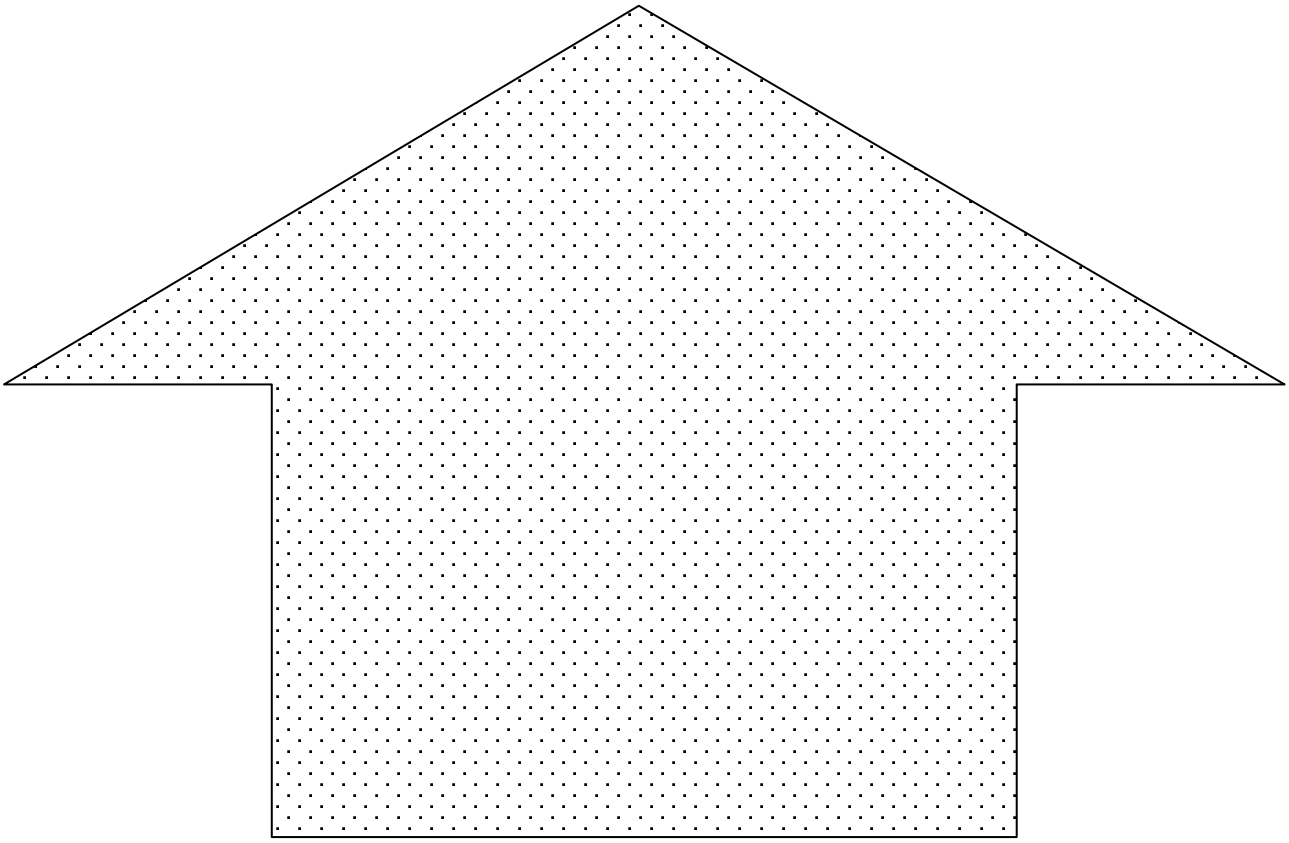

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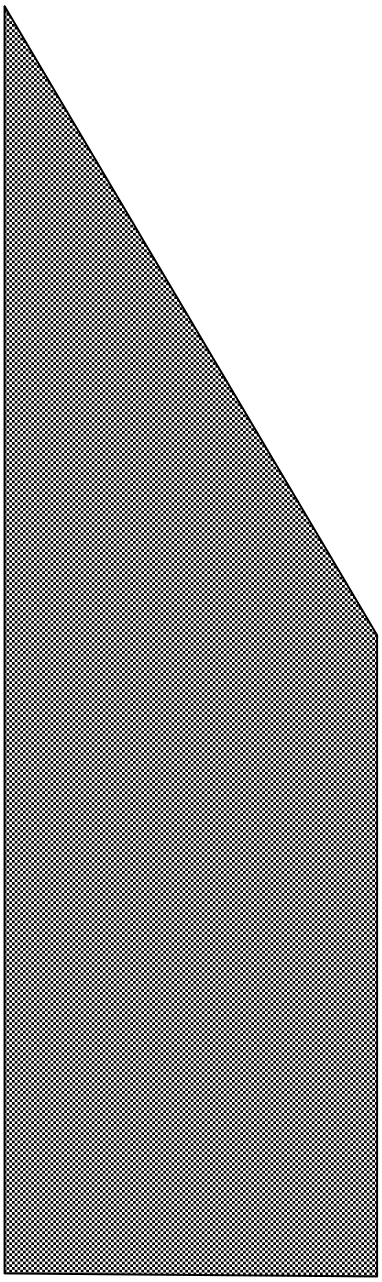

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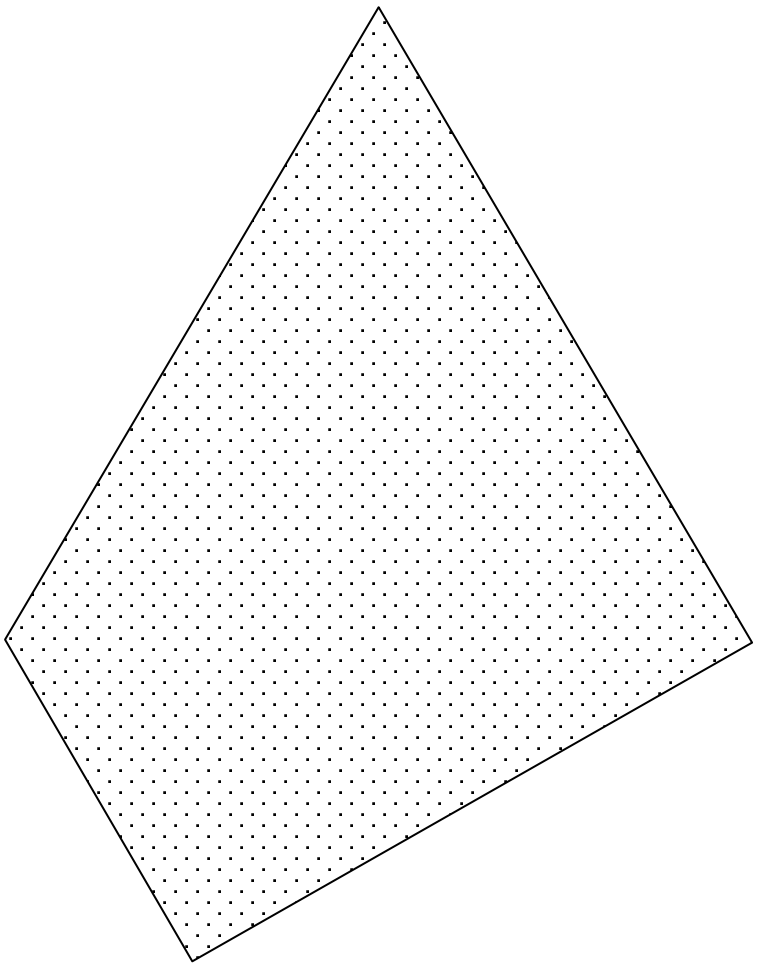

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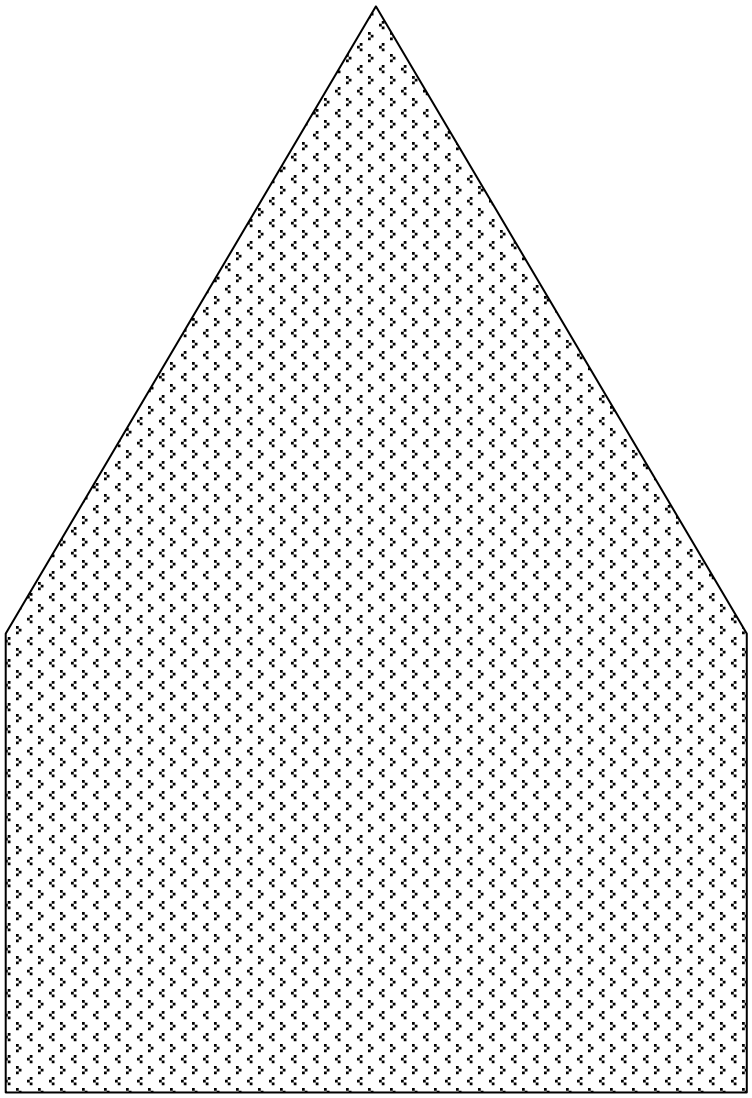

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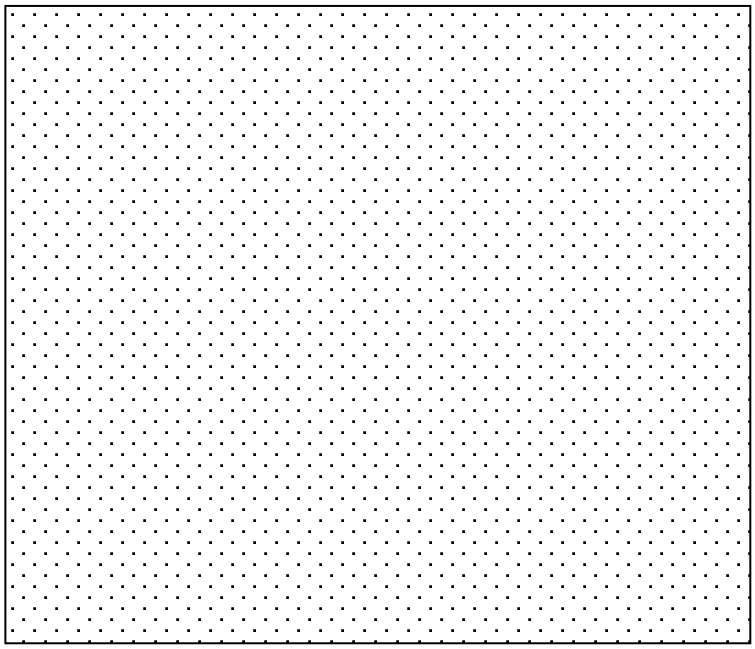

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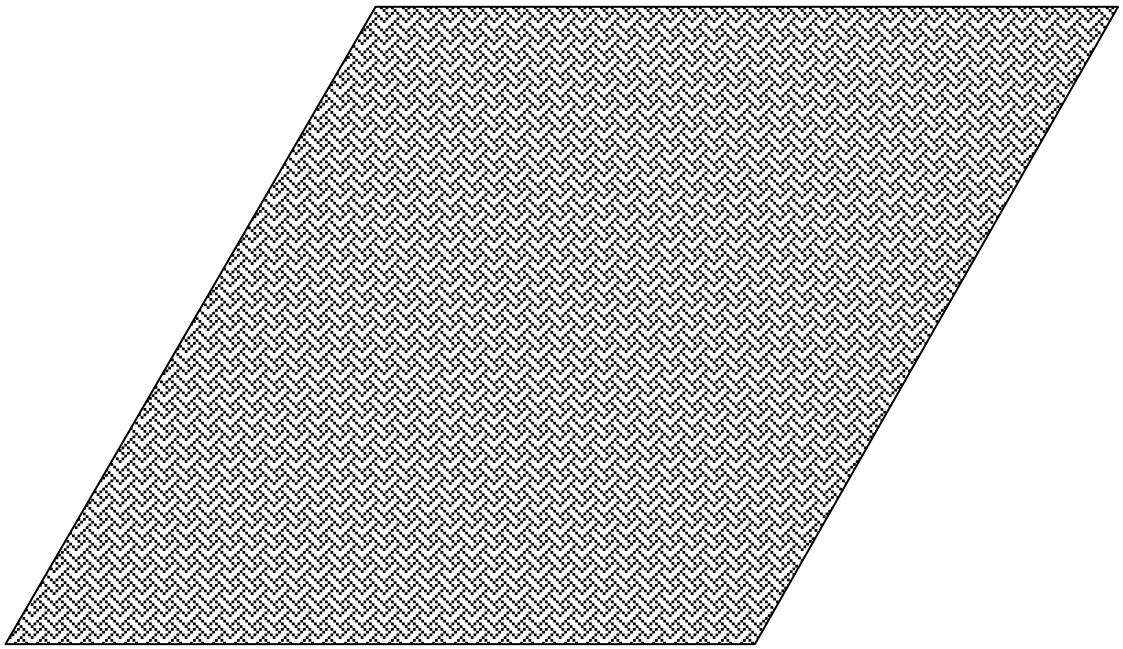

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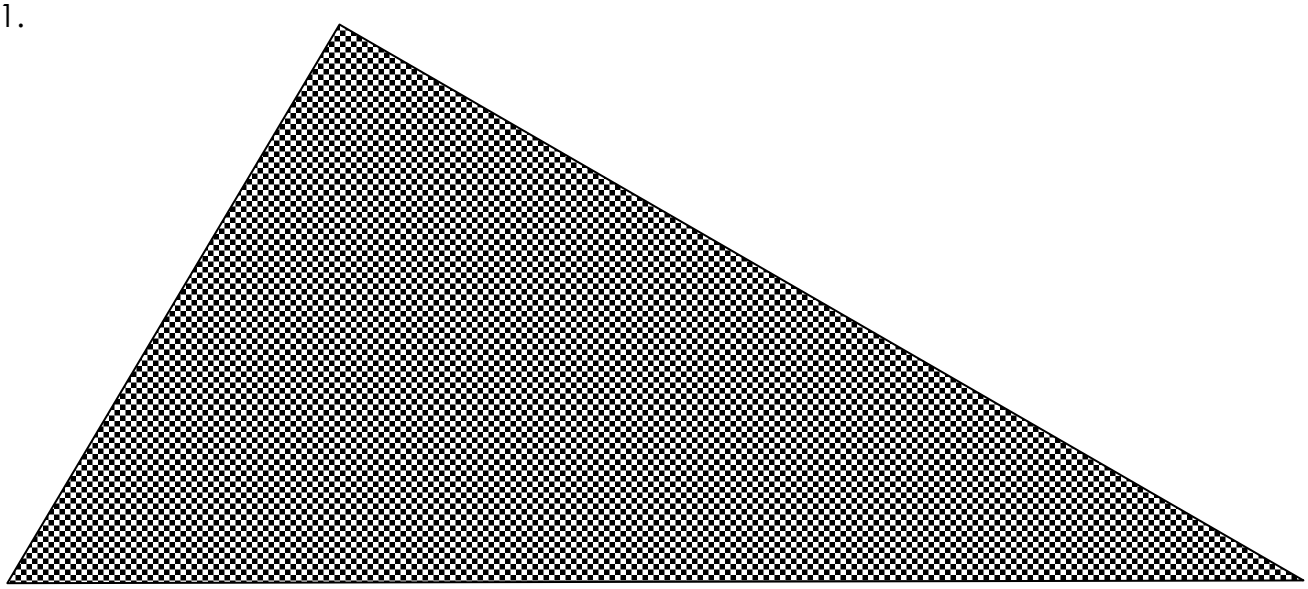

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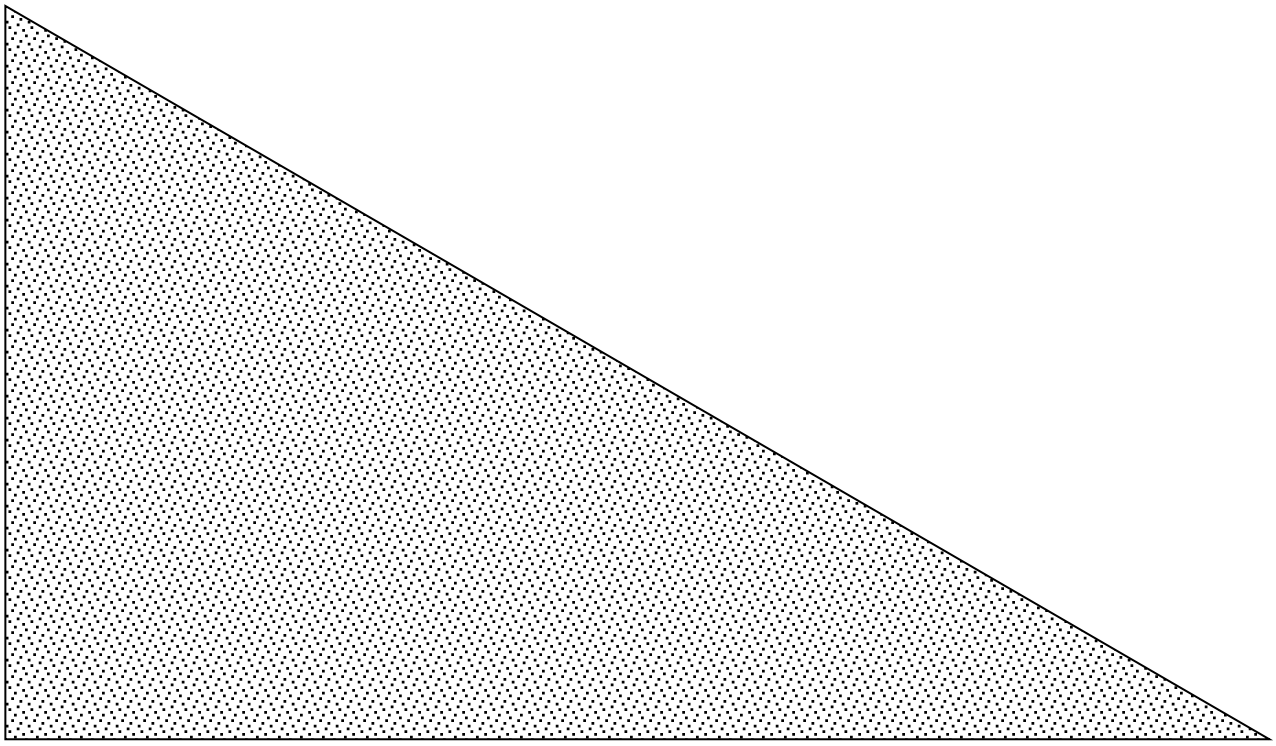

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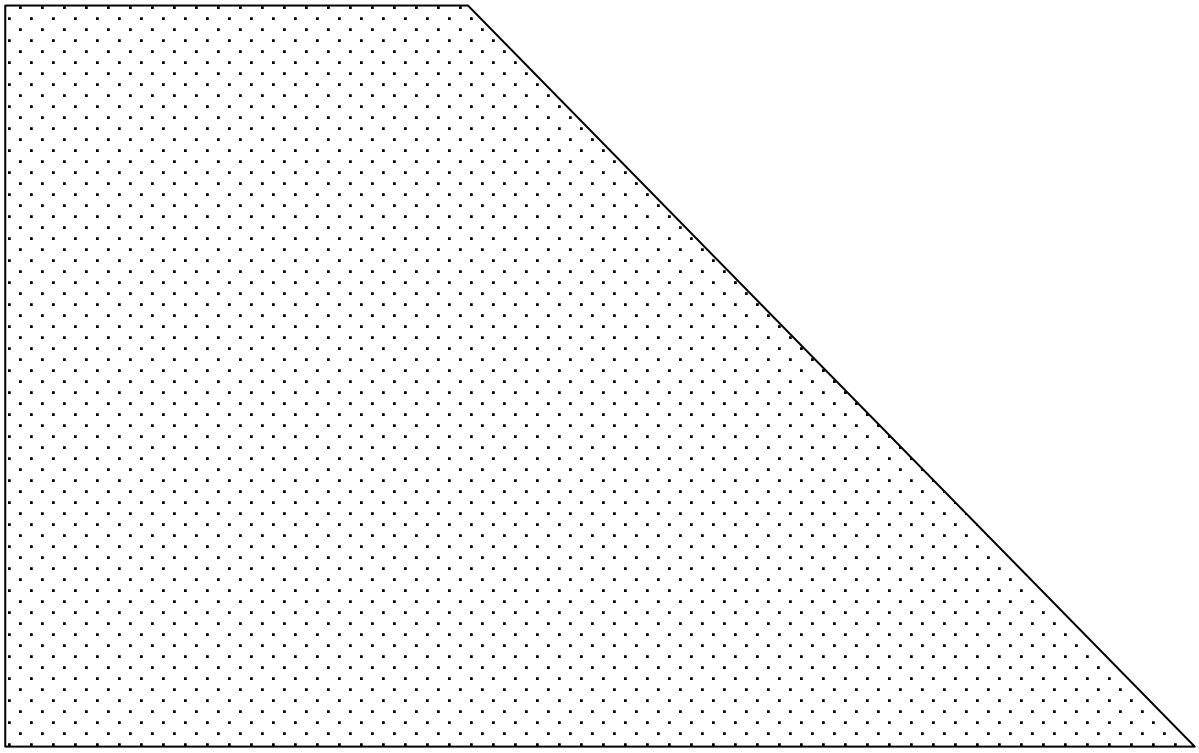

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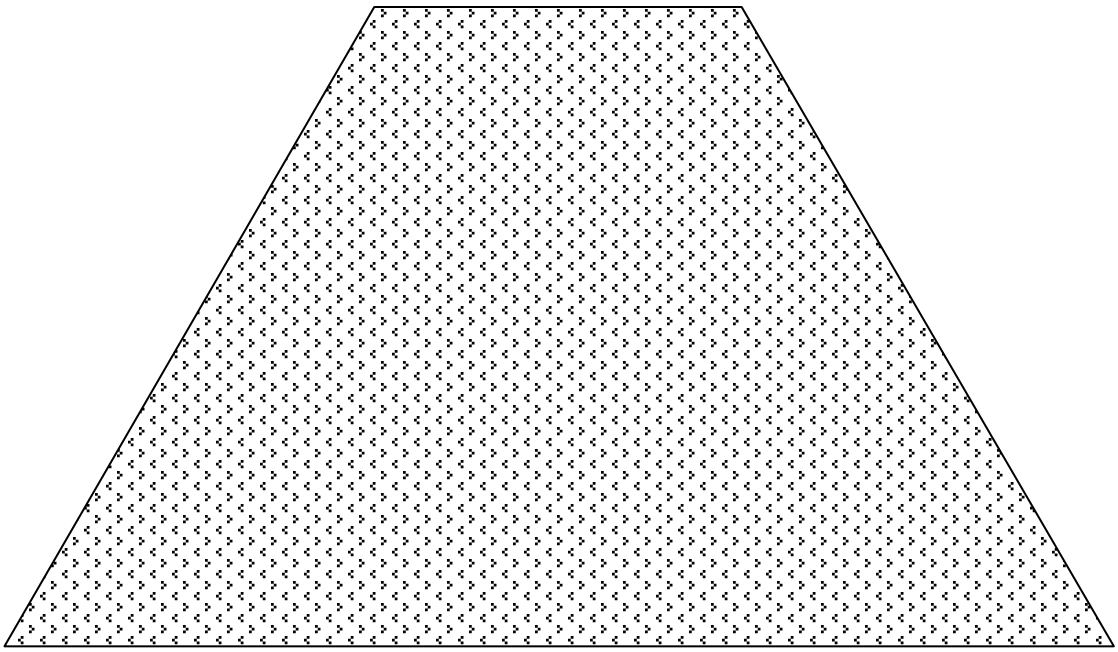

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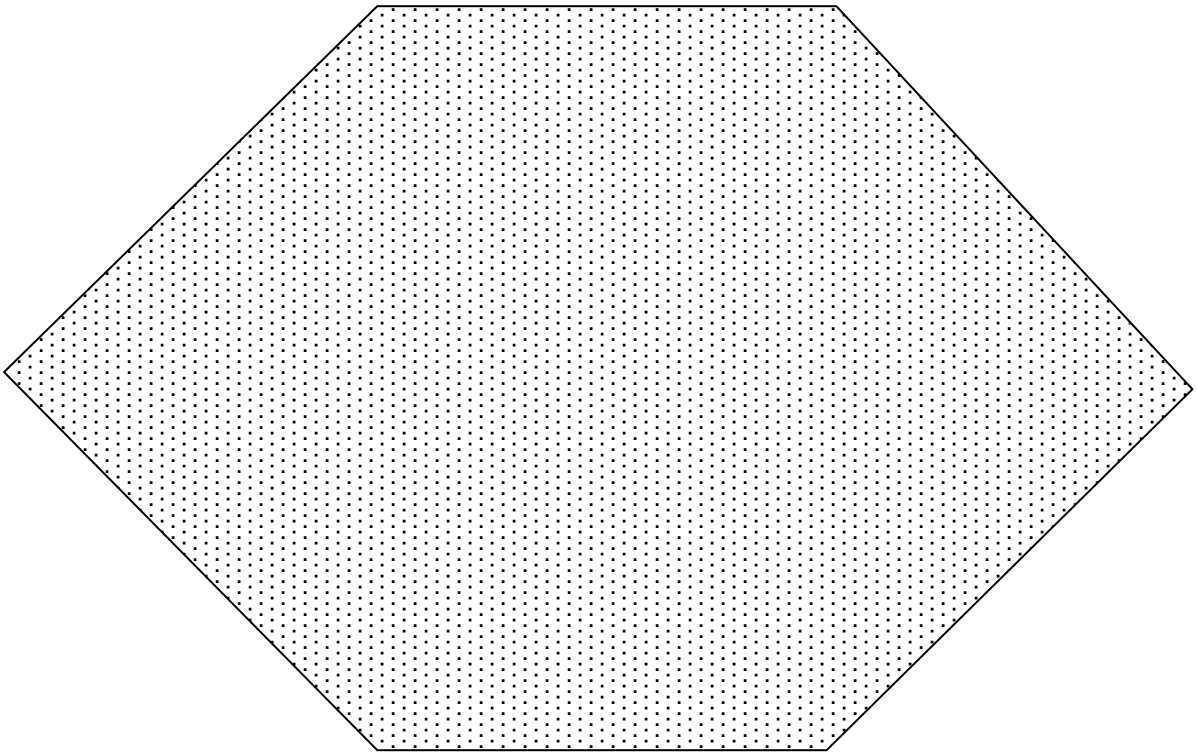

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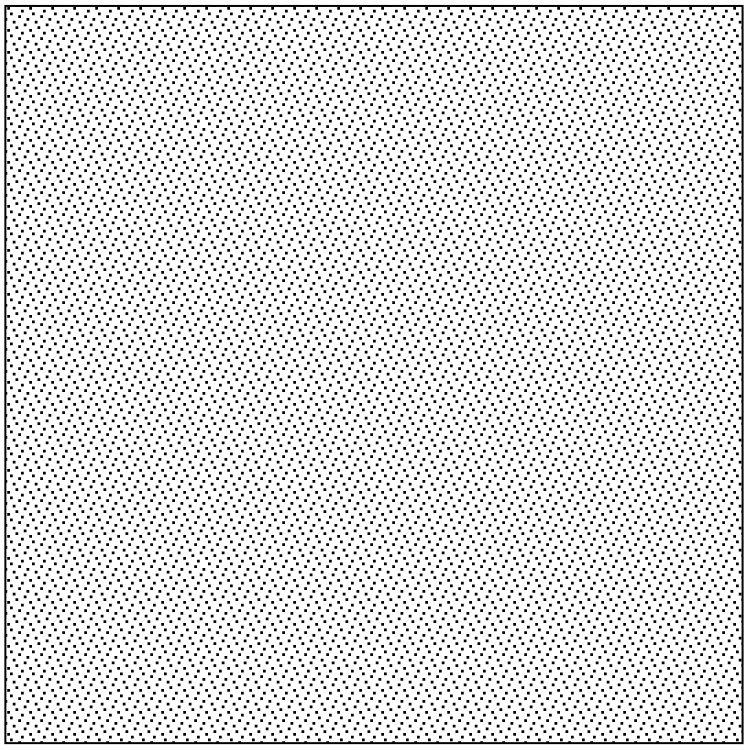

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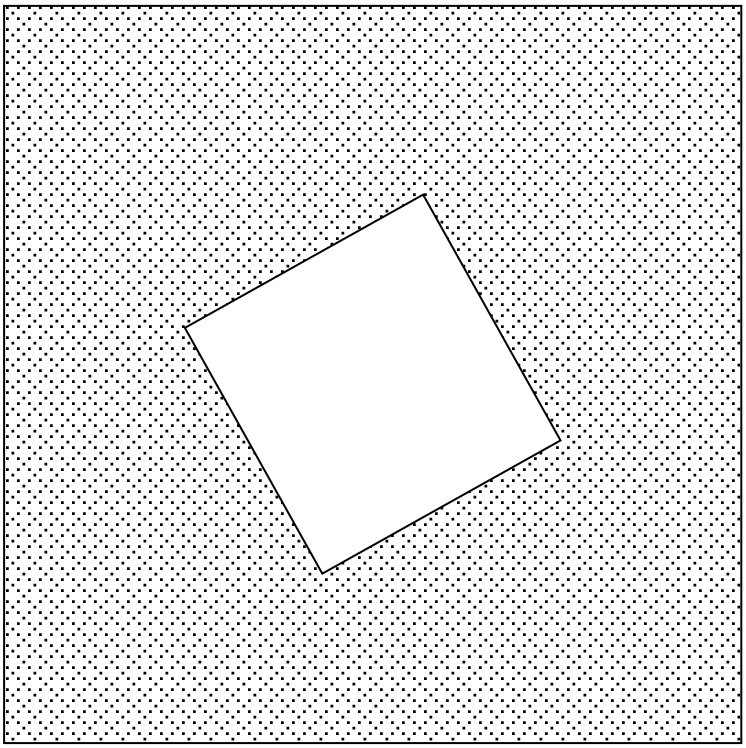

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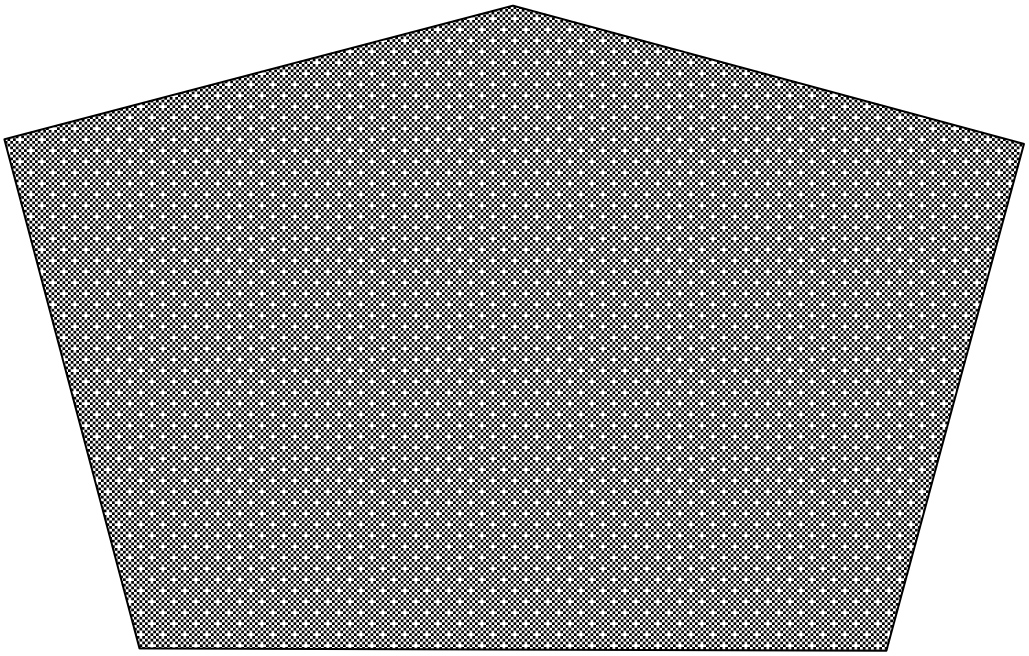

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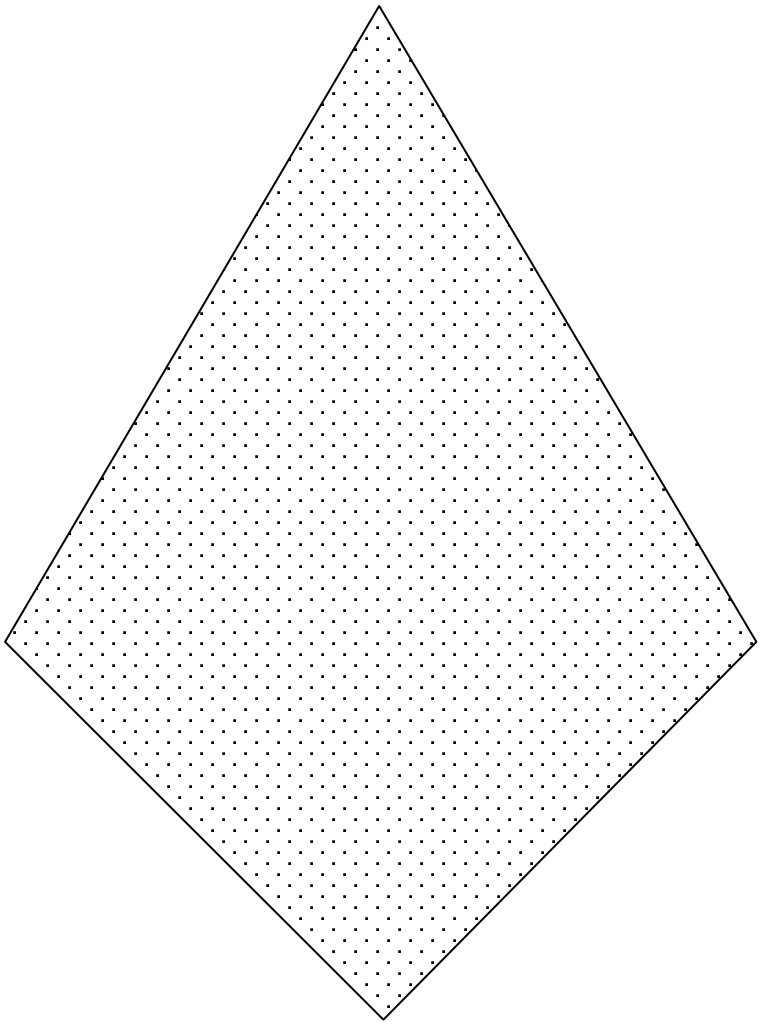

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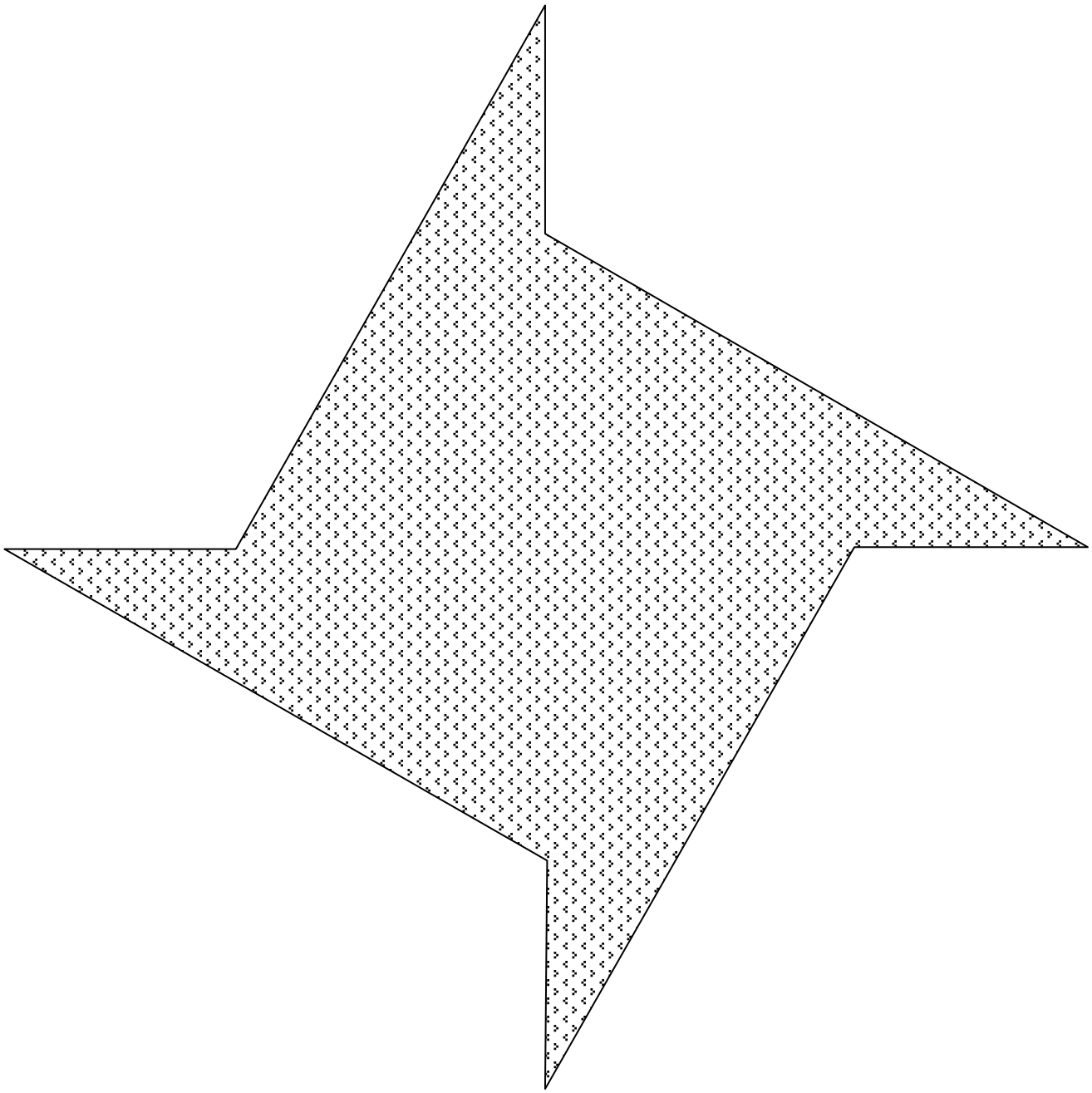

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Level 3

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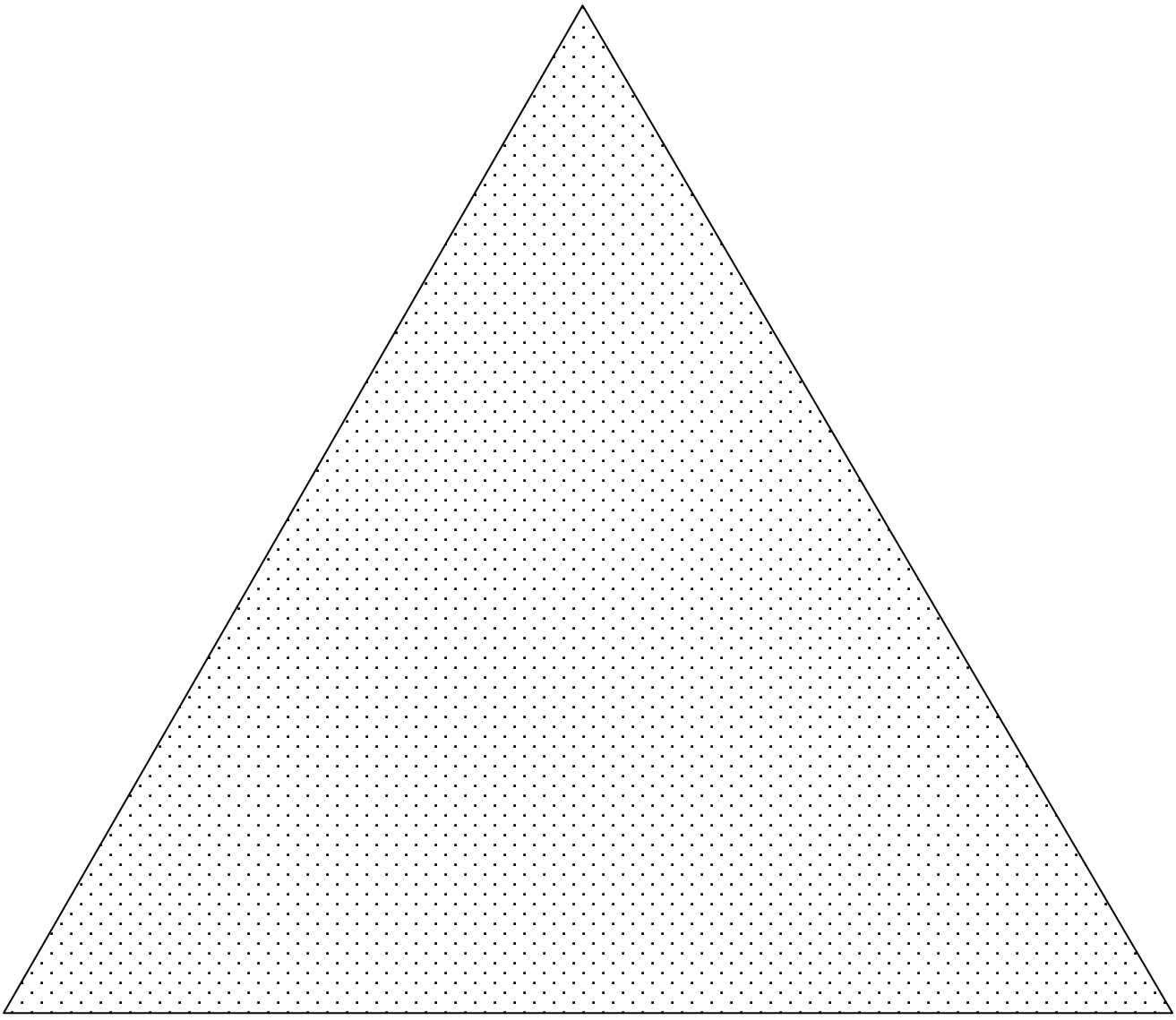

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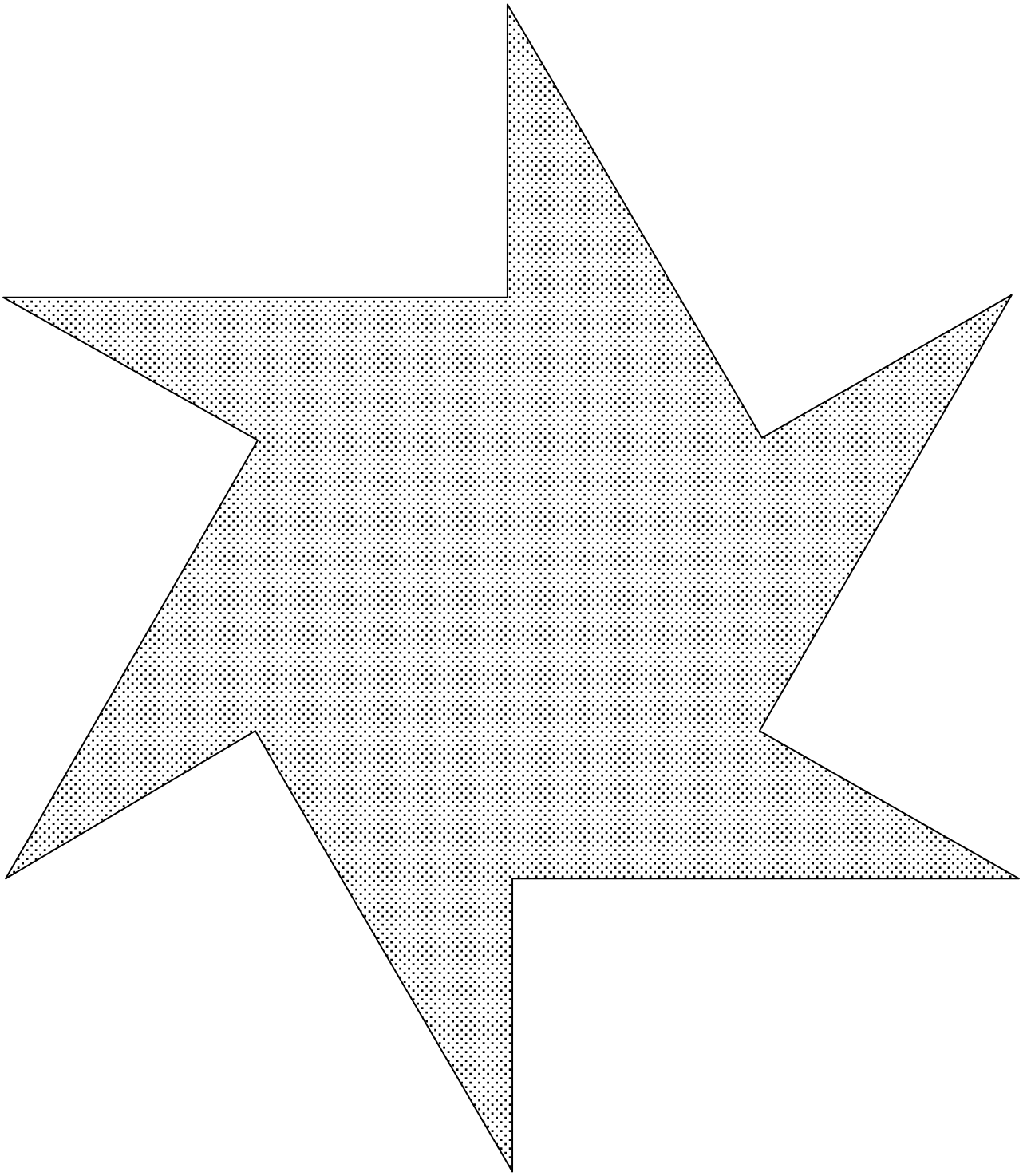

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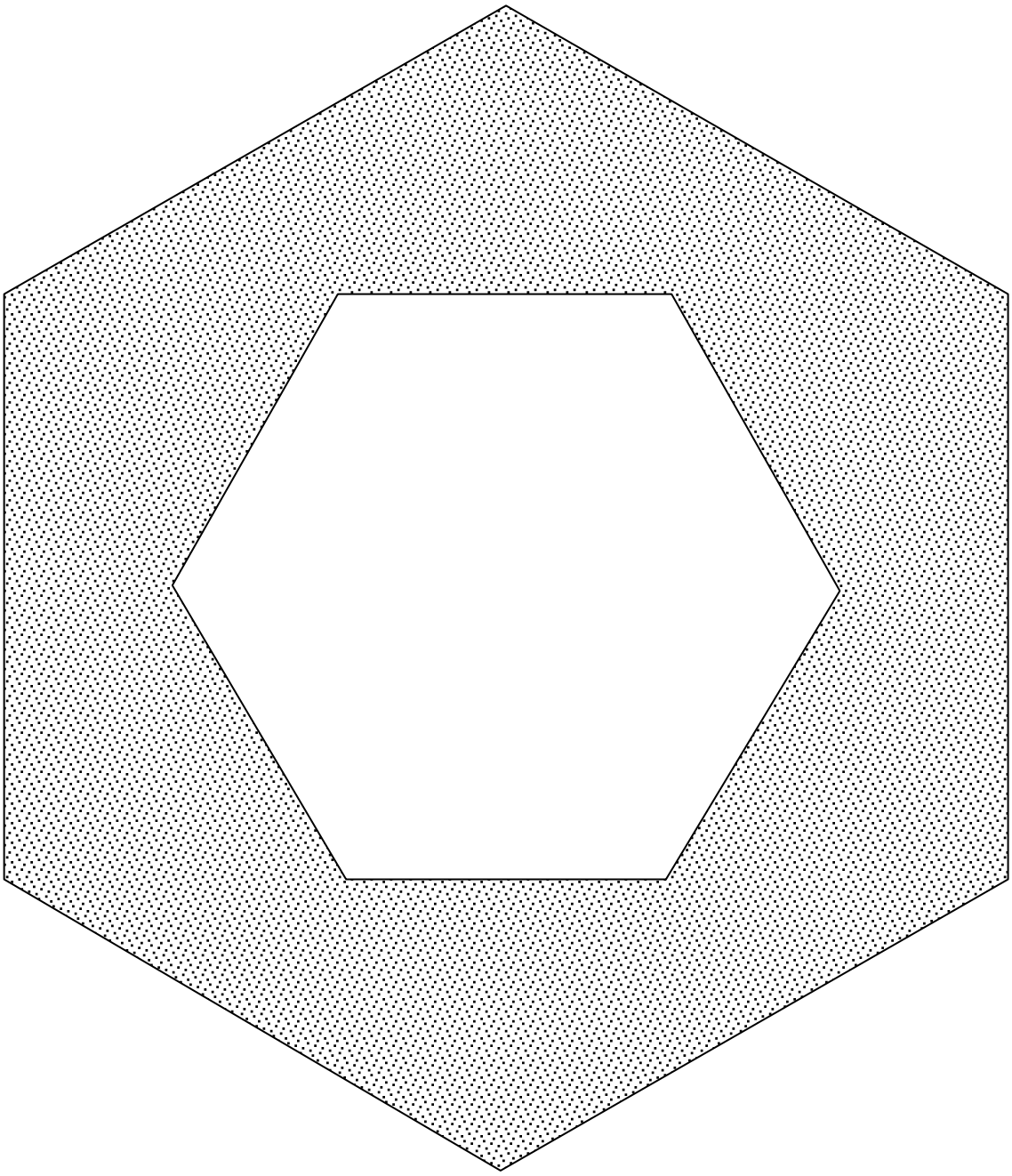

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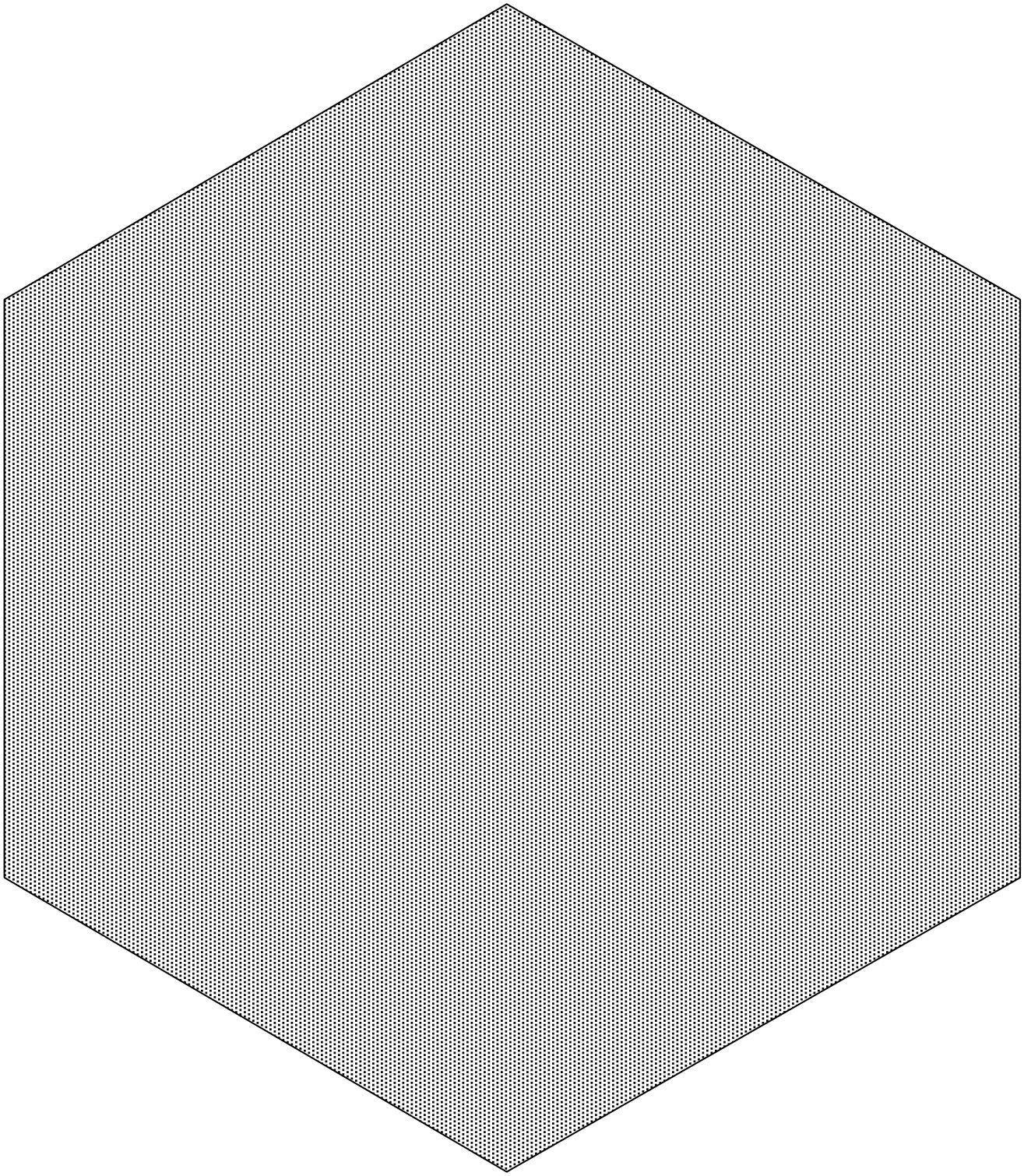

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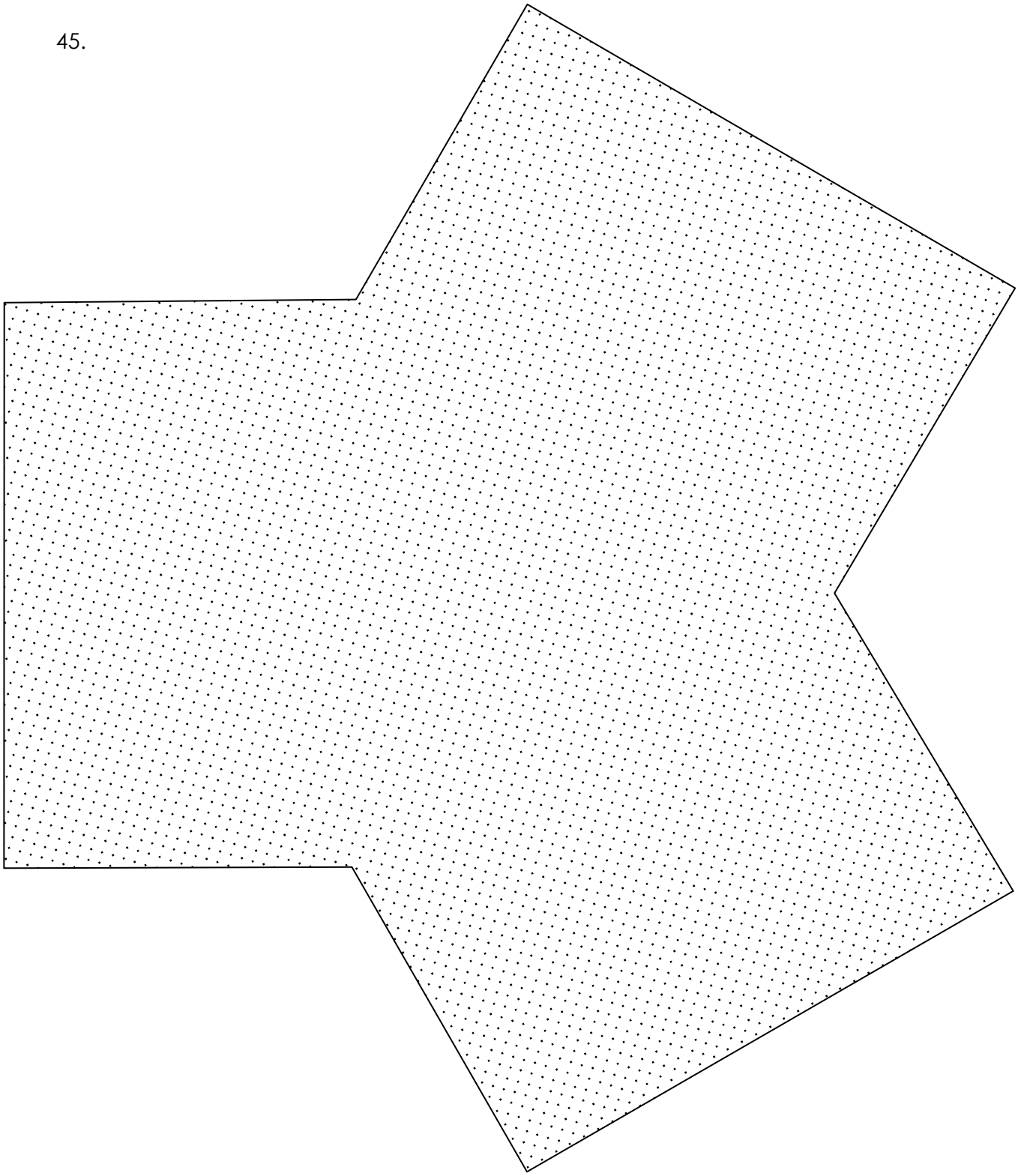

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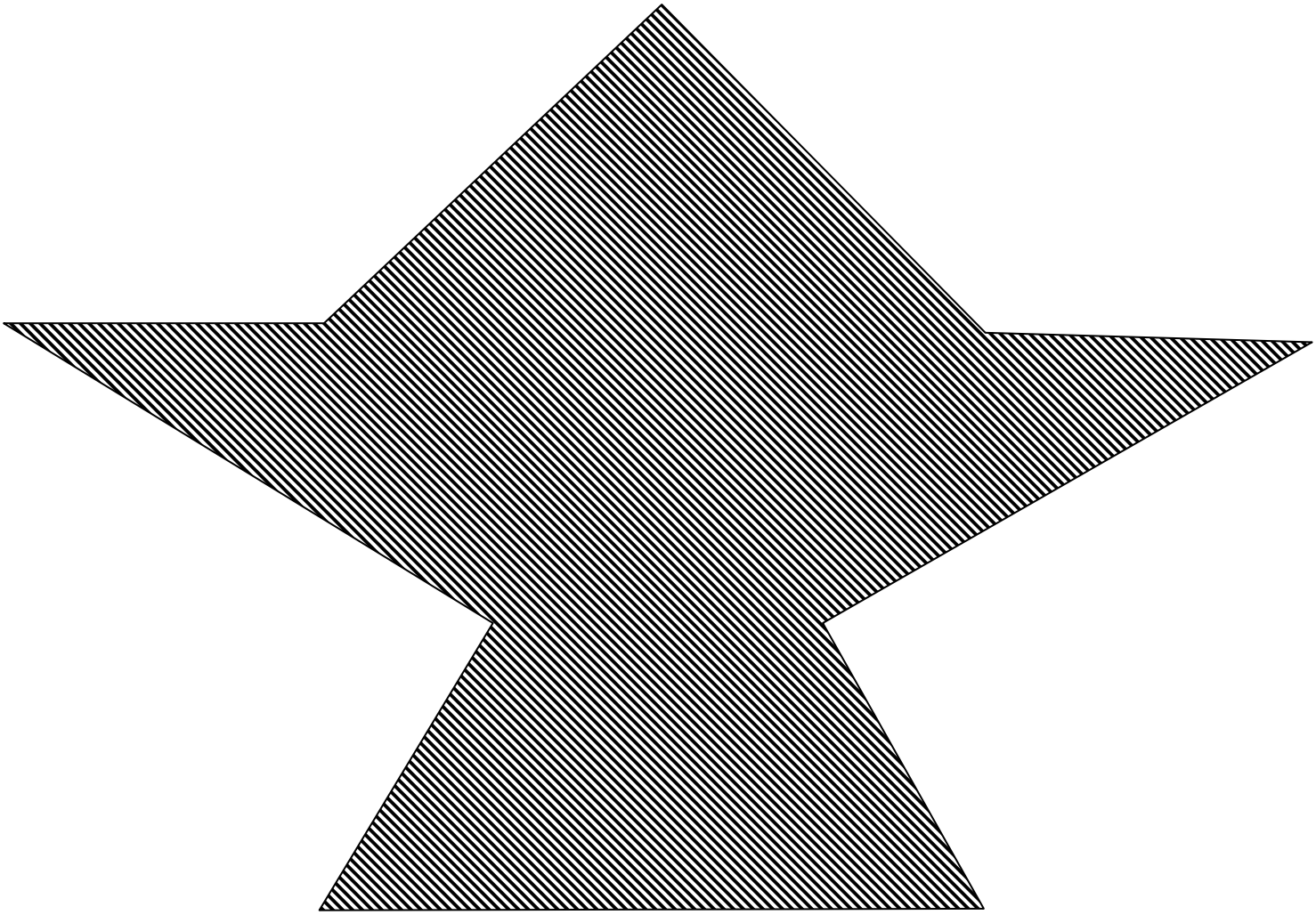

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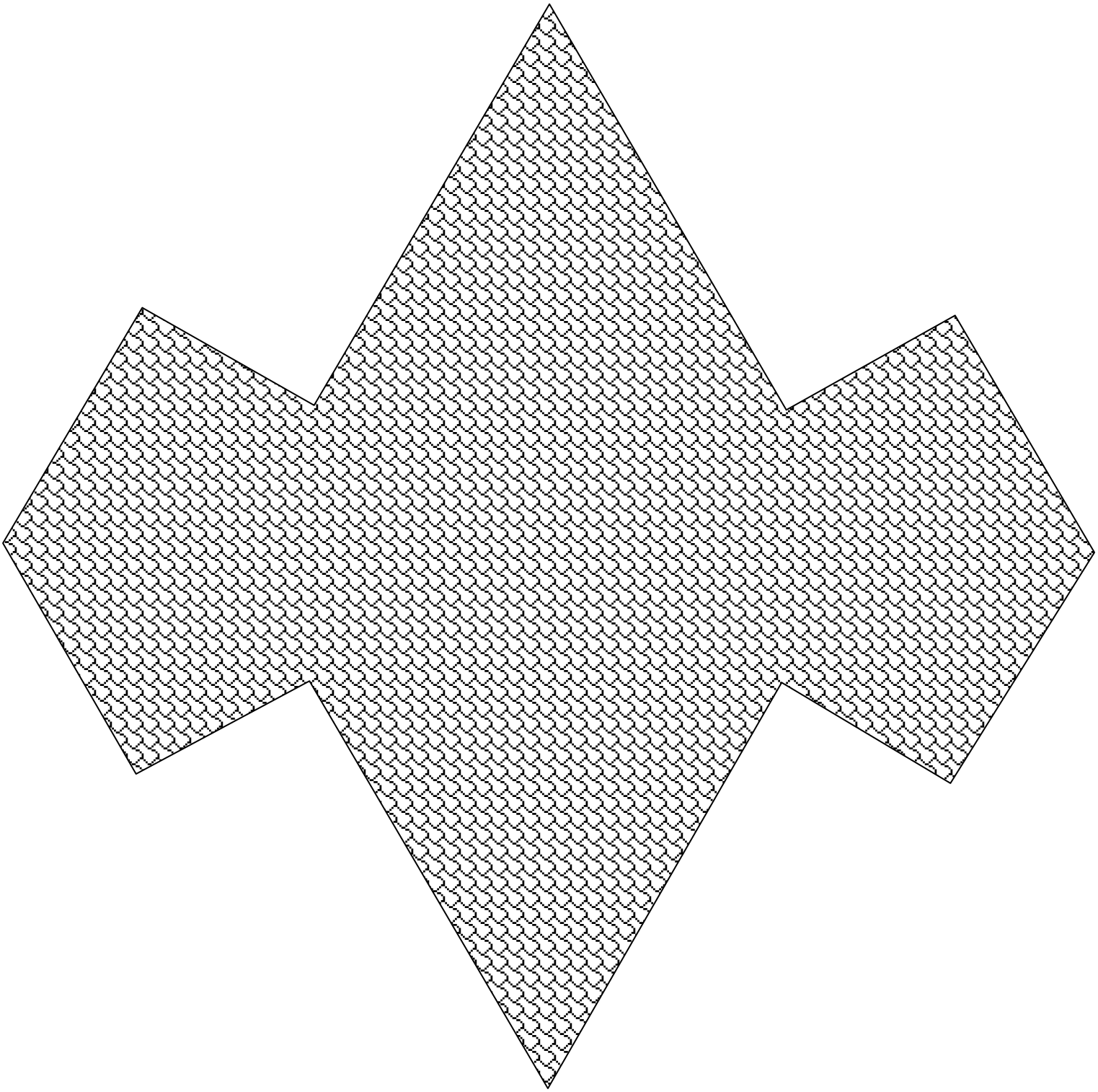

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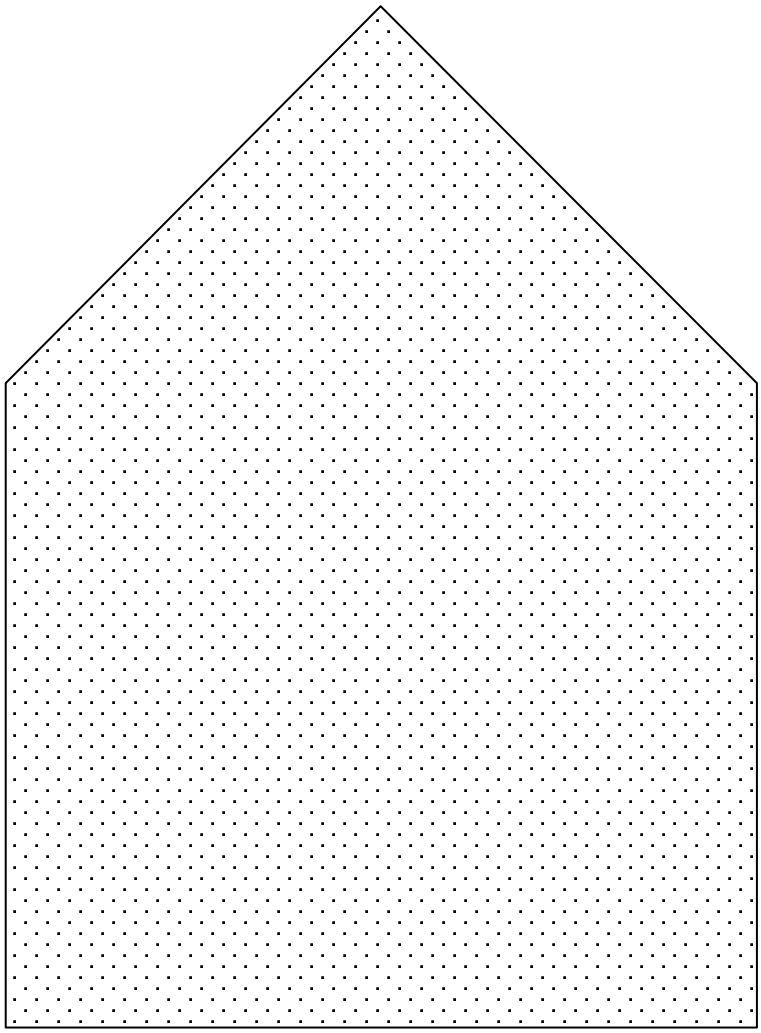

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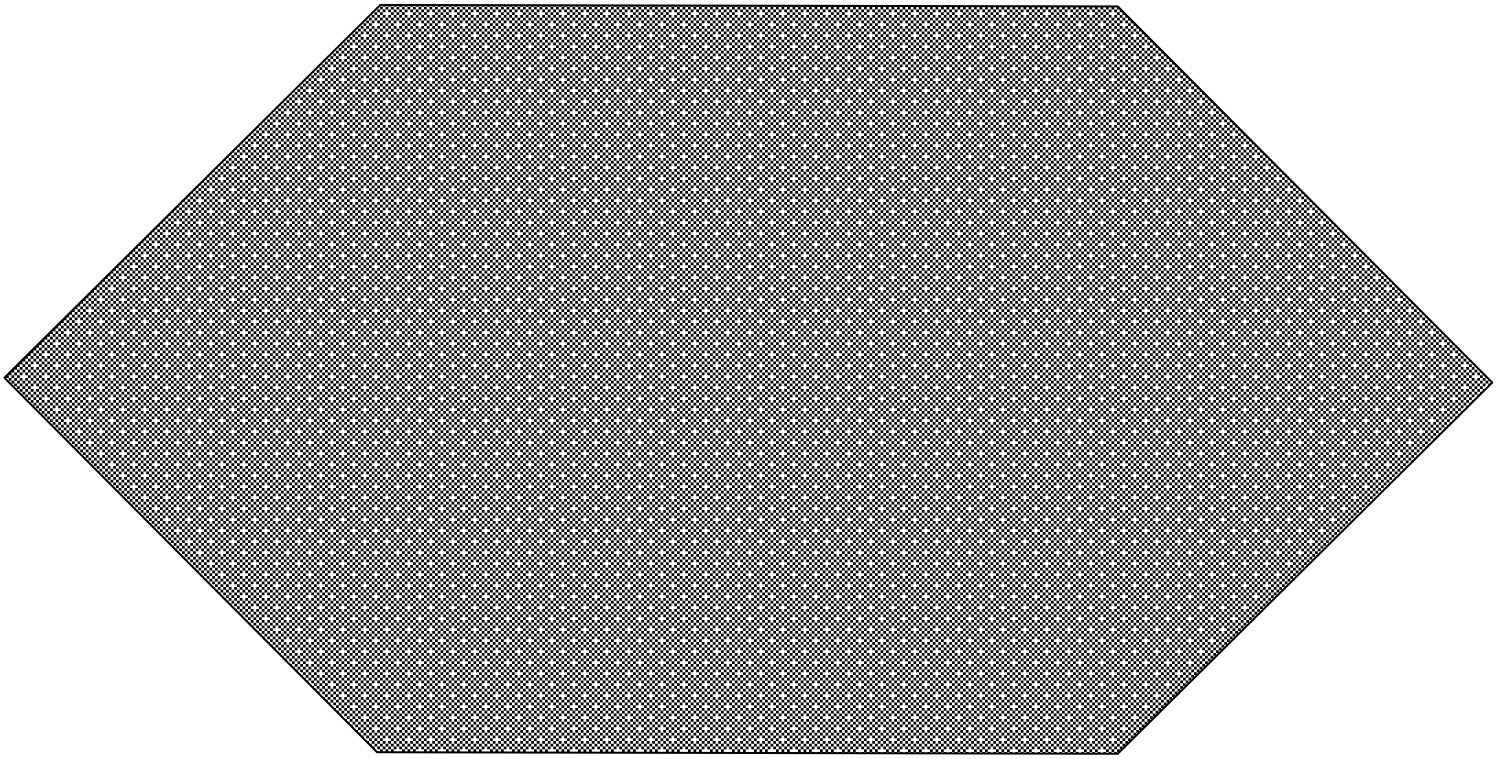

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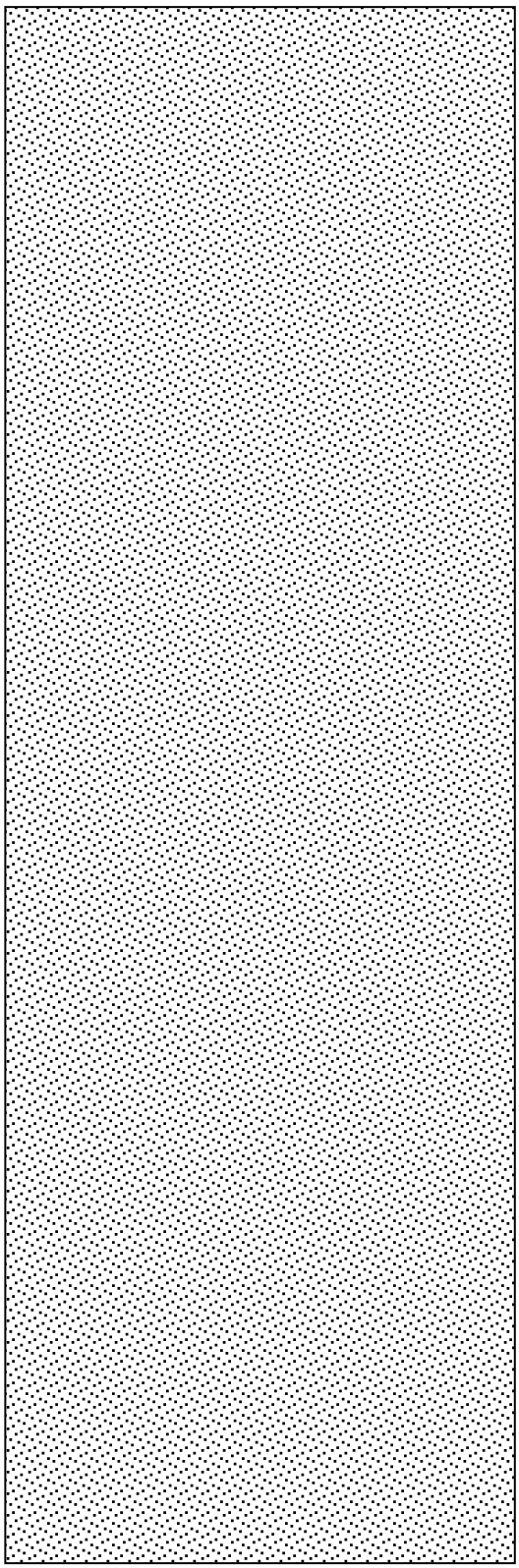

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Group B

Level 1

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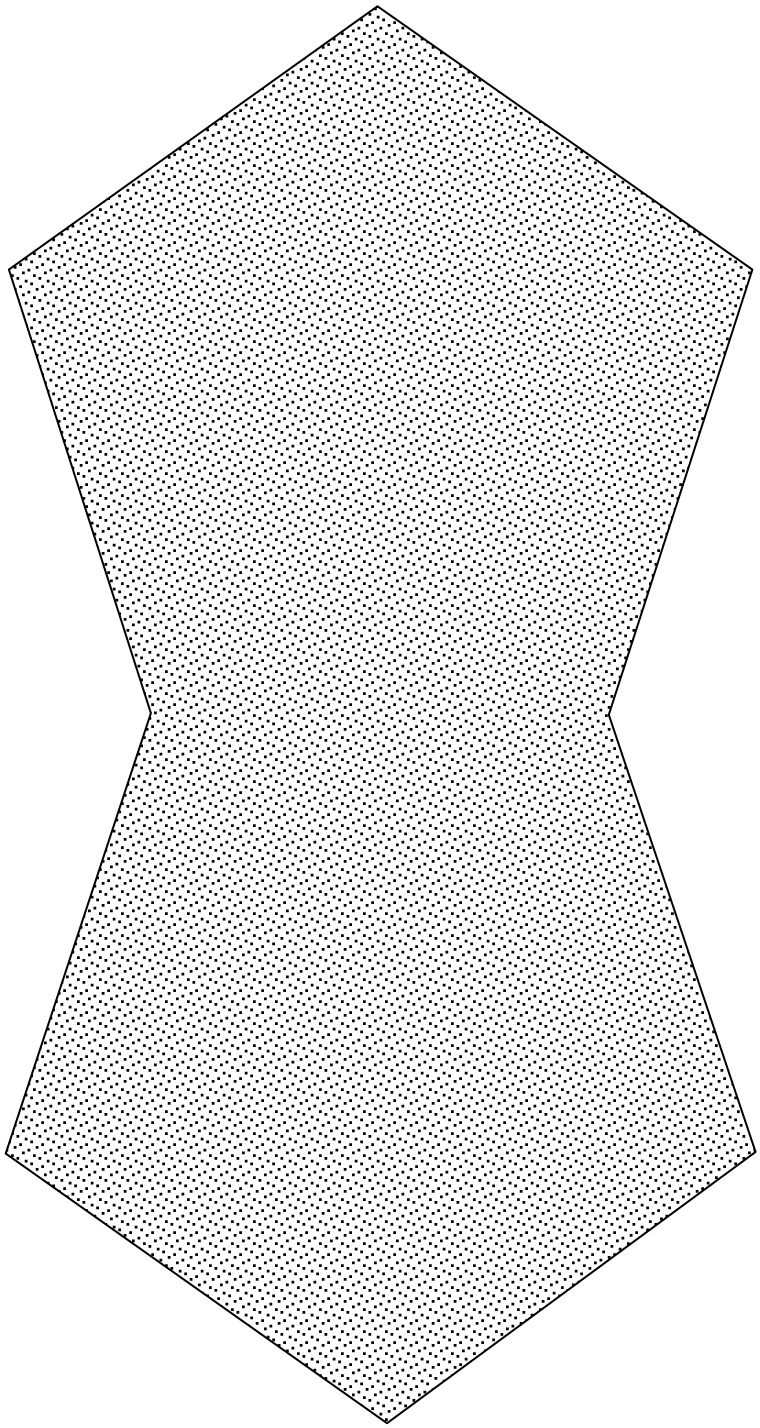

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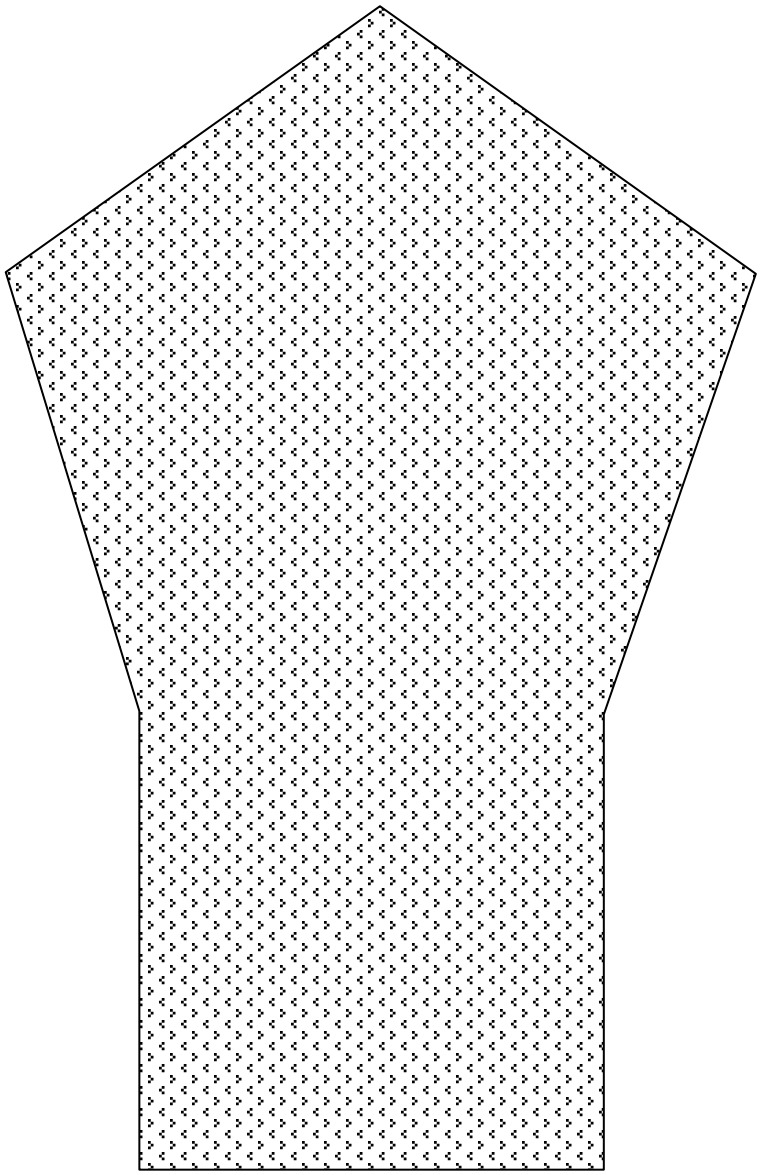

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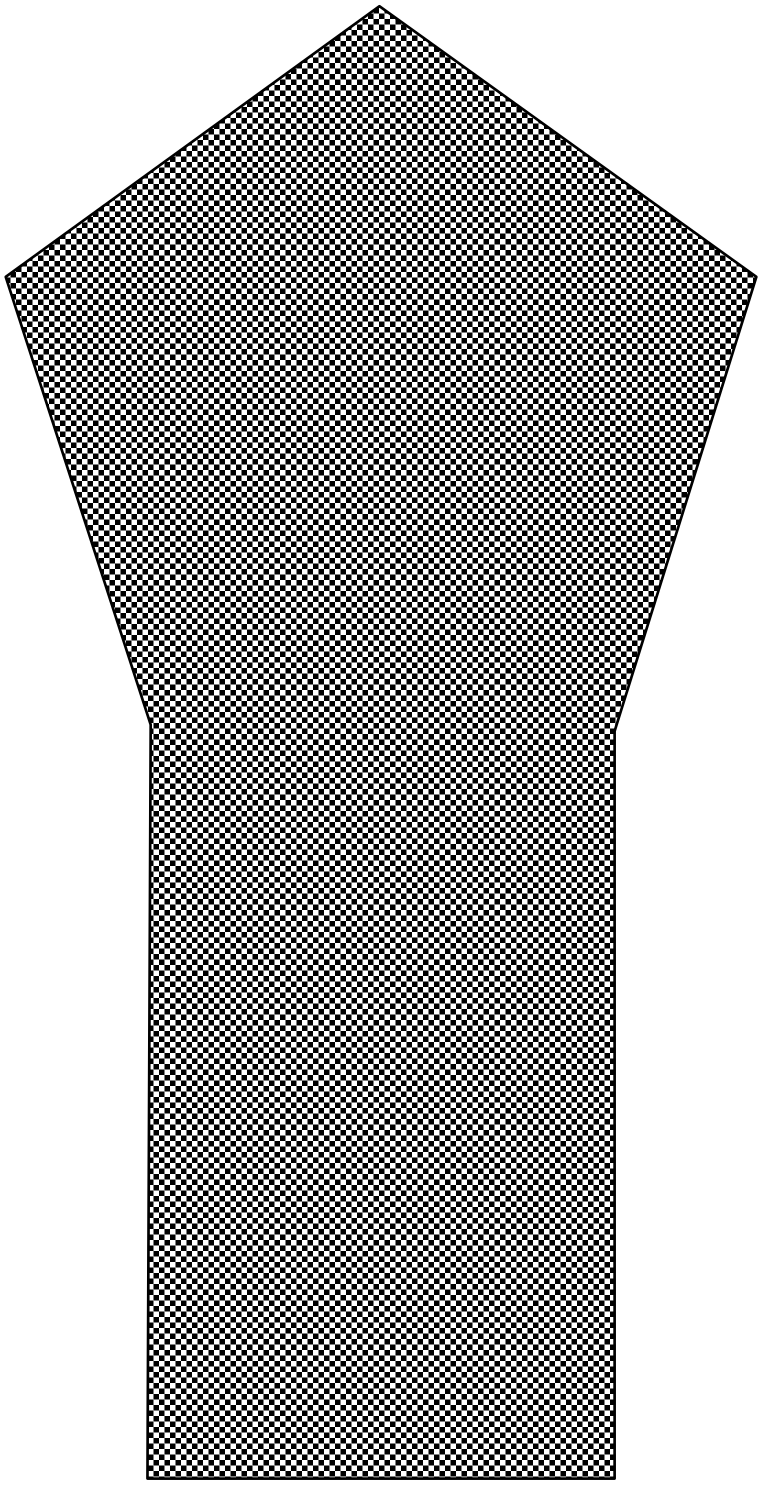

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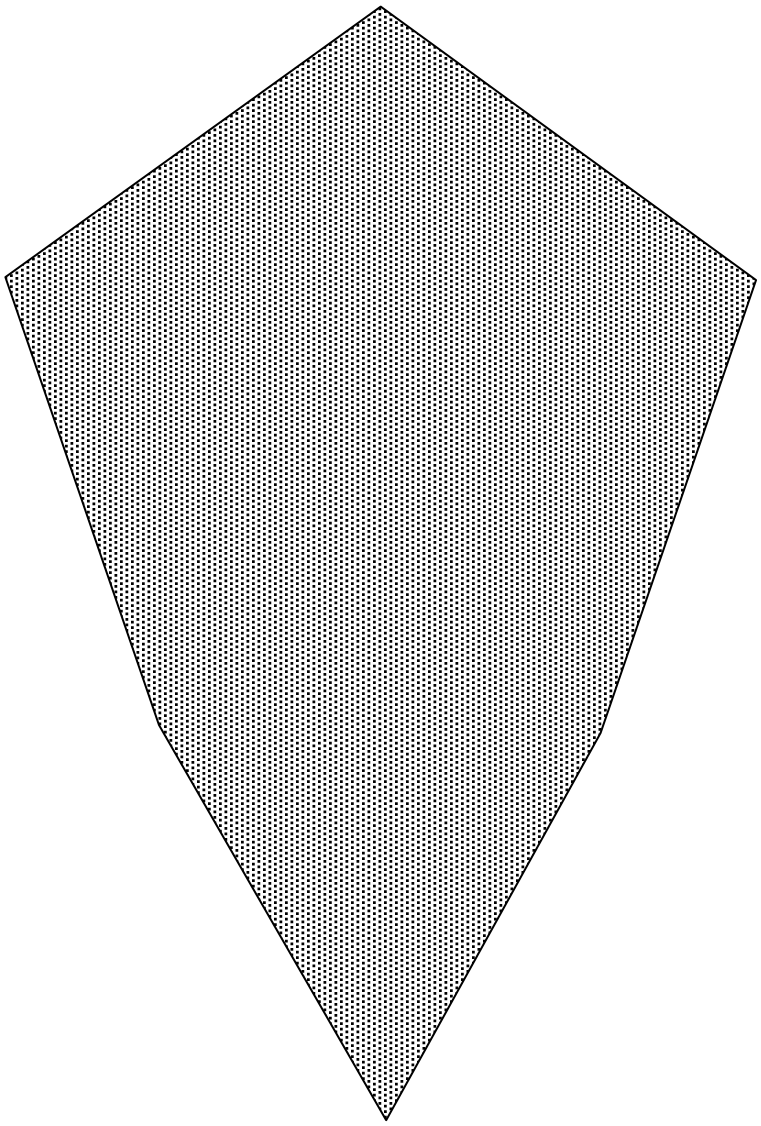

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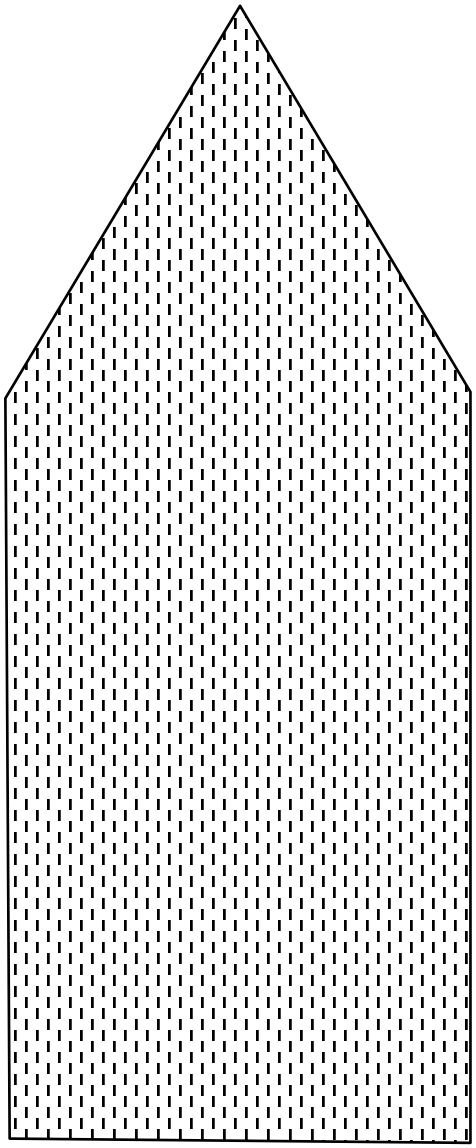

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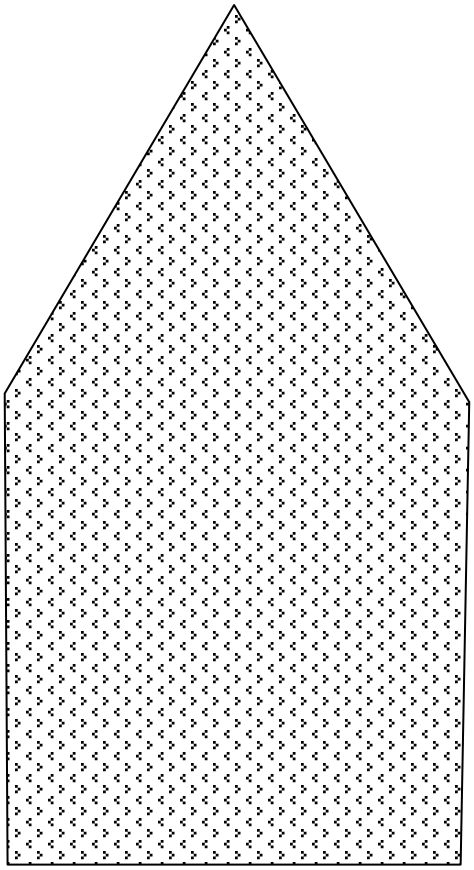

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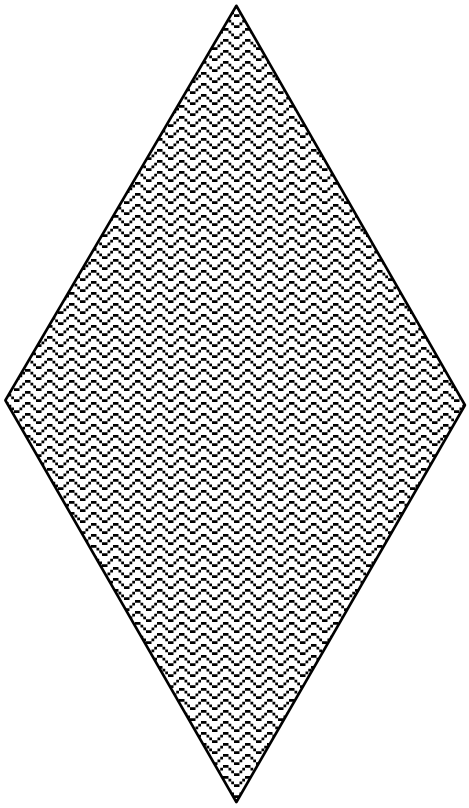

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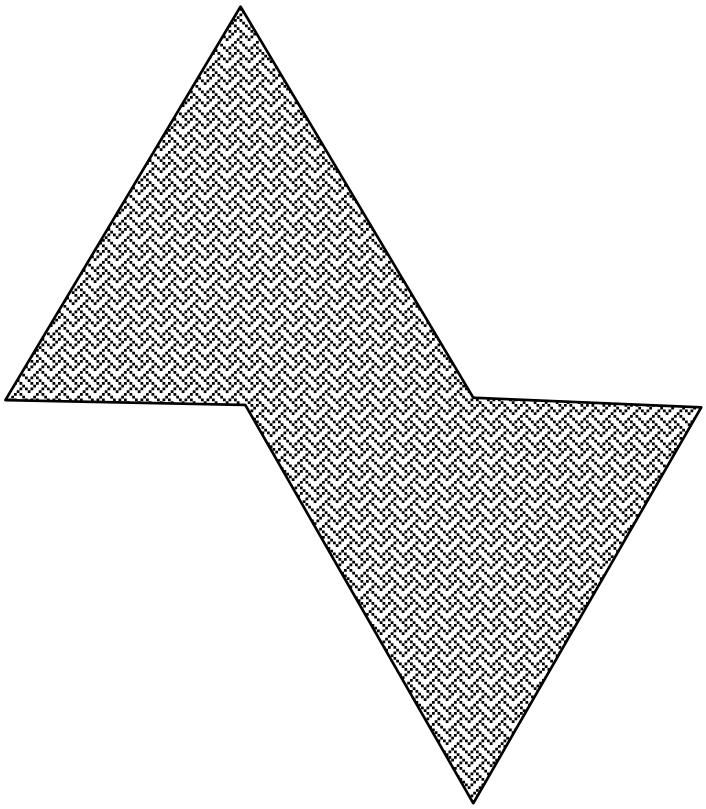

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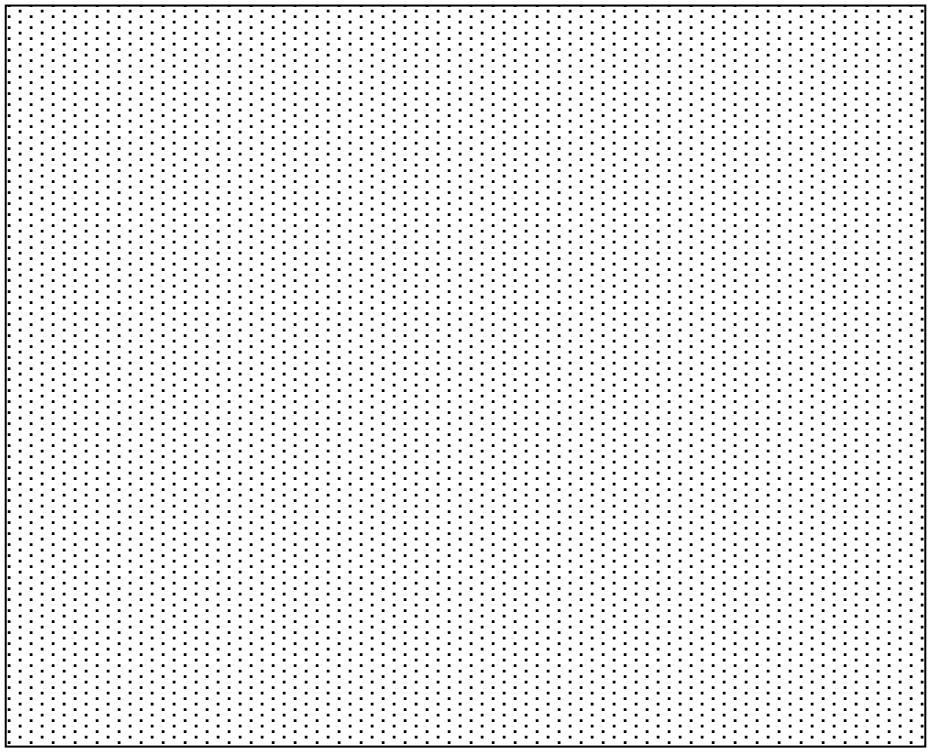

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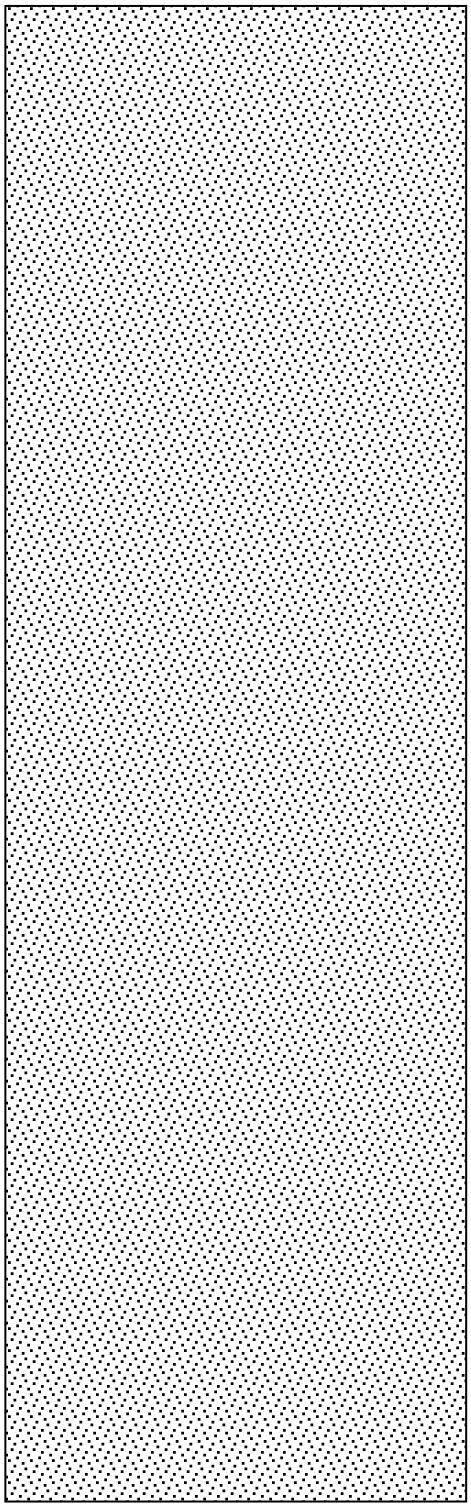

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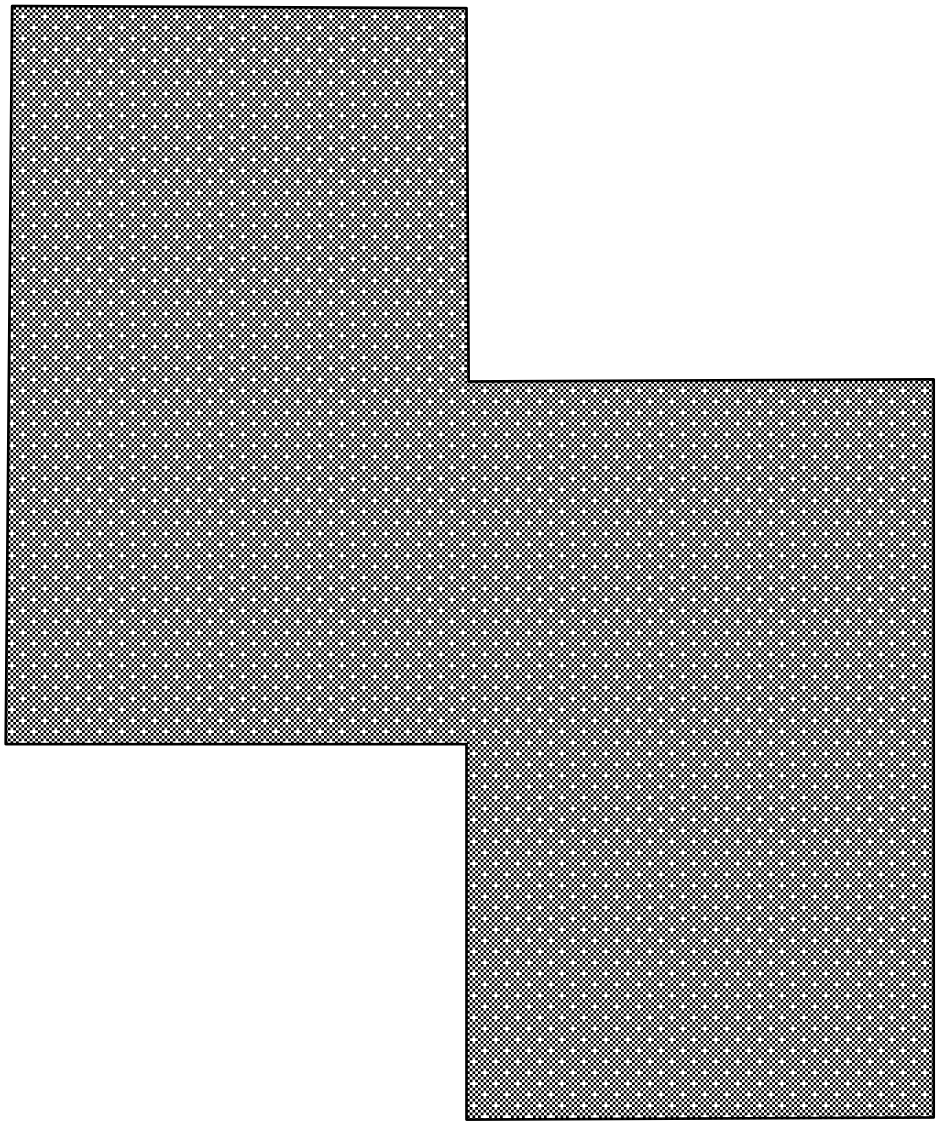

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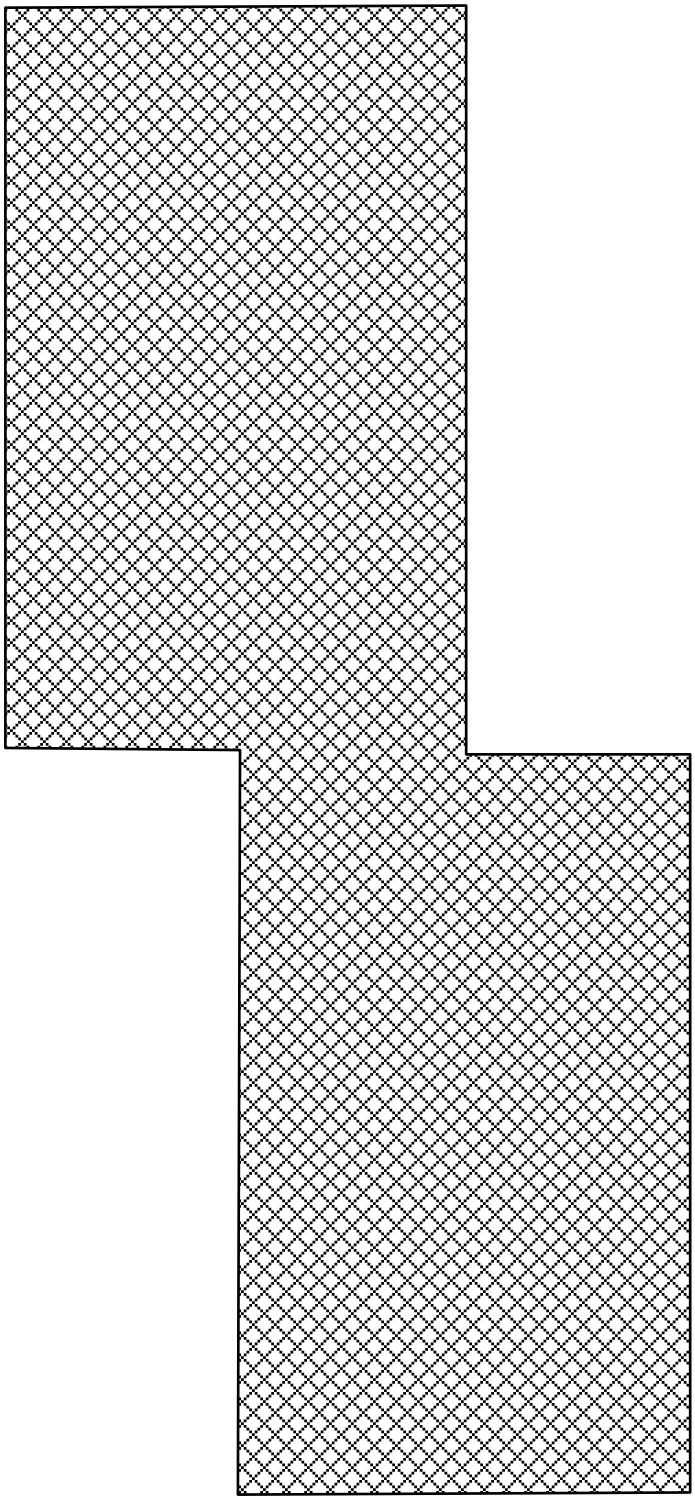

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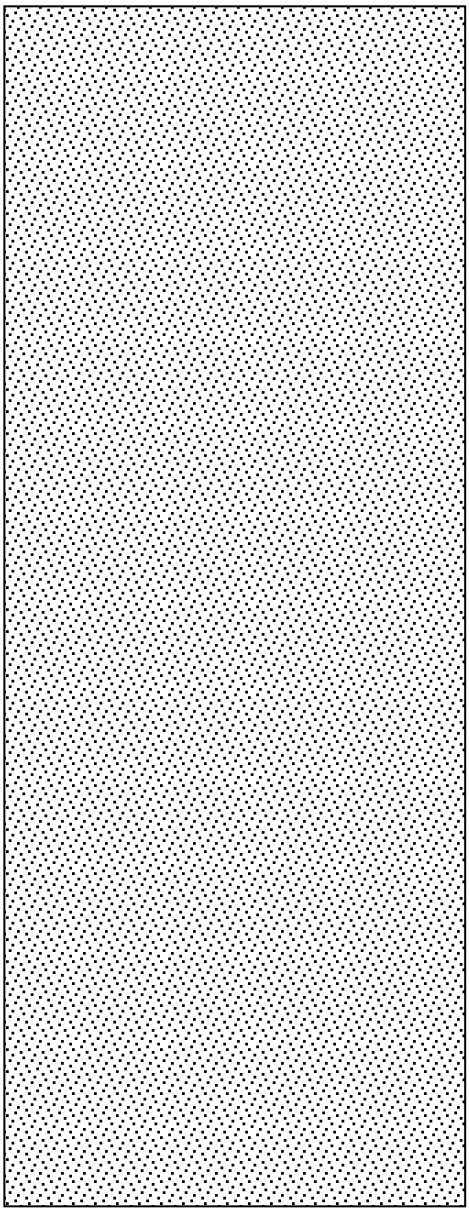

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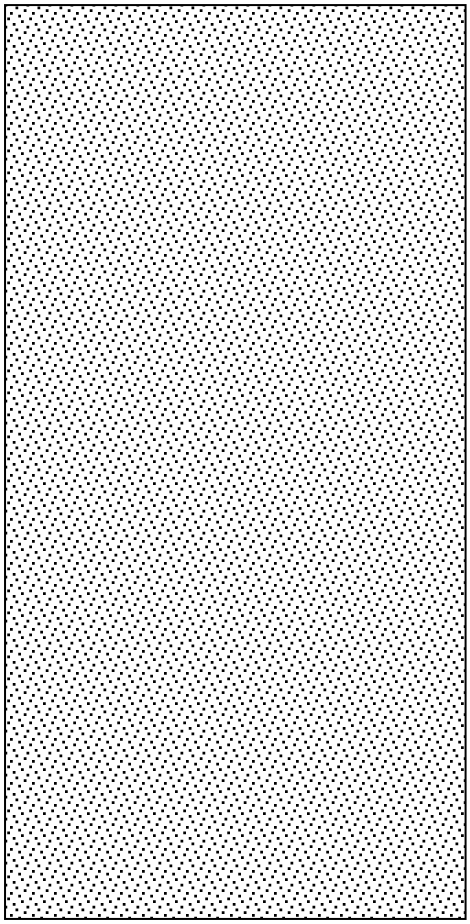

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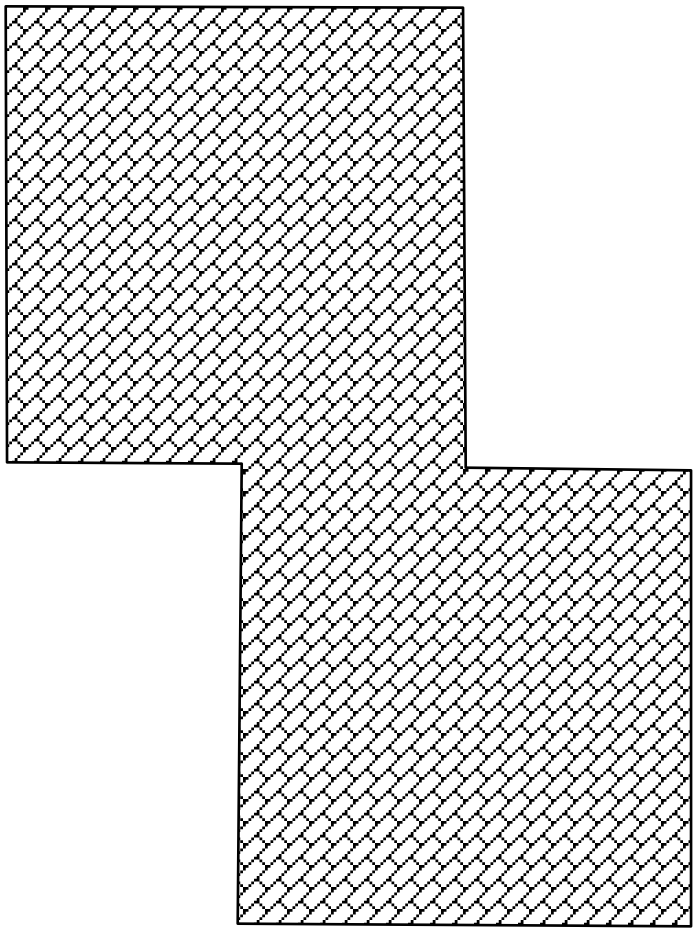

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Group B

Level 2

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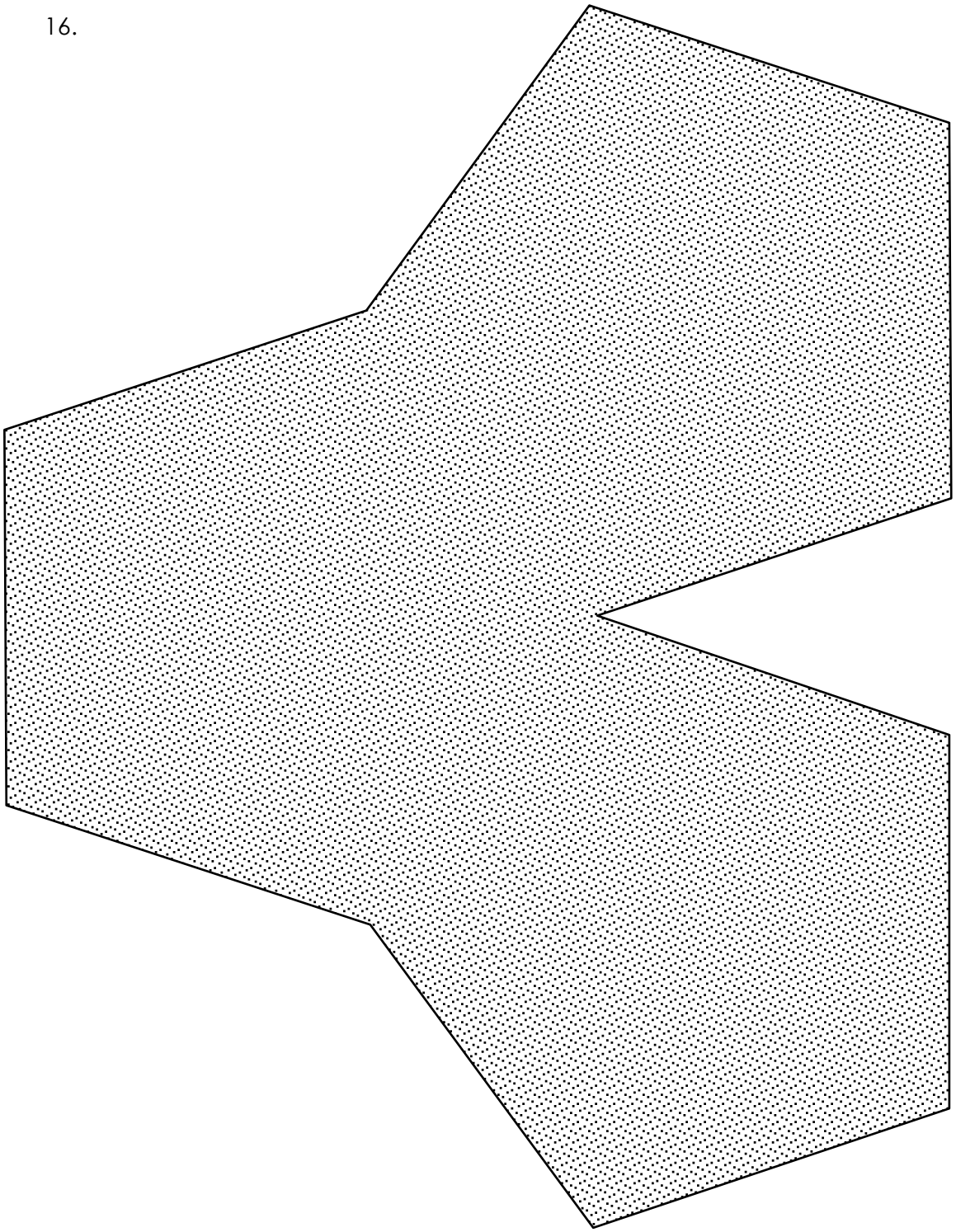

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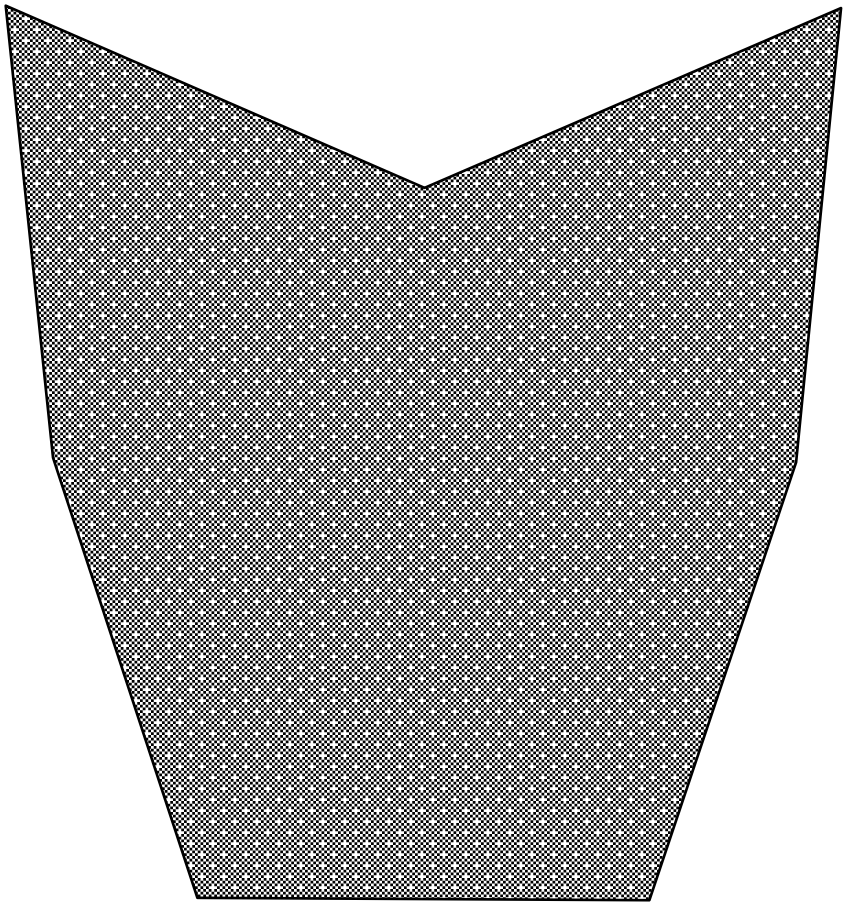

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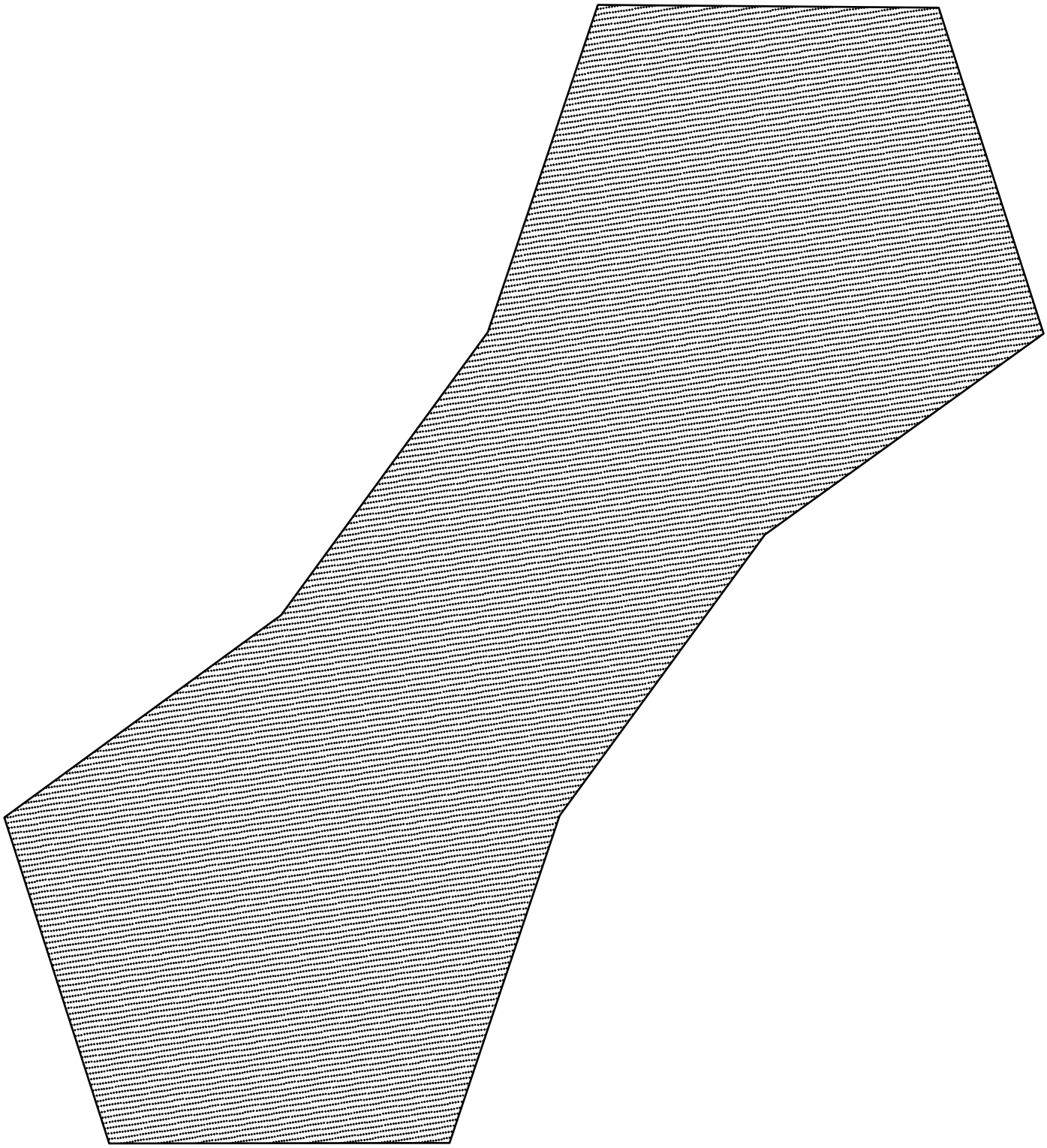

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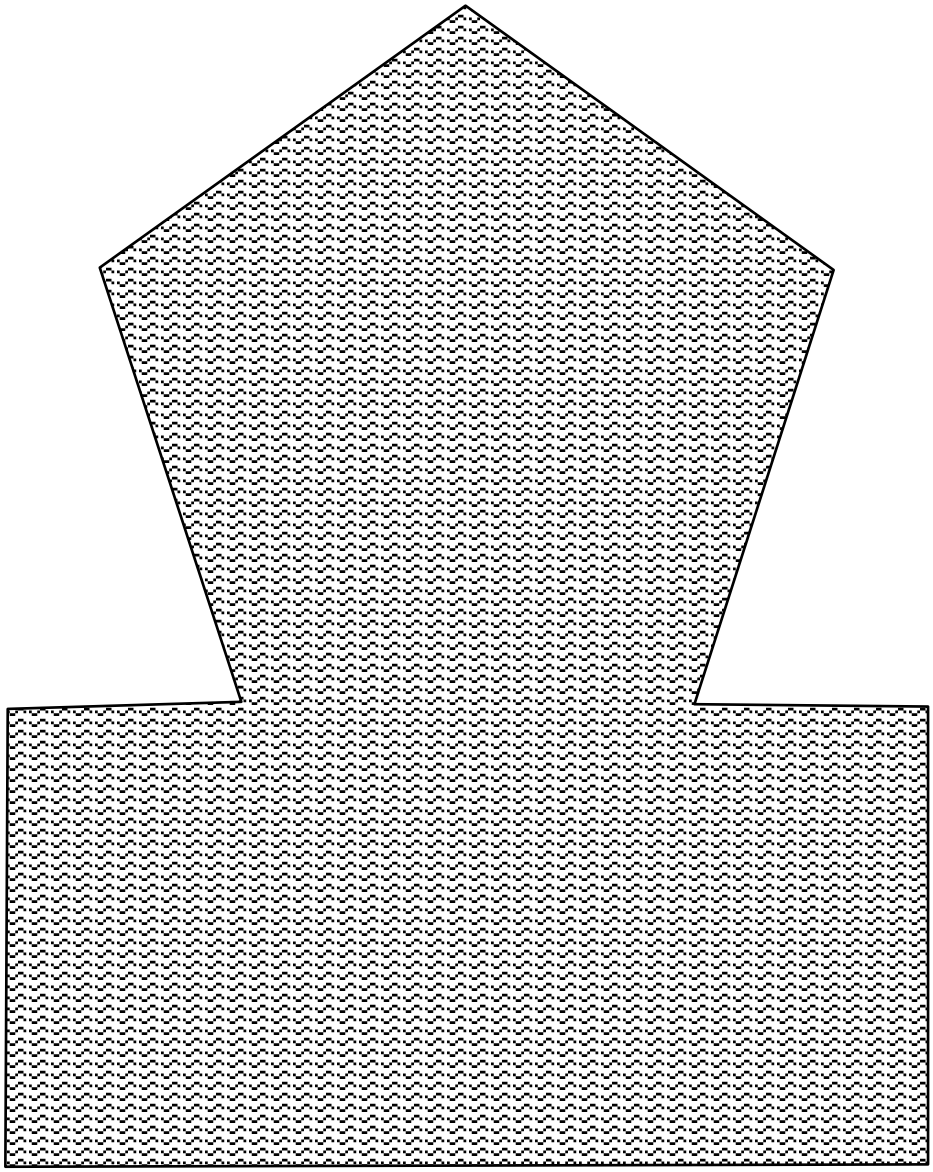

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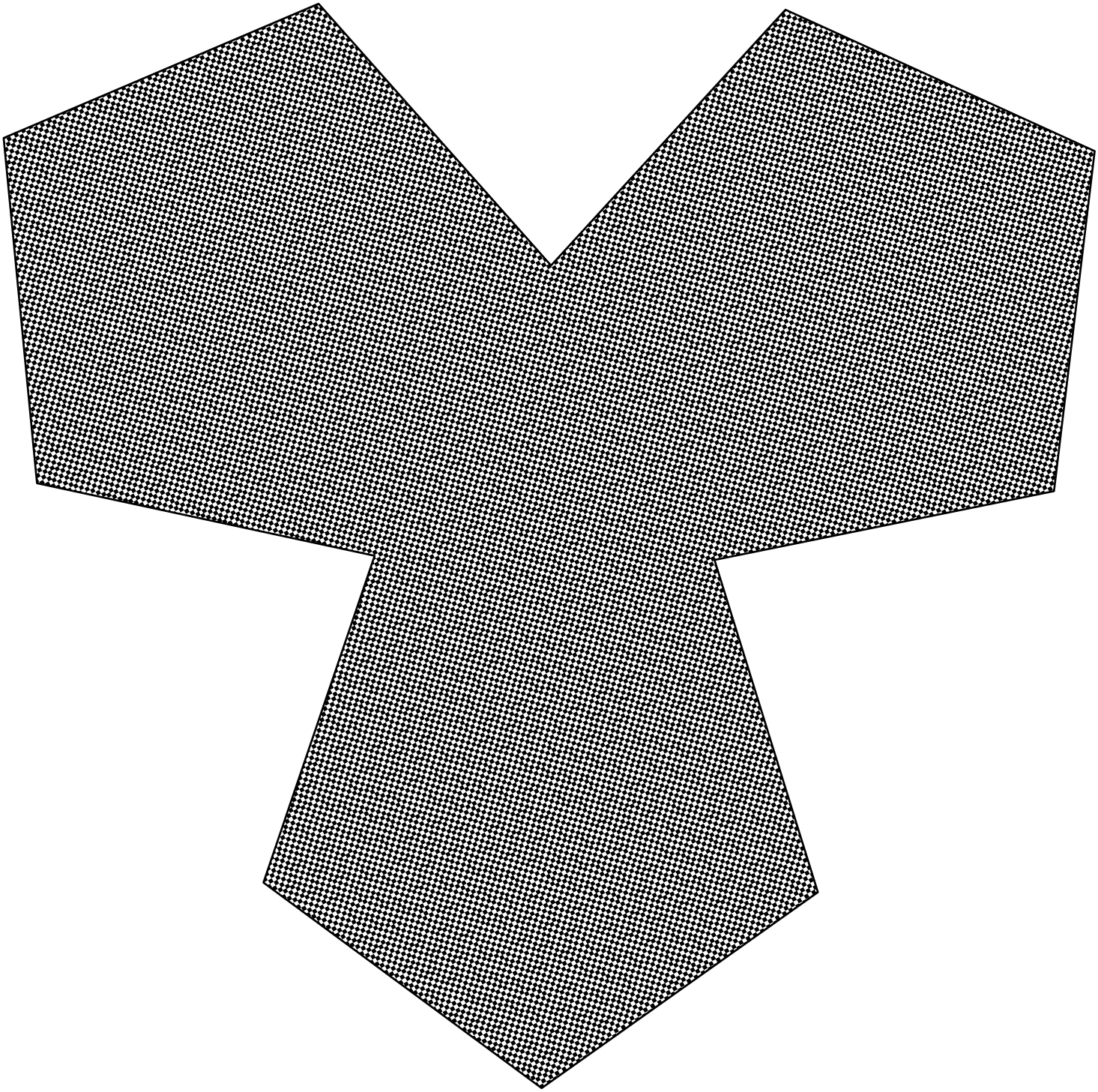

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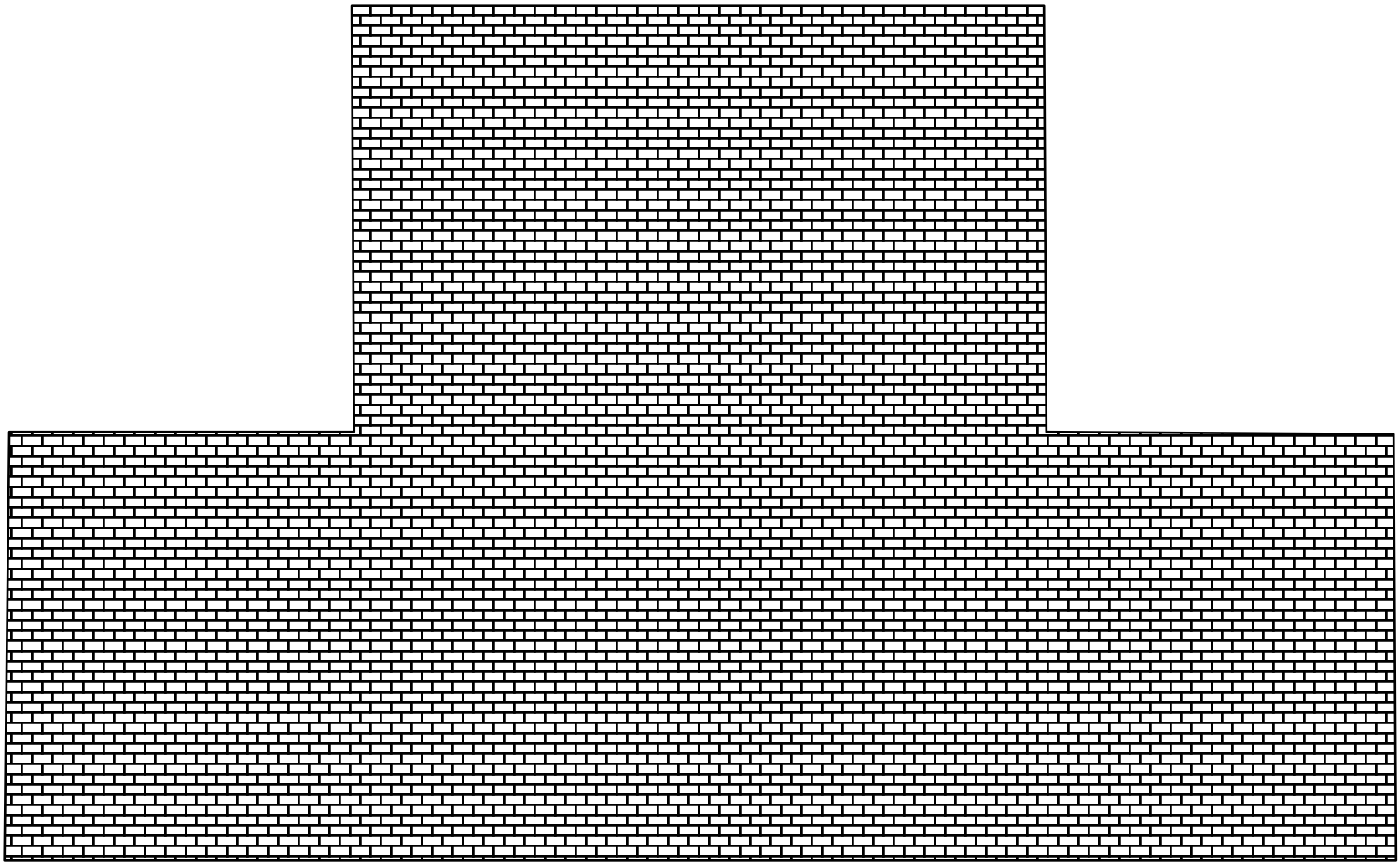

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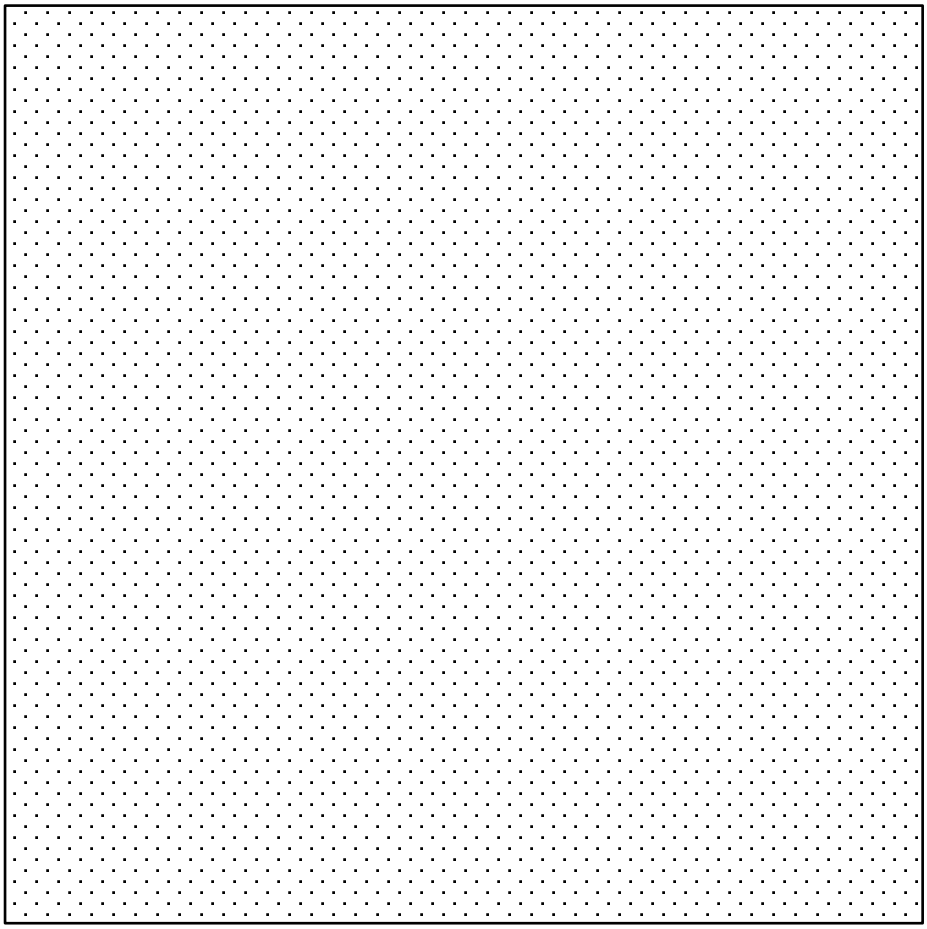

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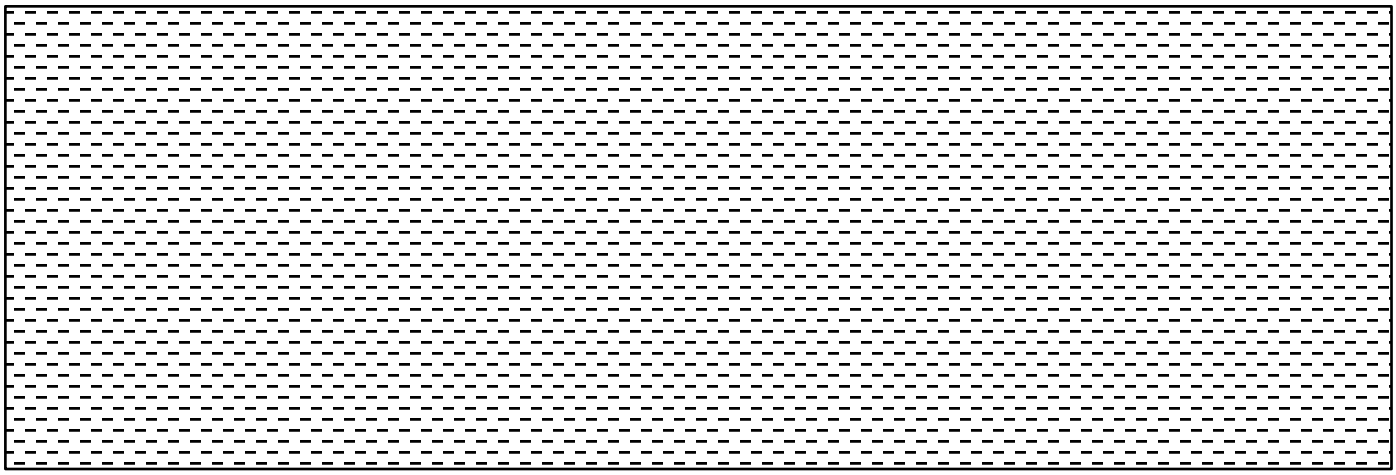

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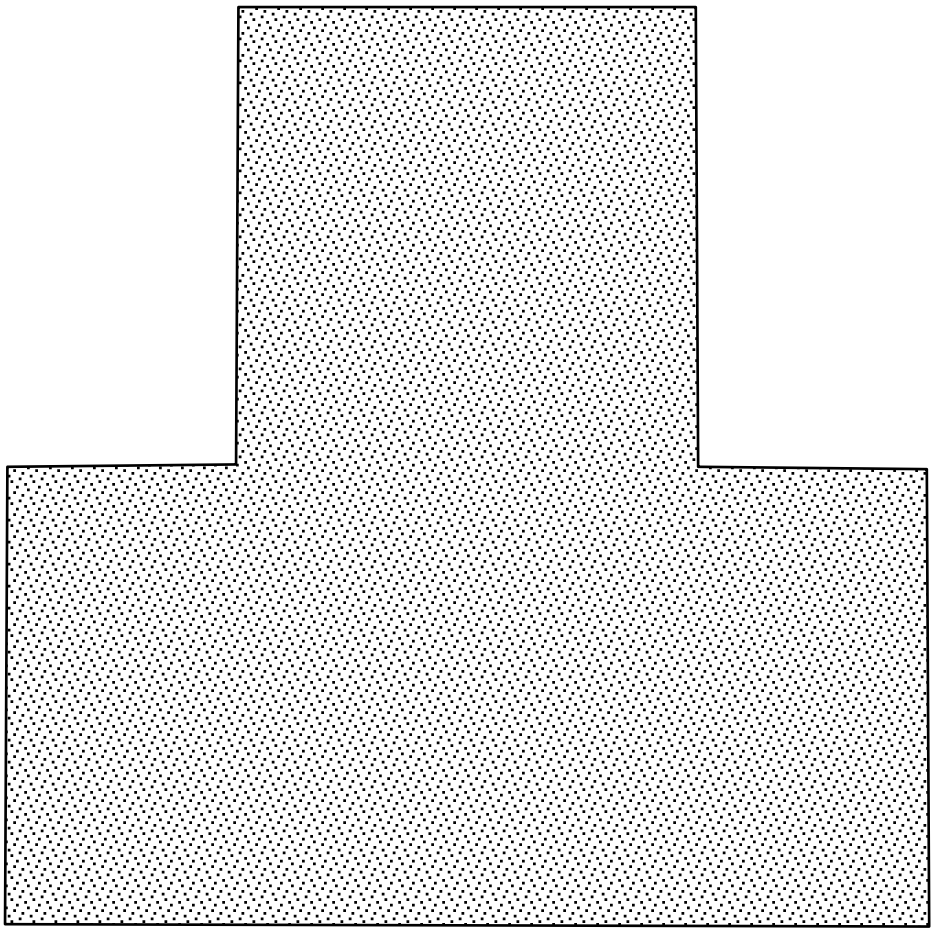

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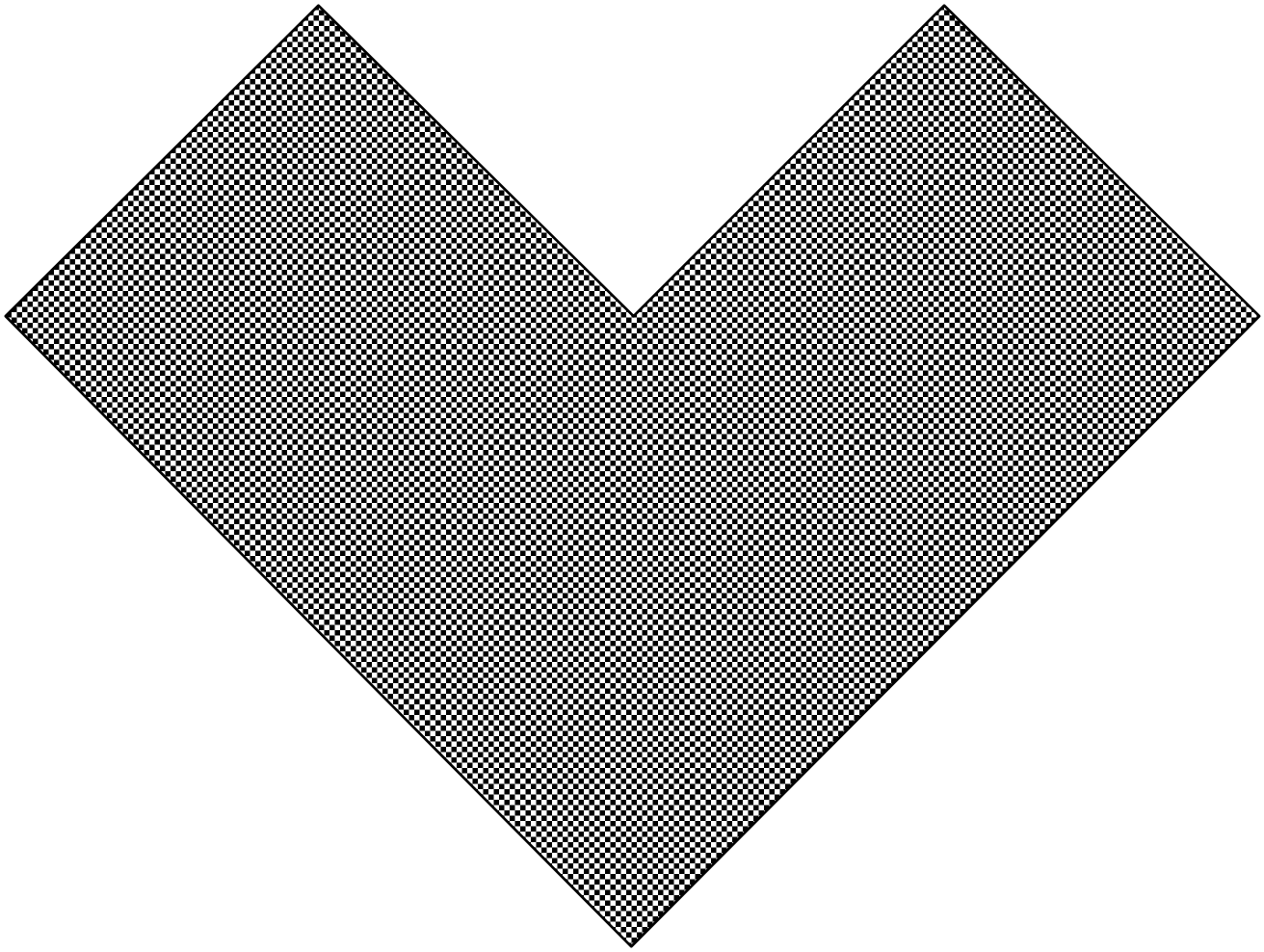

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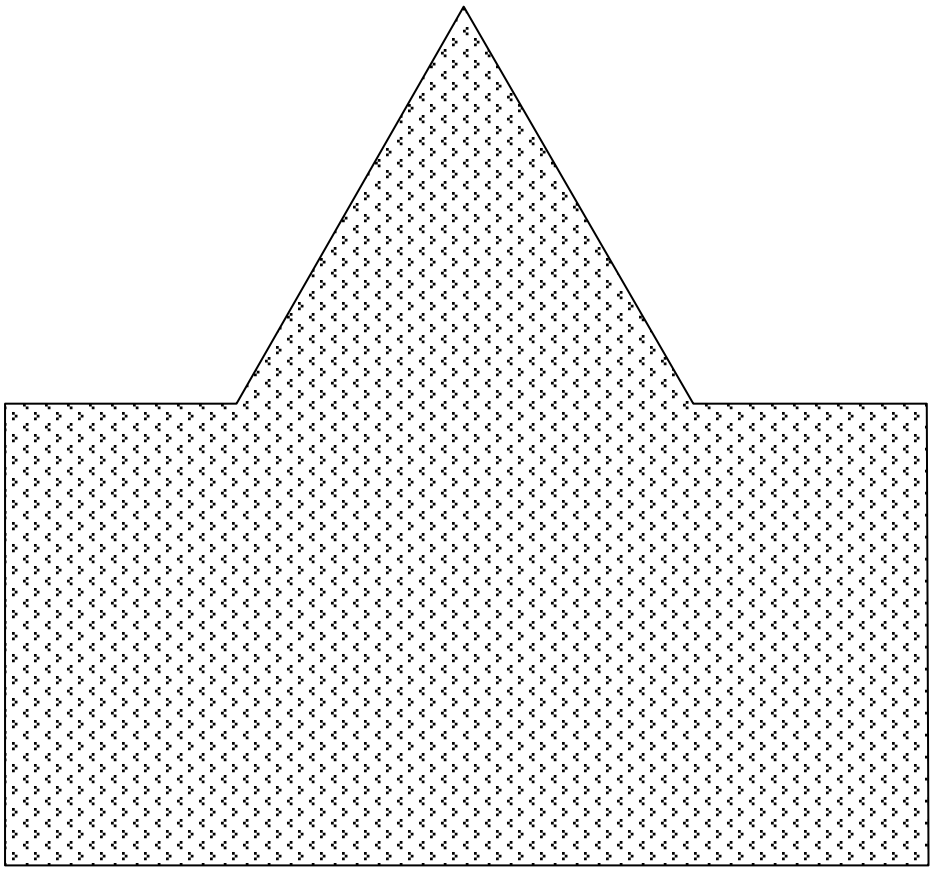

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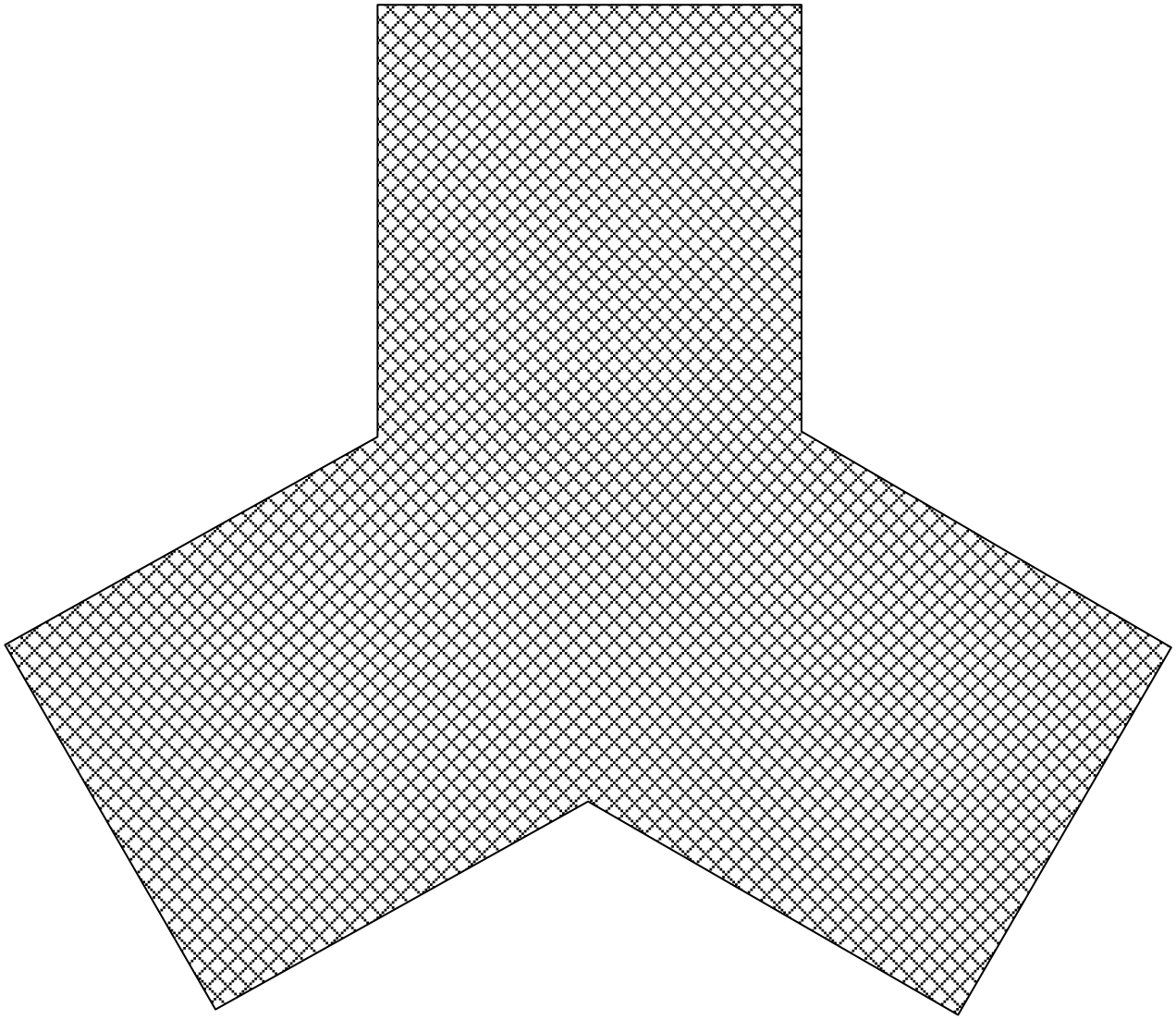

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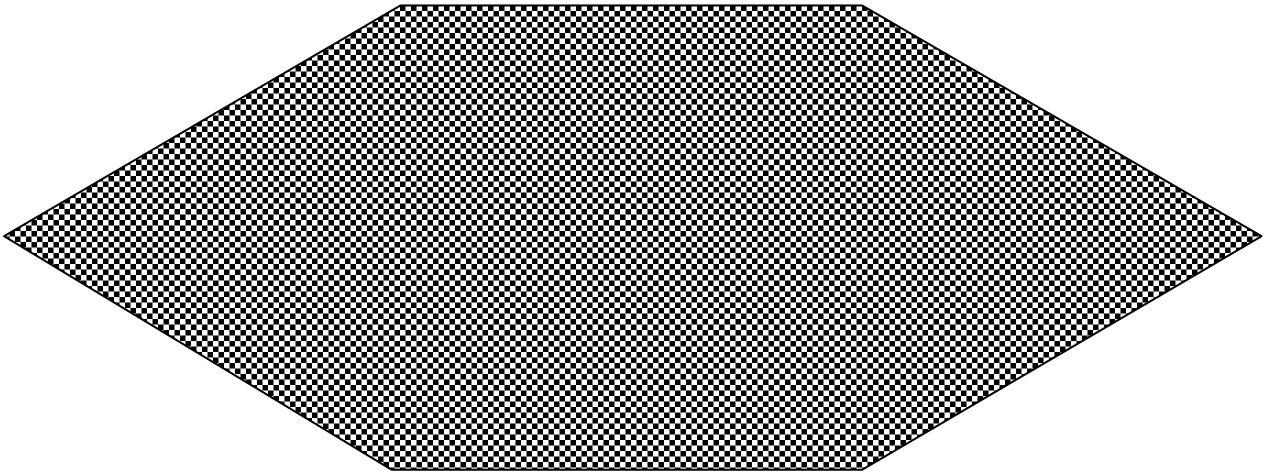

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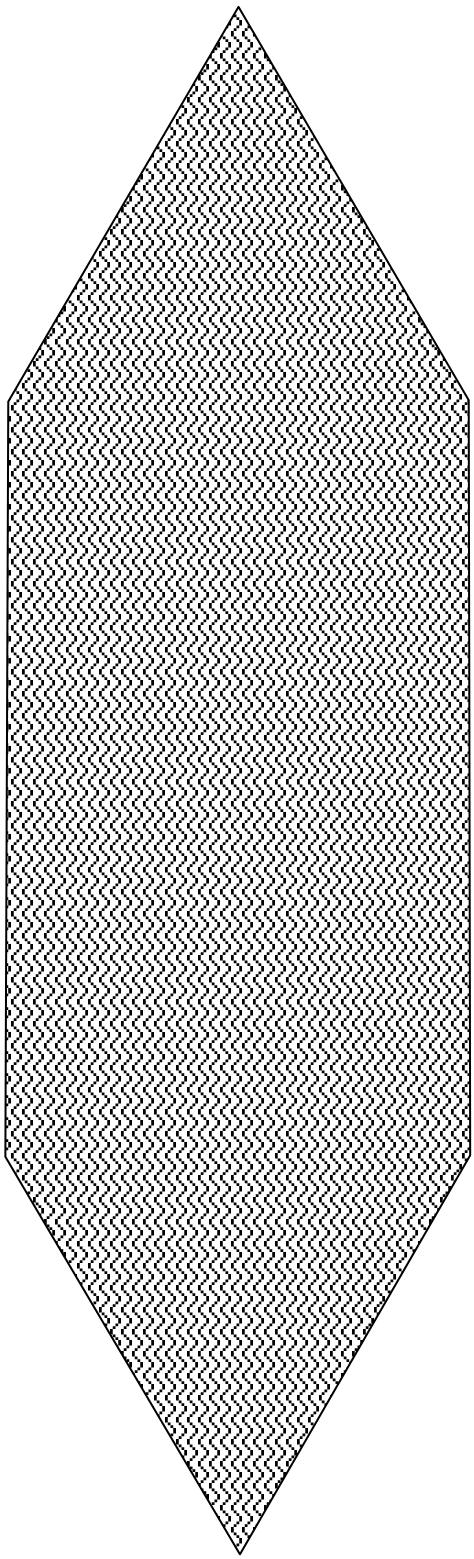

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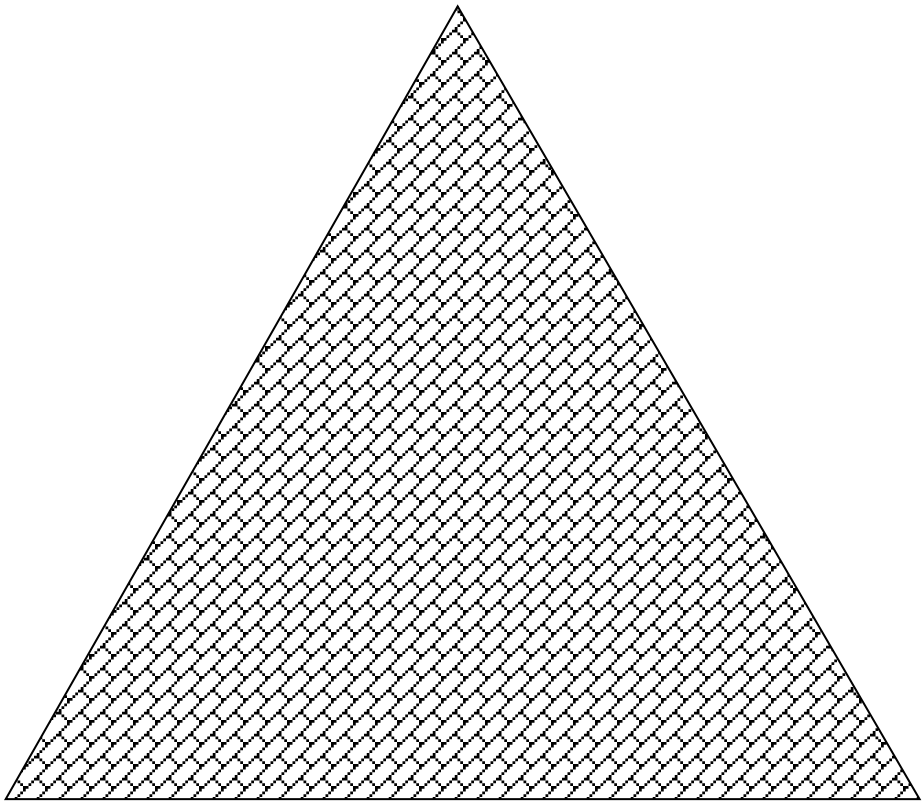

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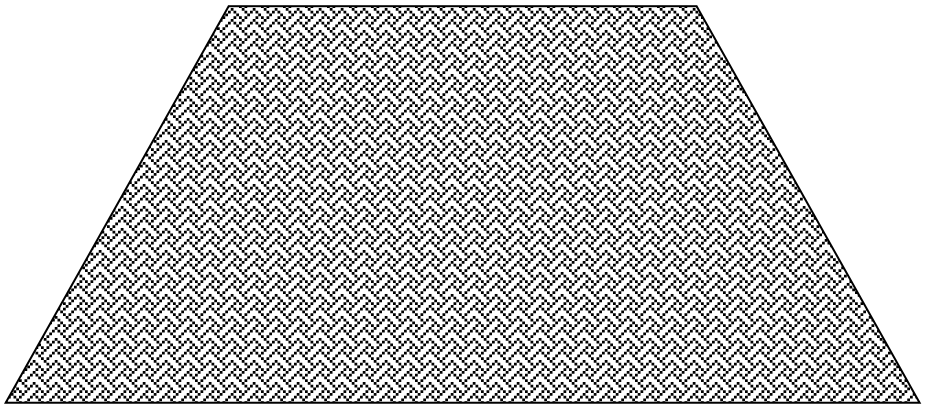

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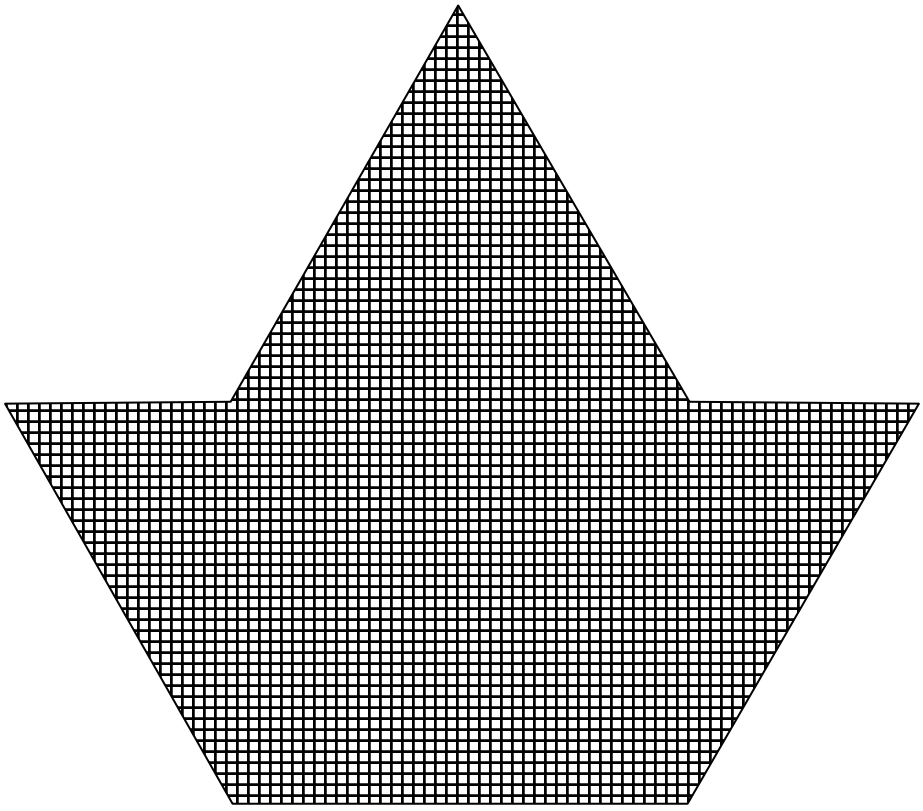

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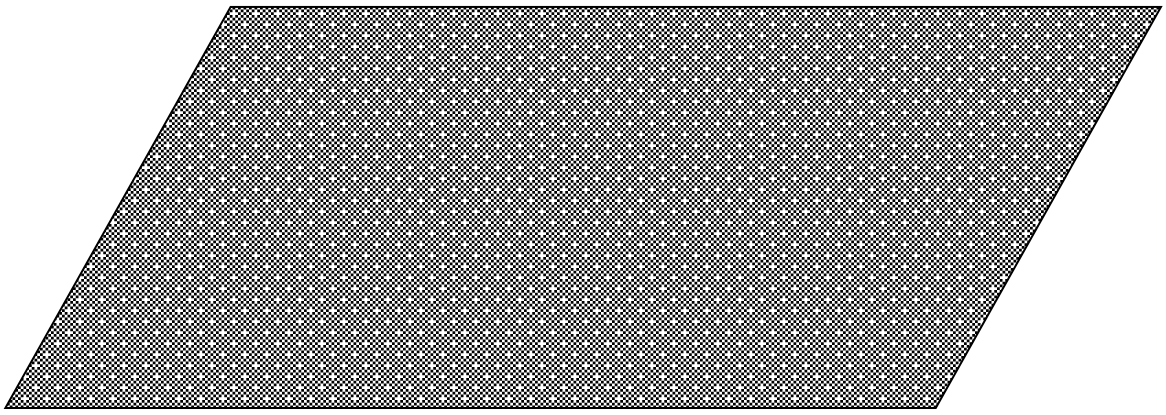

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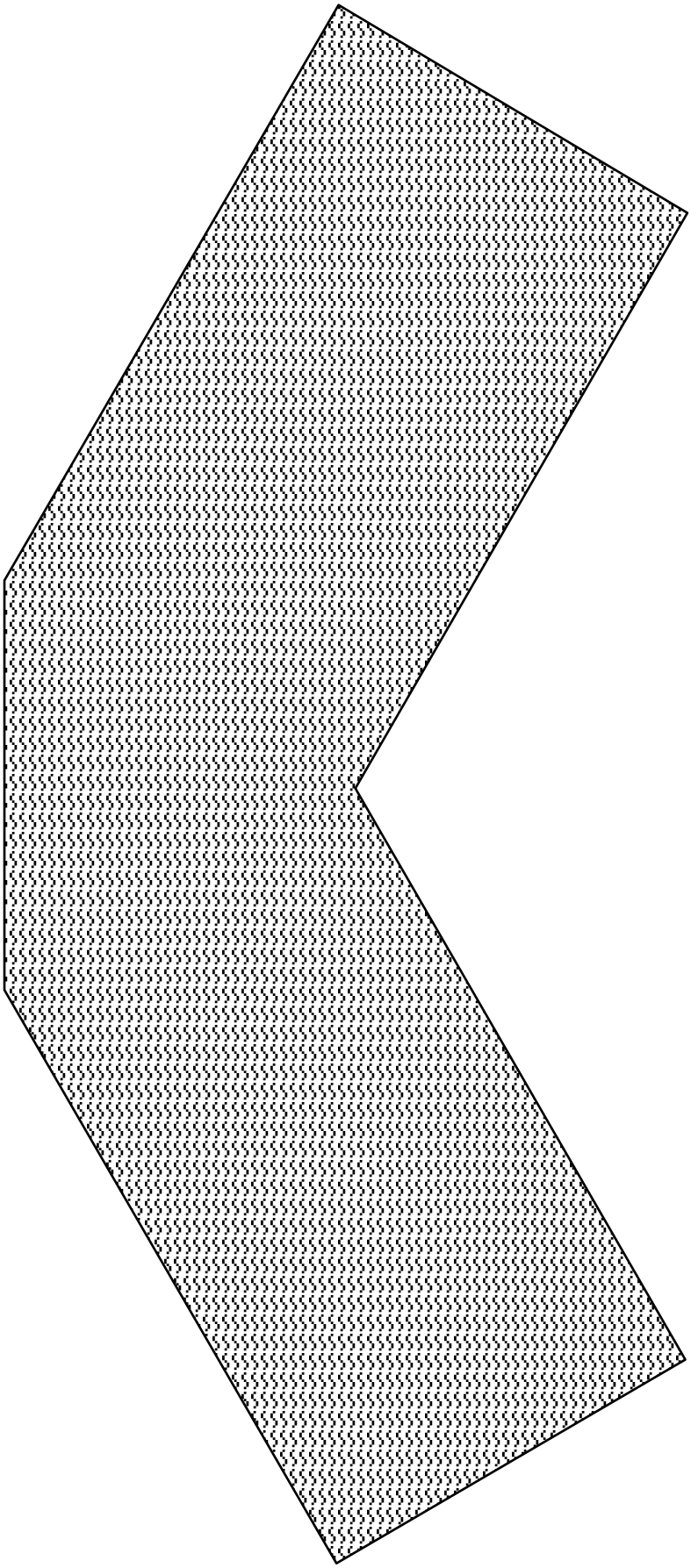

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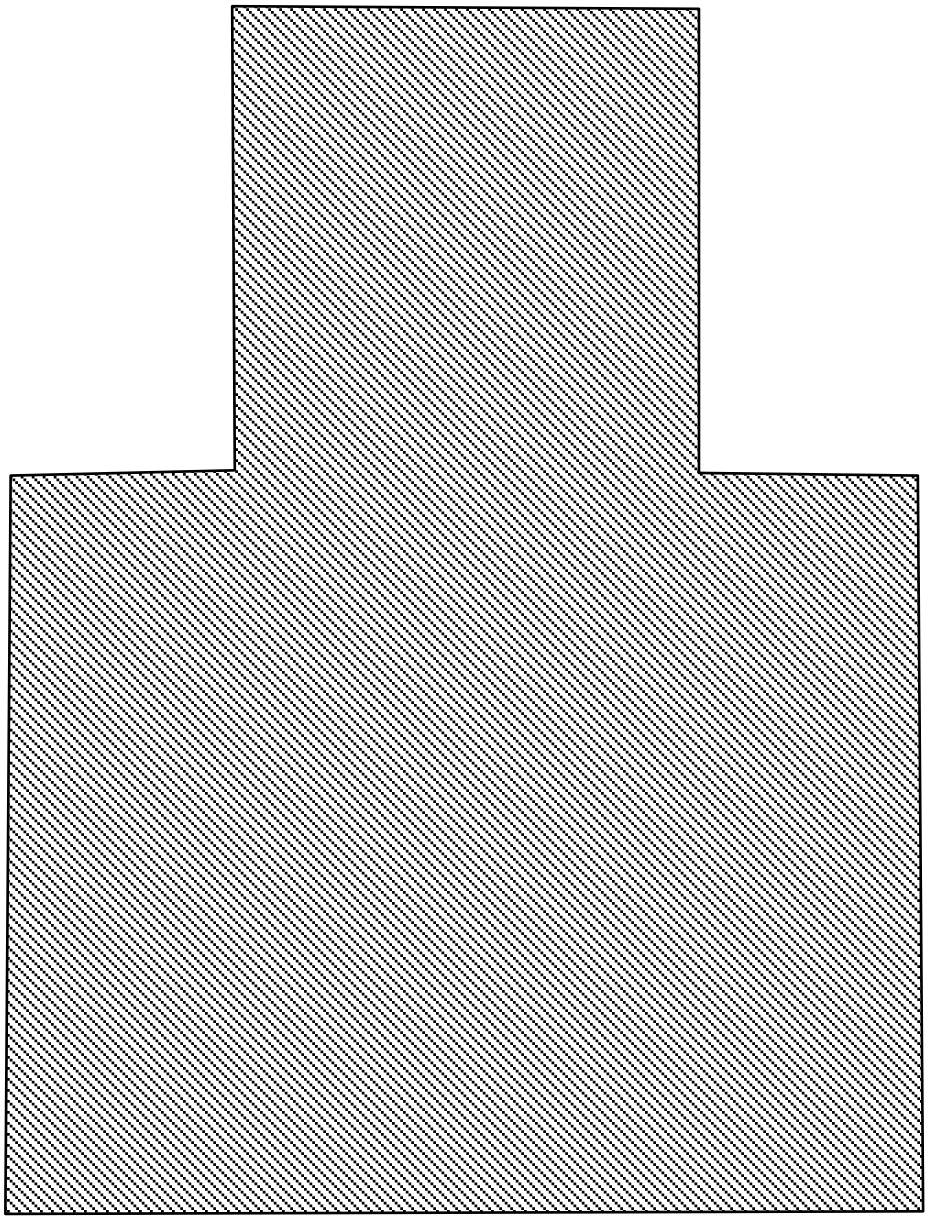

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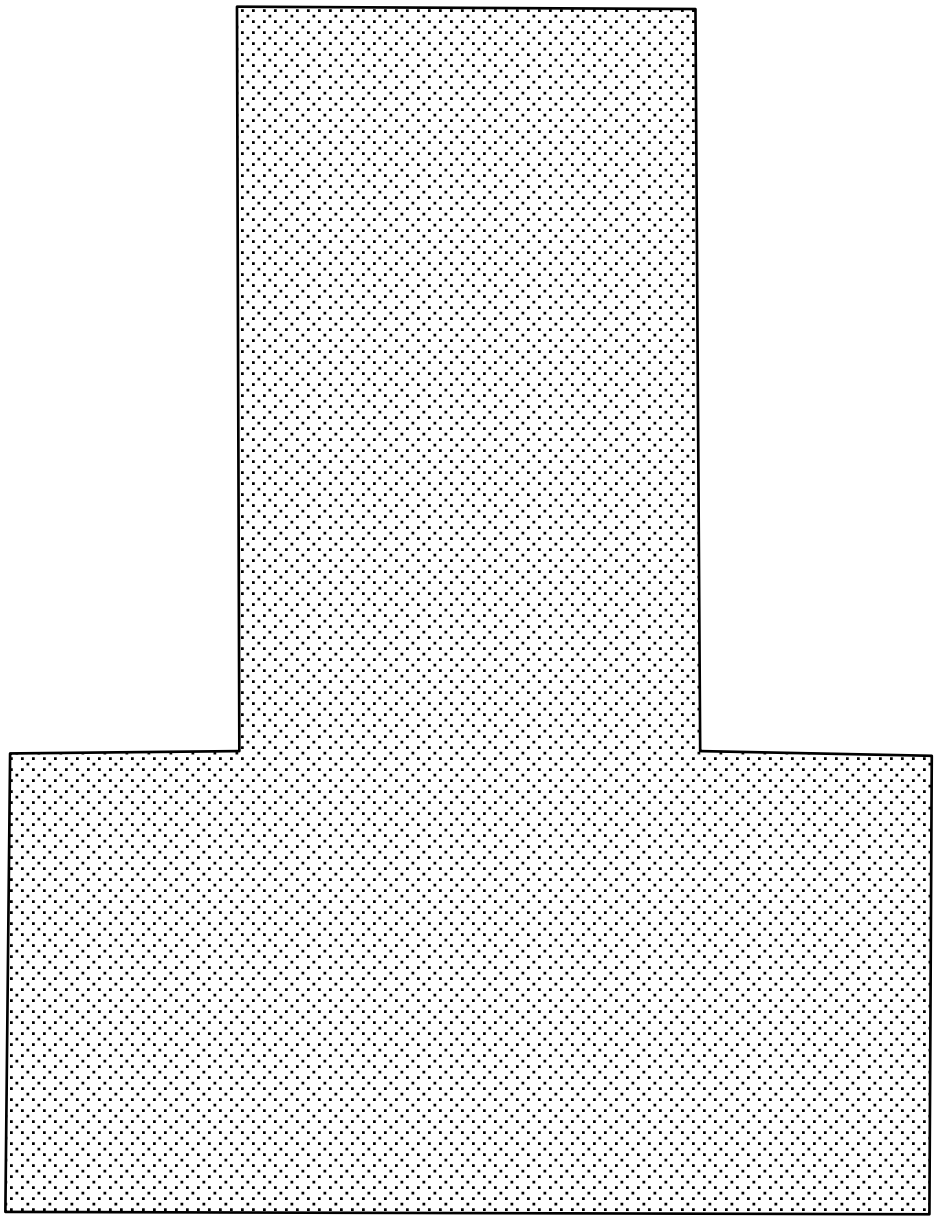

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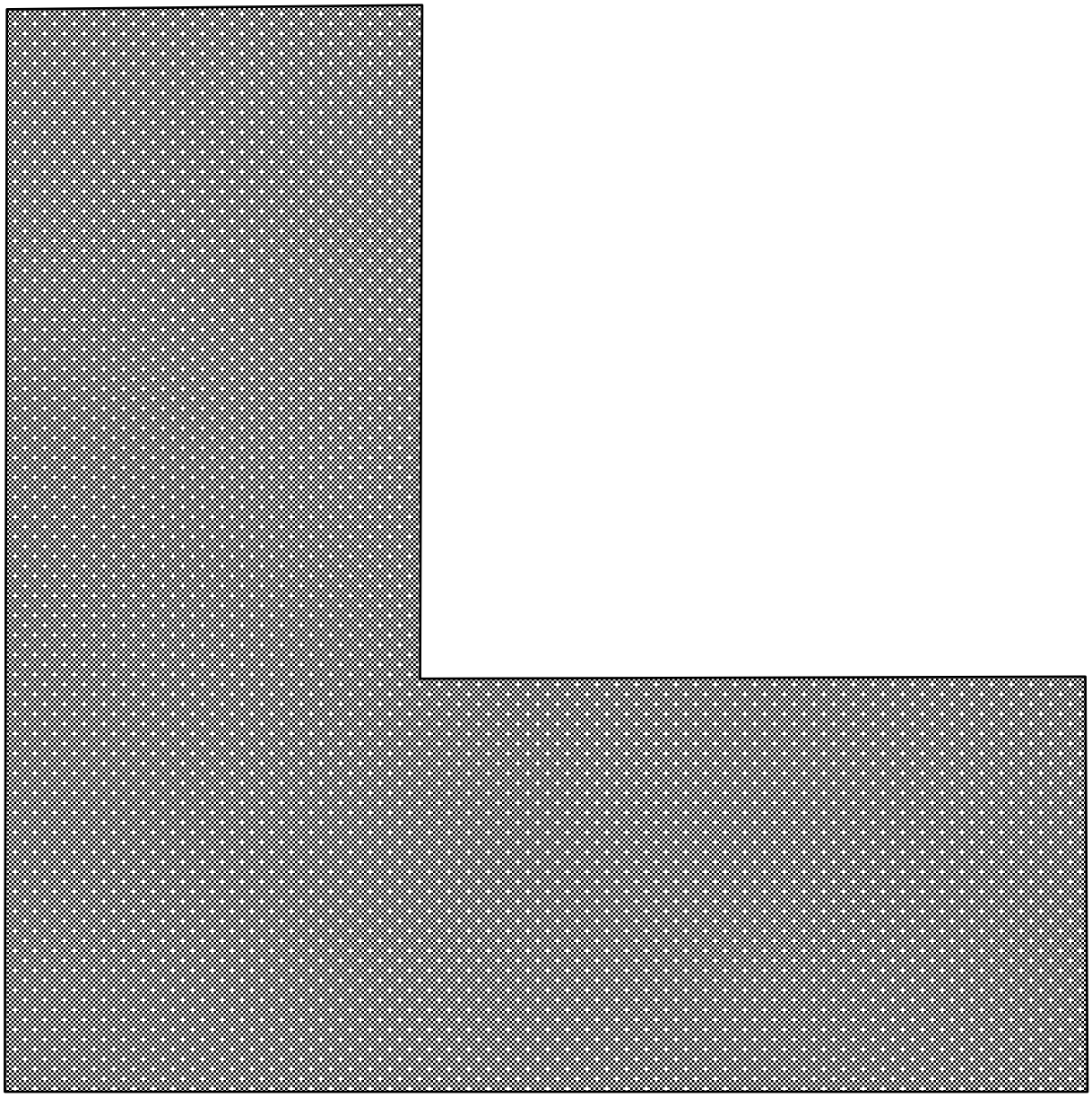

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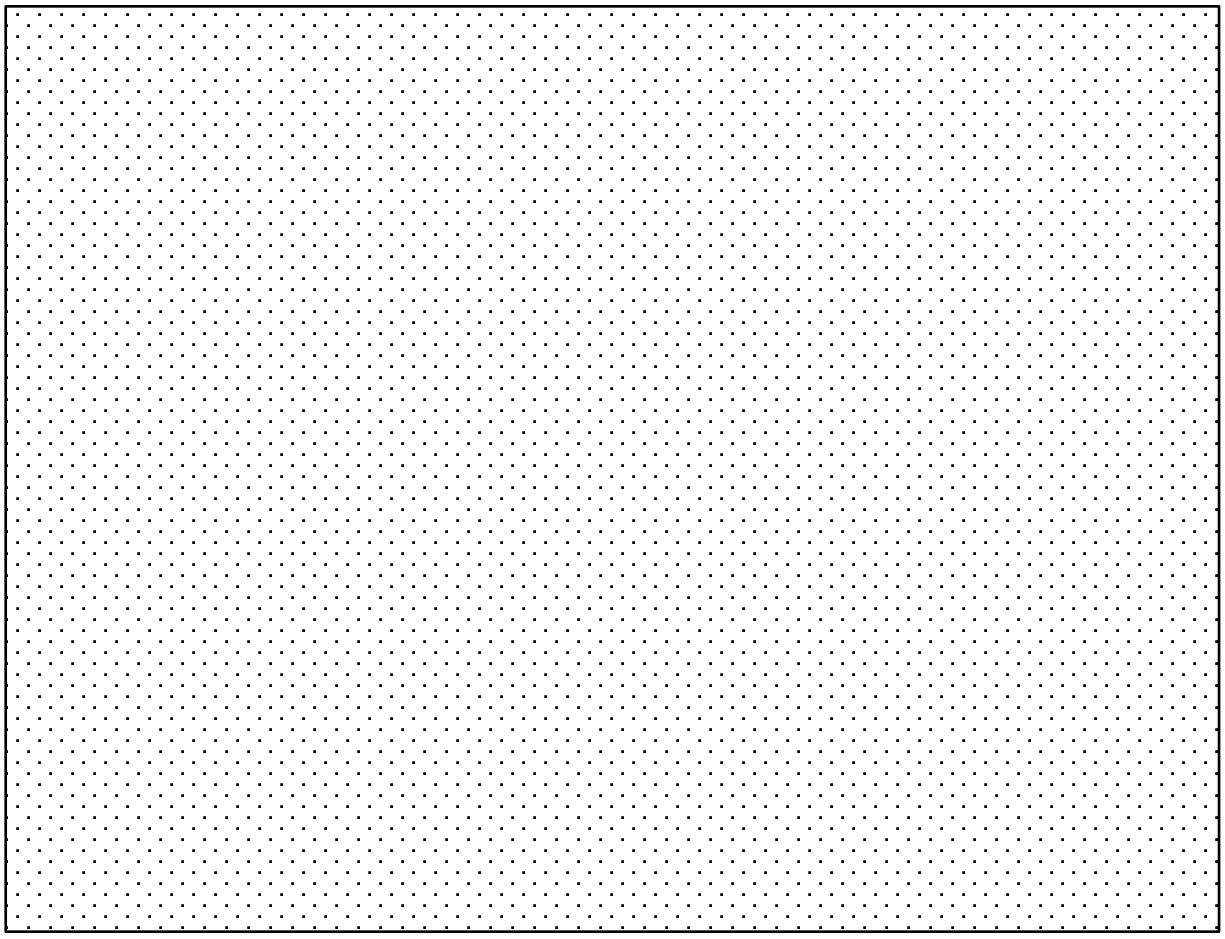

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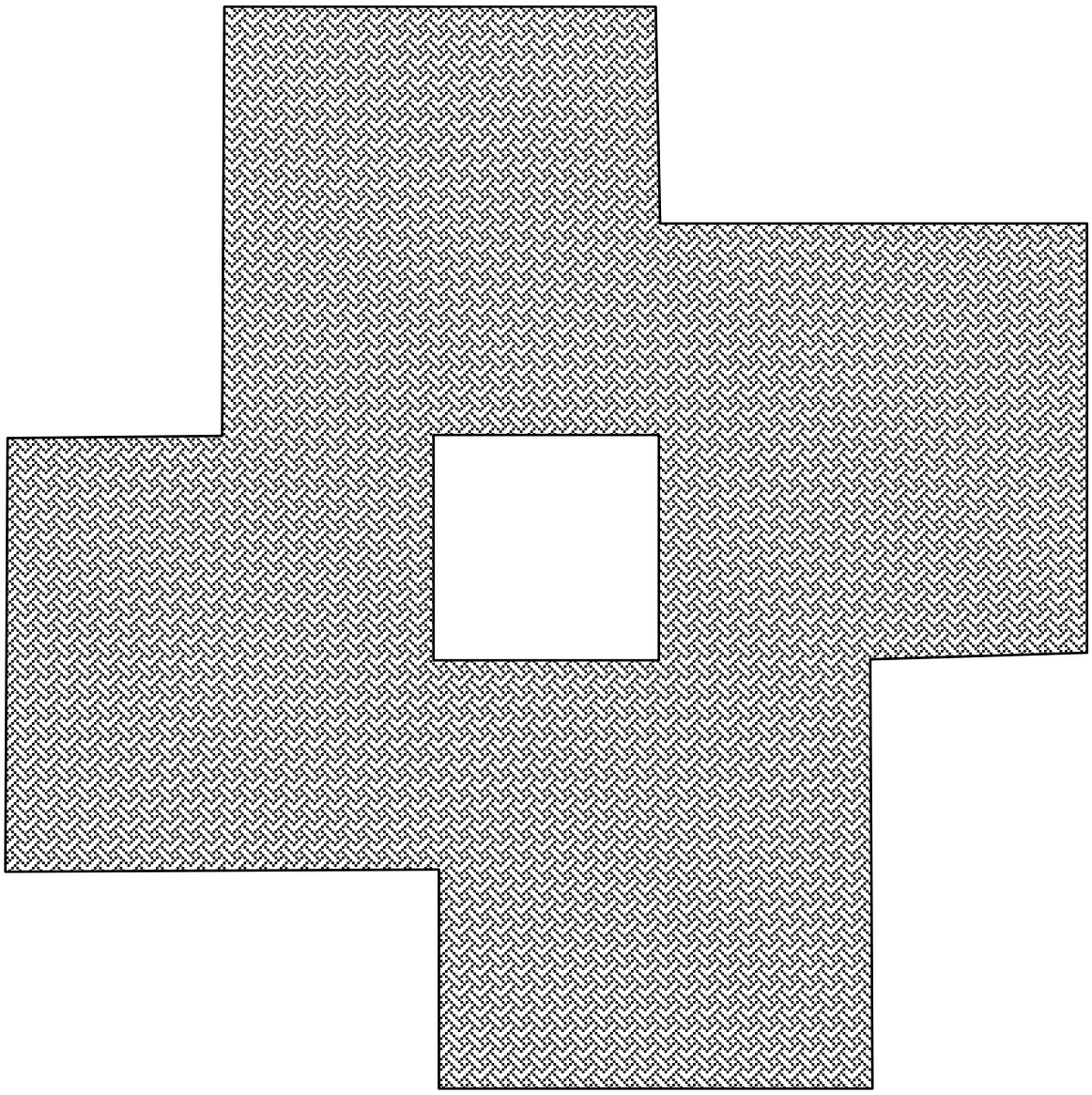

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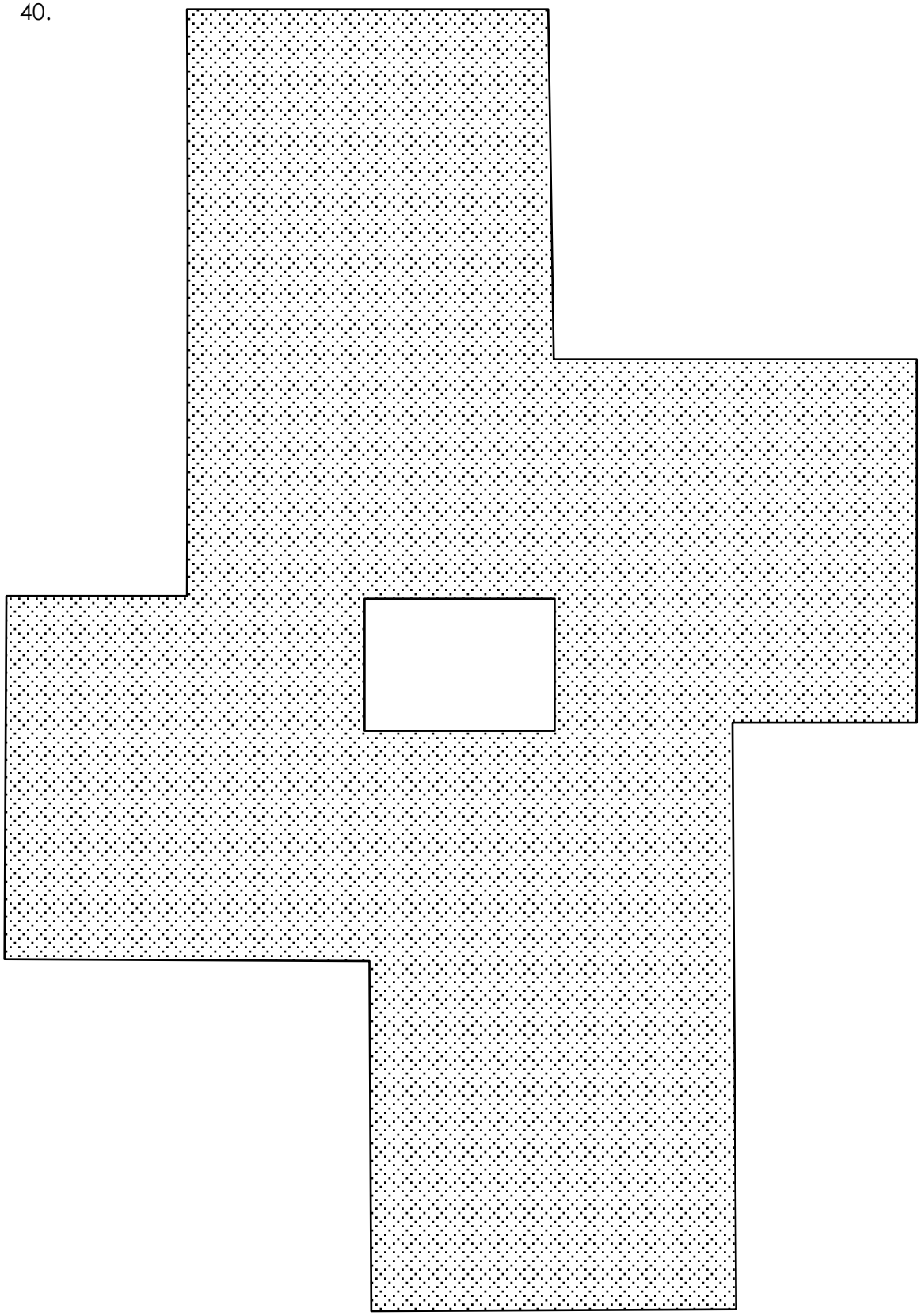

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Group B

Level 3

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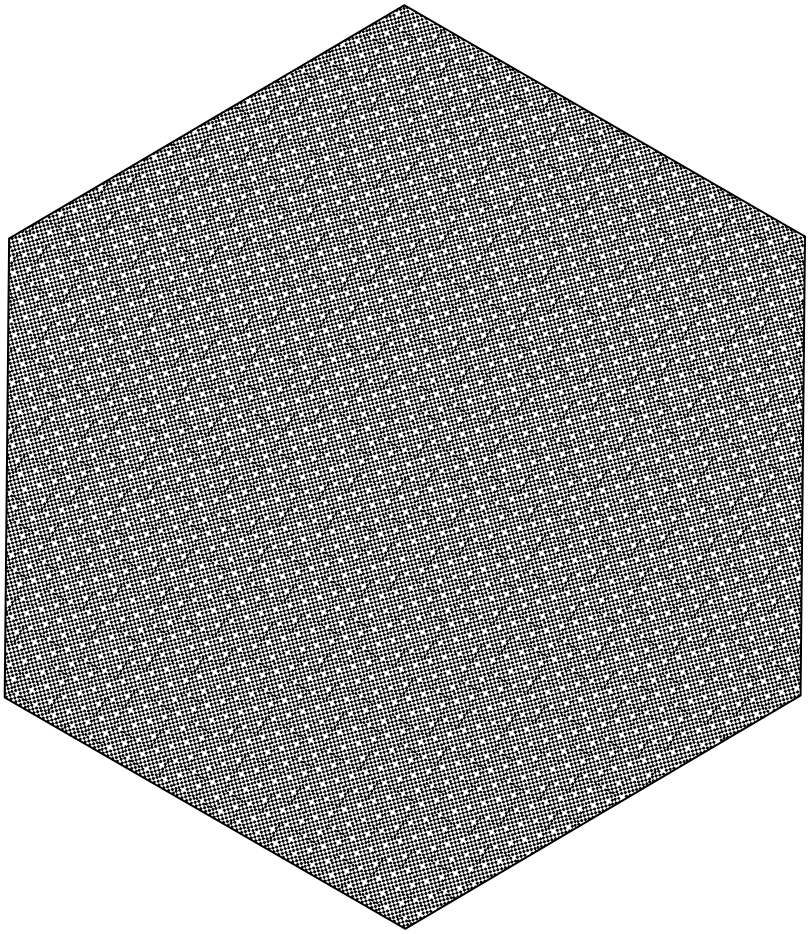

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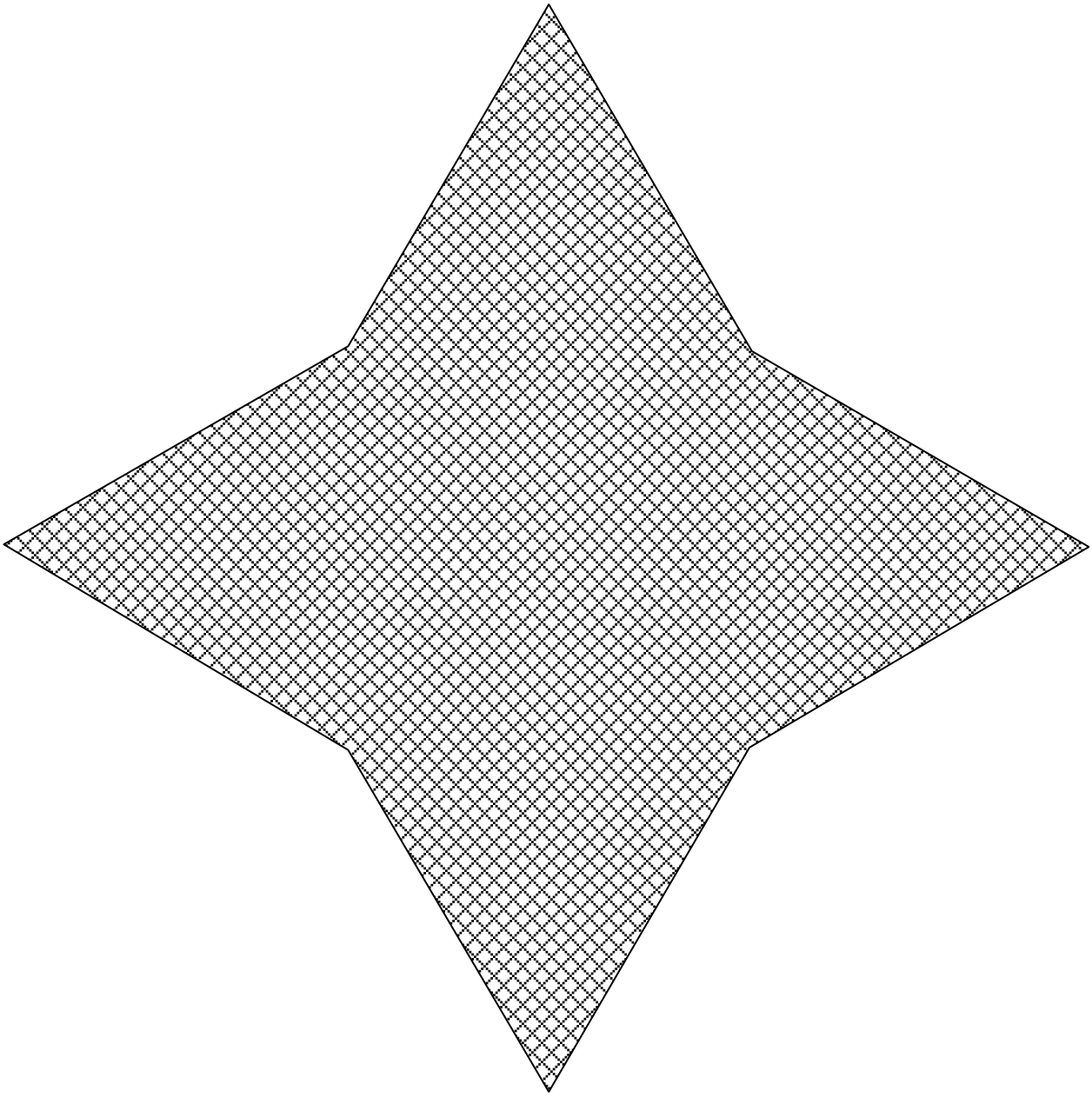

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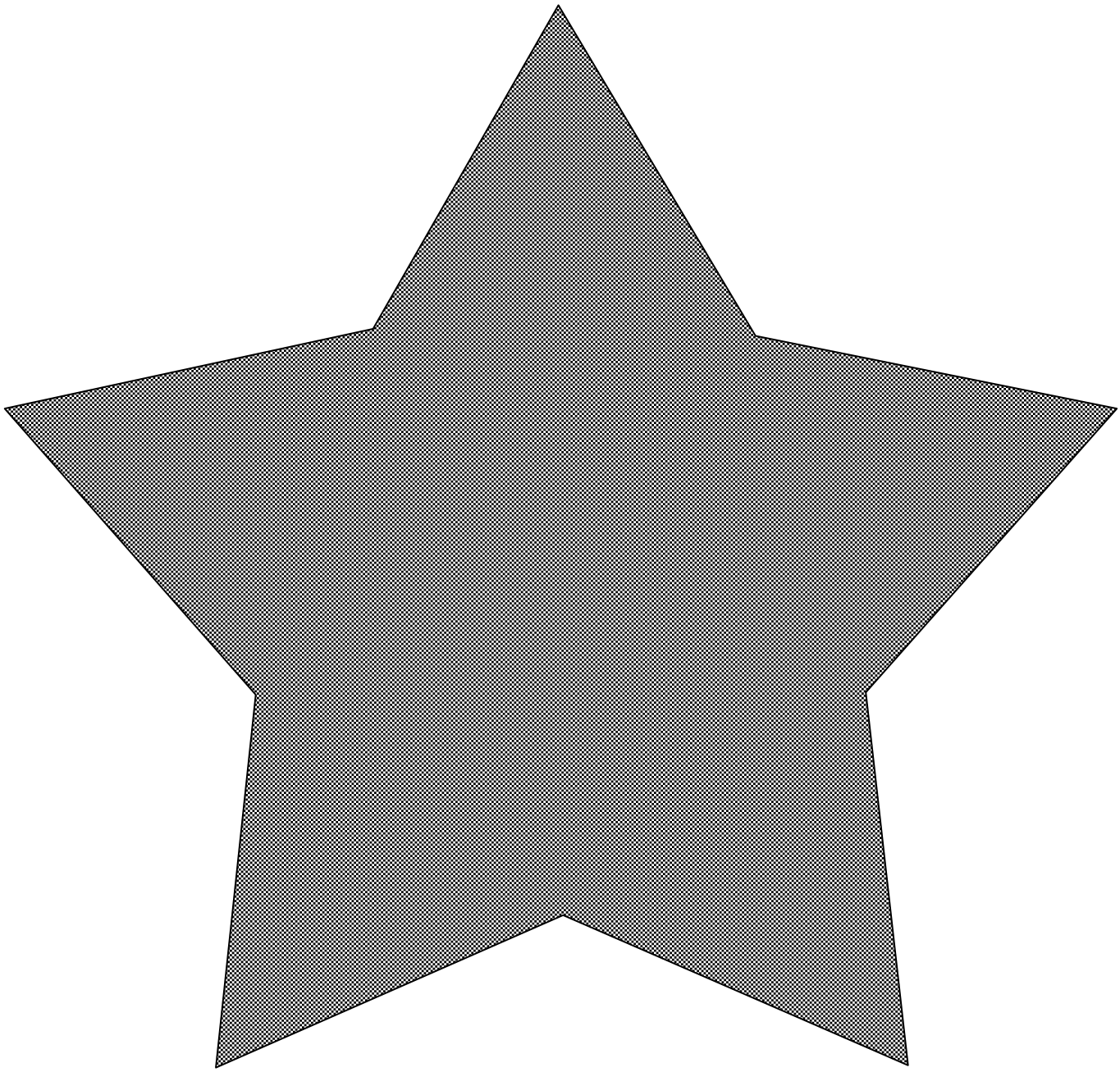

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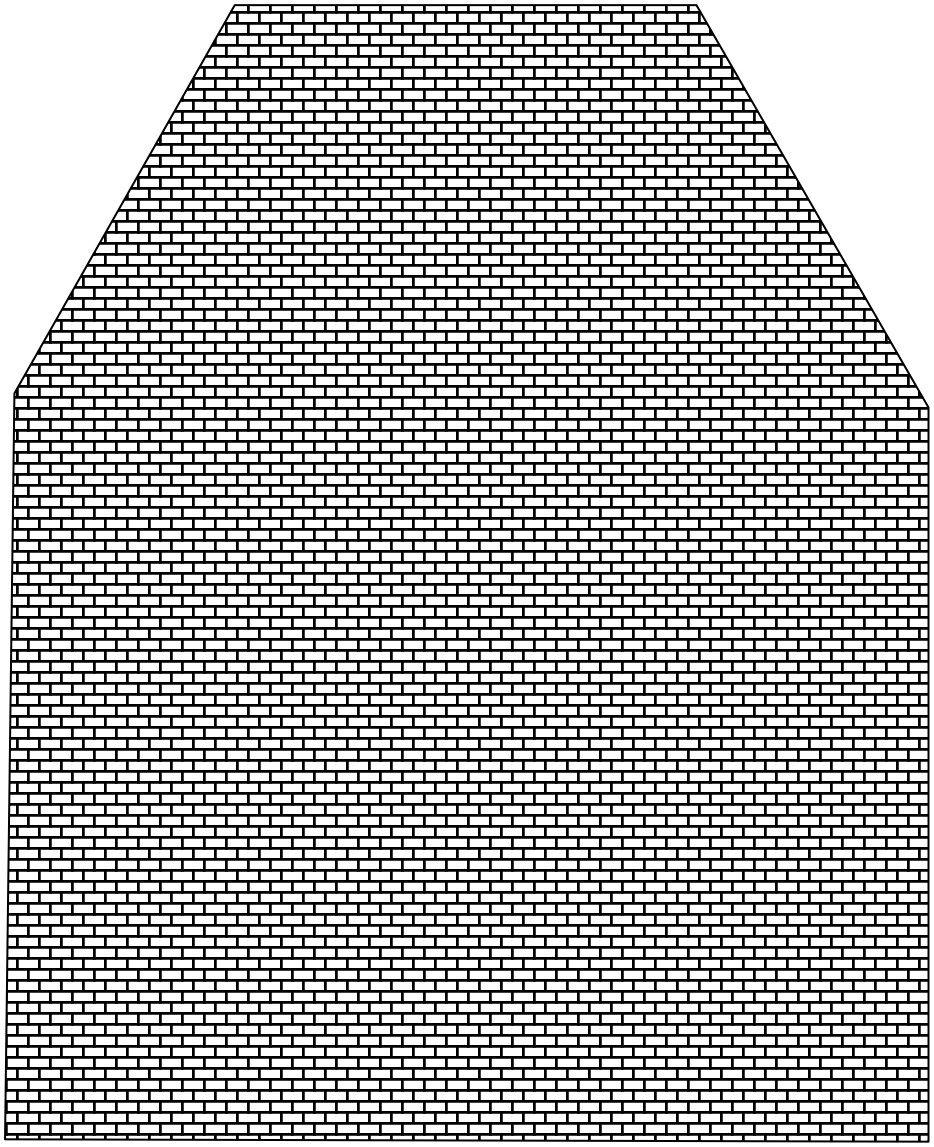

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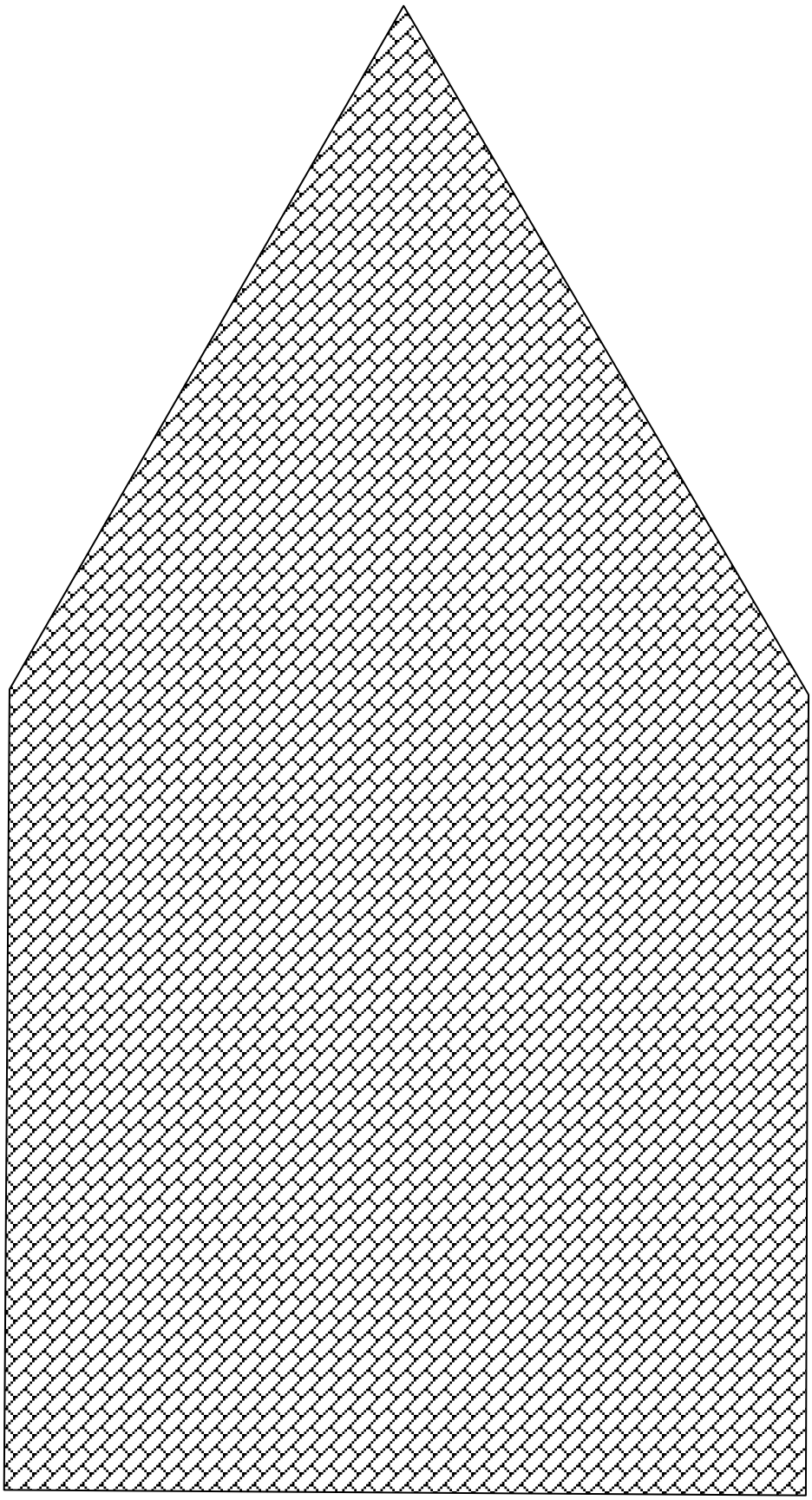

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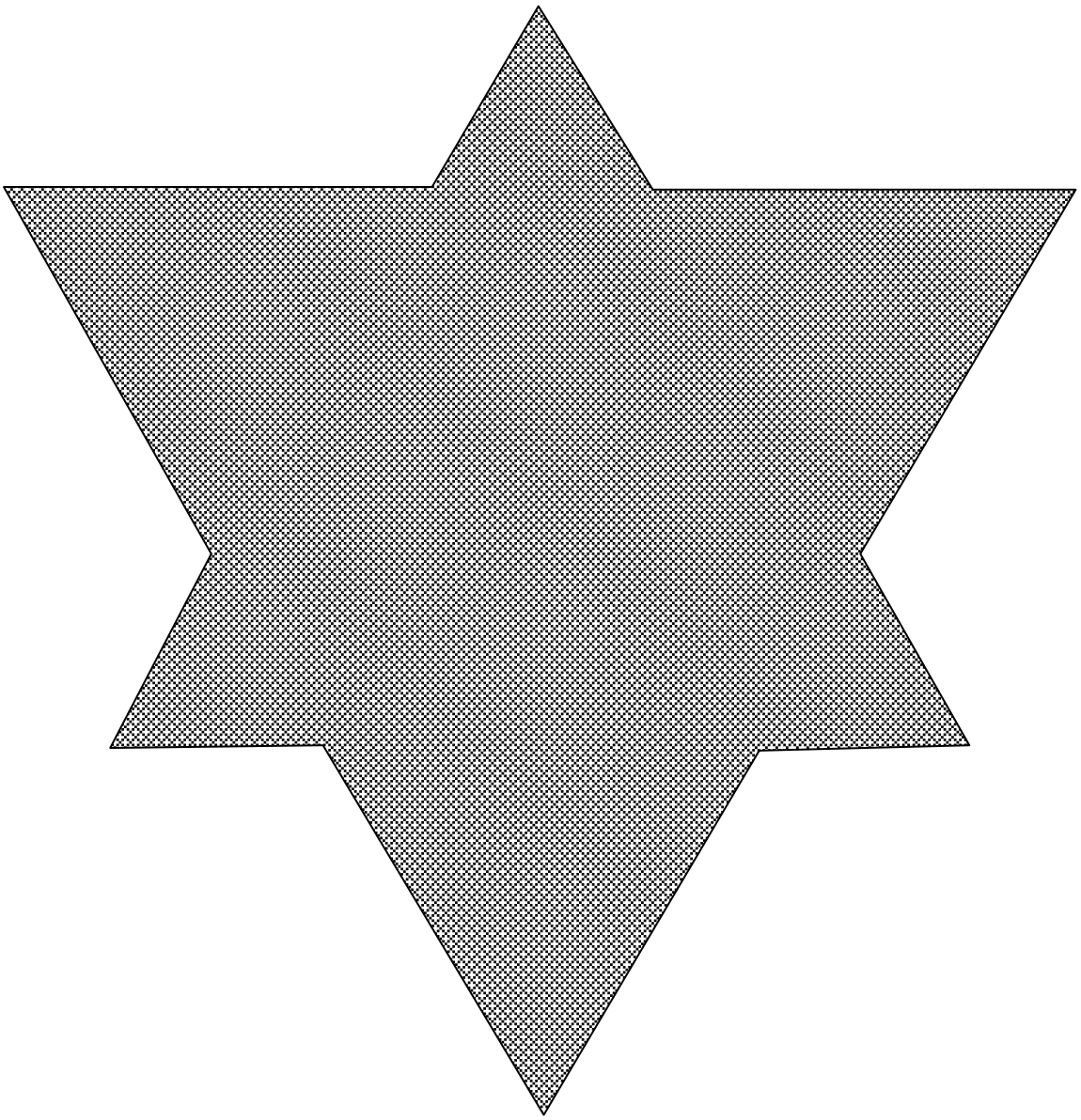

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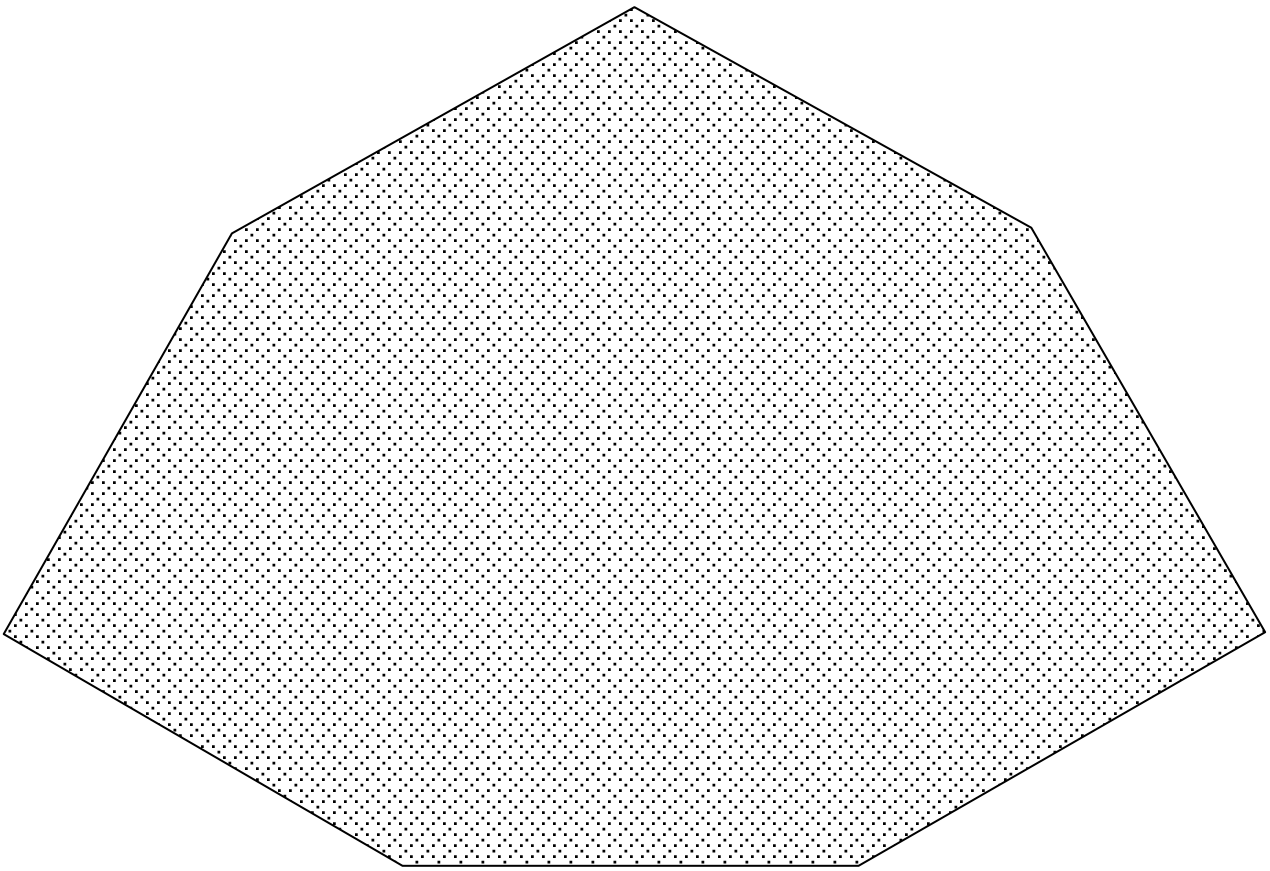
47.



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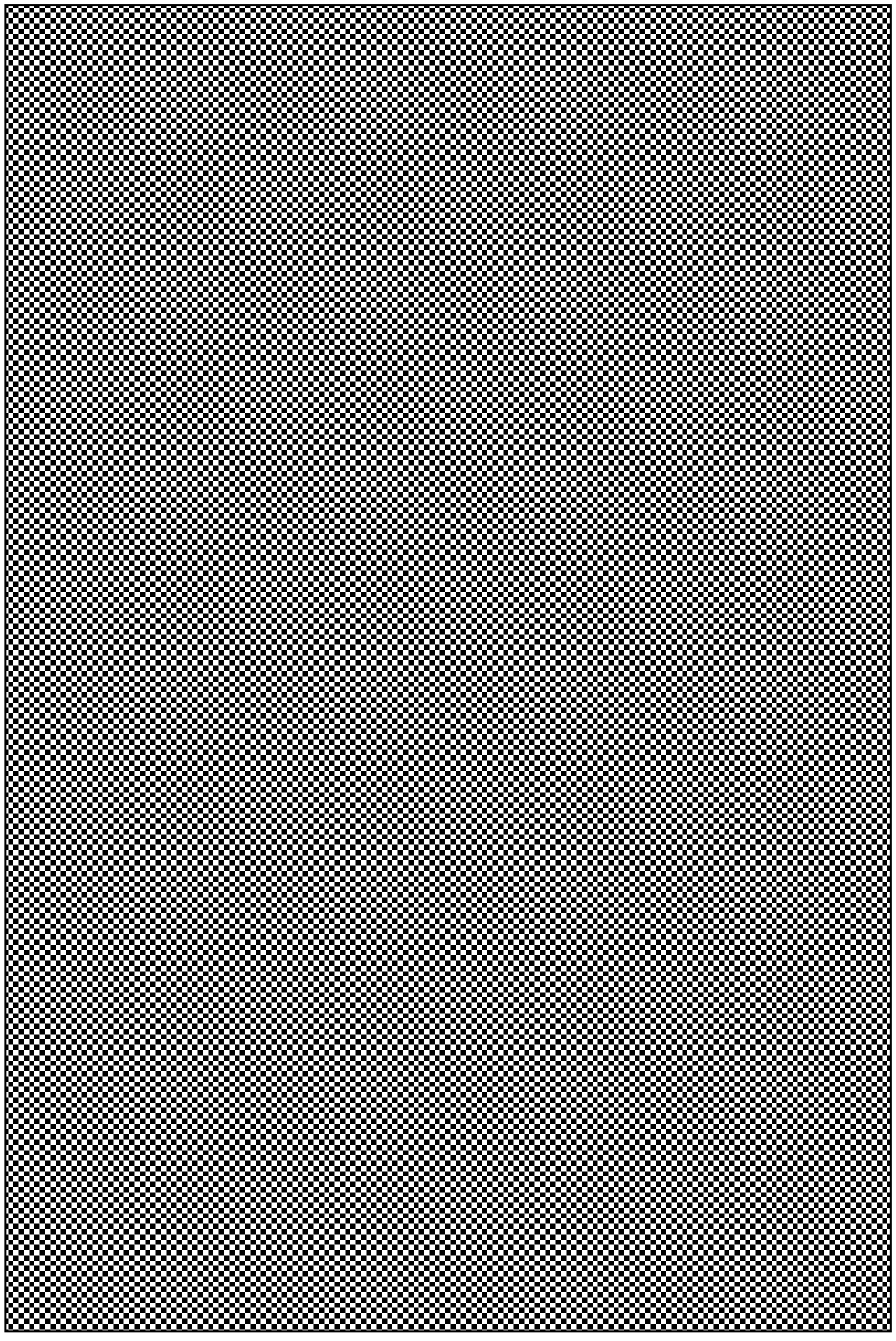

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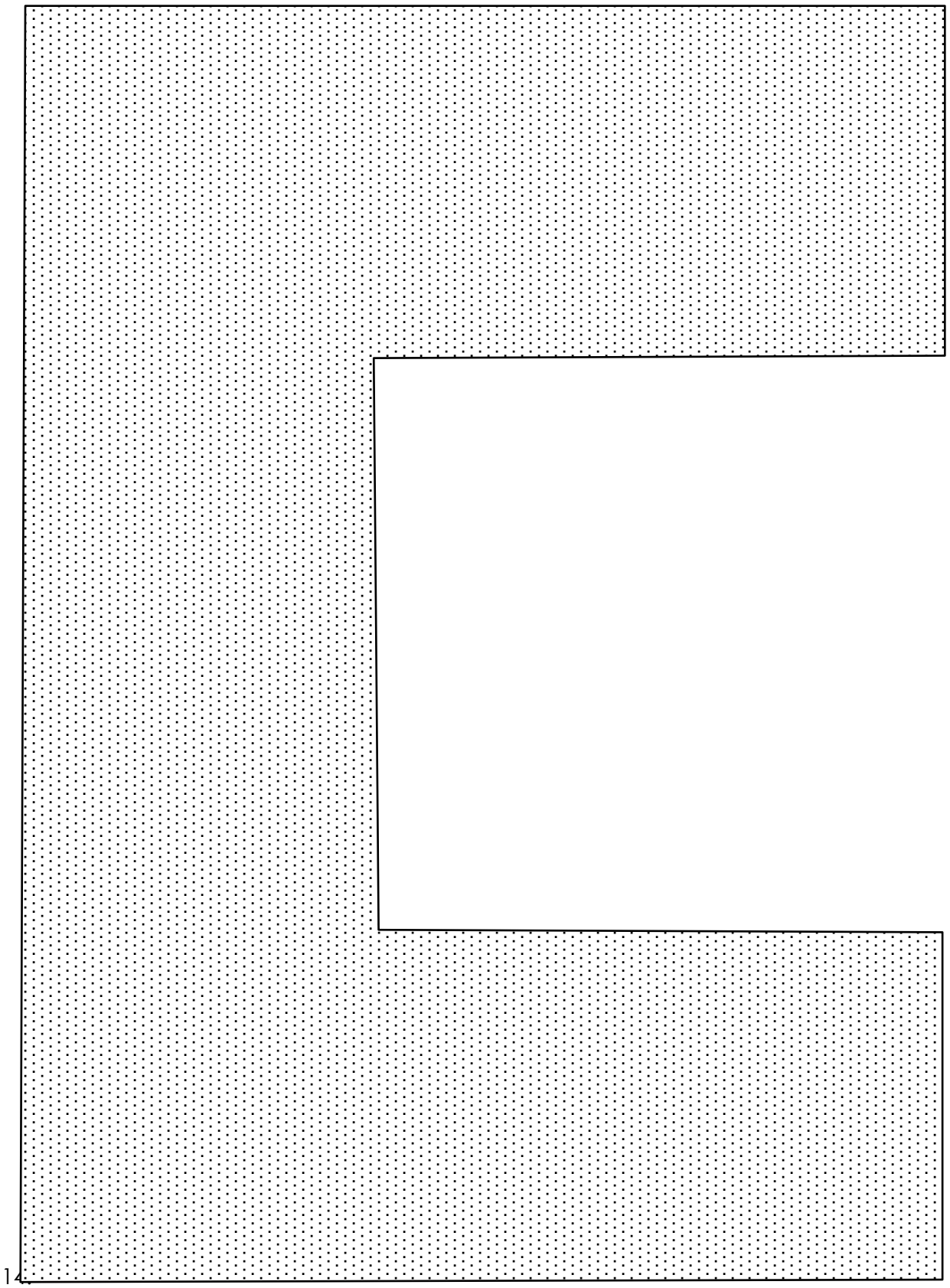

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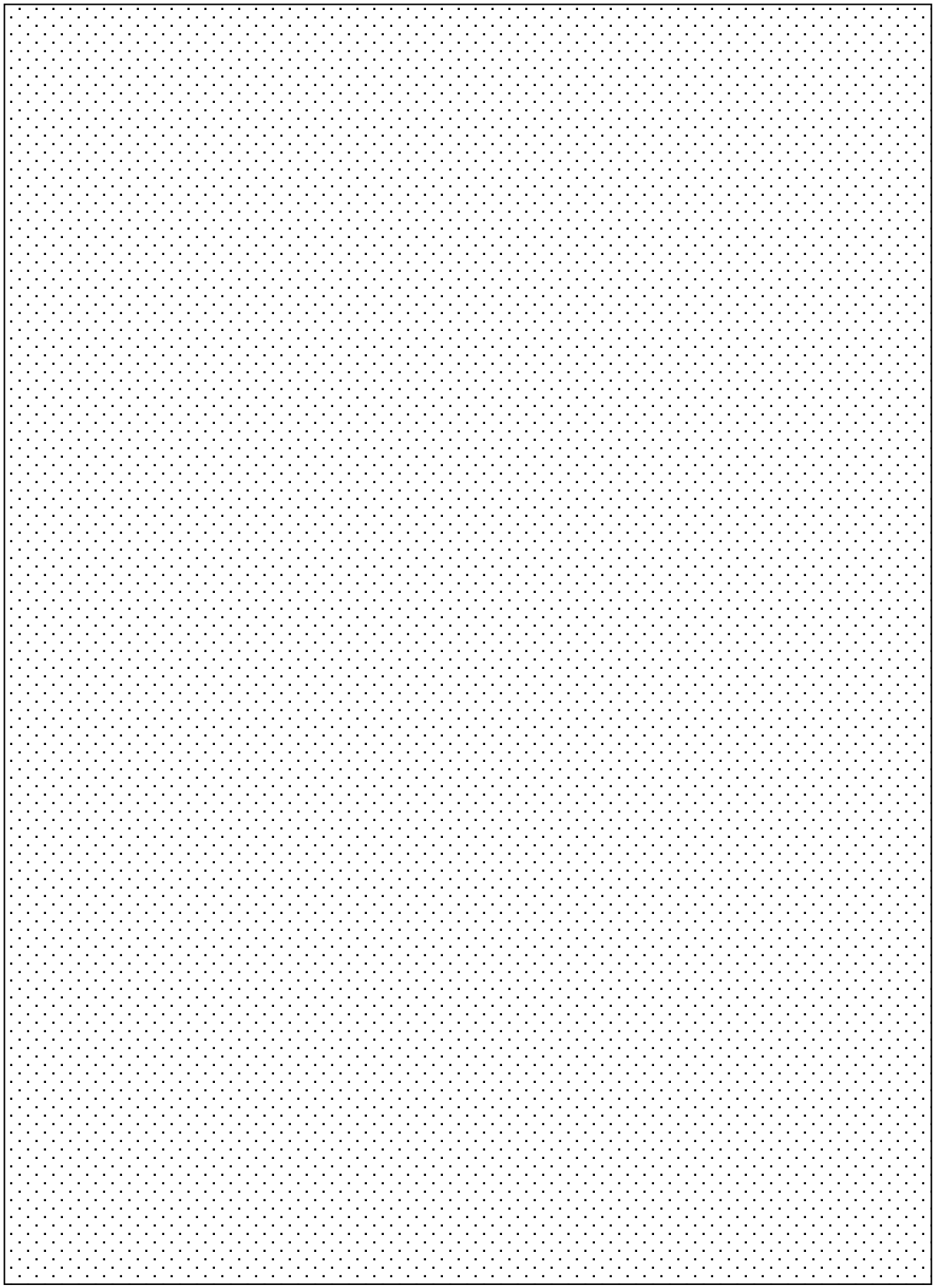

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ANSWER KEY

Note that colors of pieces in the answer key may be different from the actual colors you use.

Group A, Level 1

All puzzles in this group are solved with 2 pieces.

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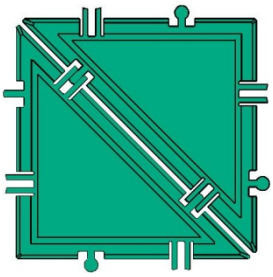

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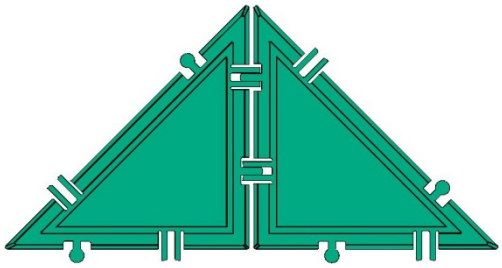

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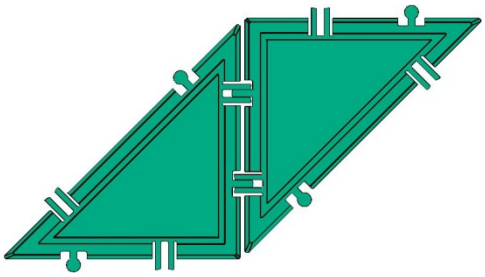

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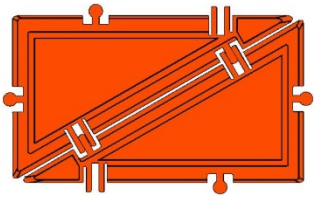

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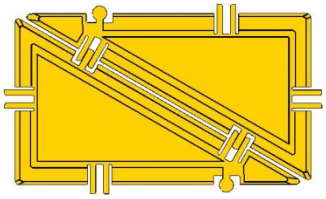

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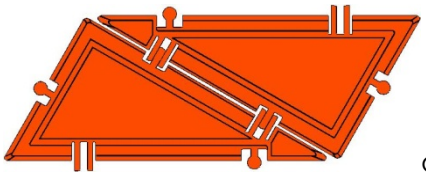

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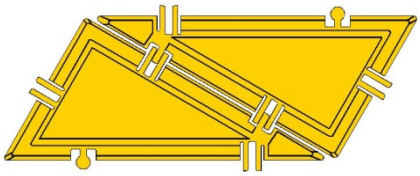

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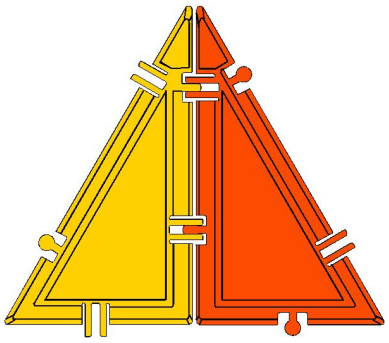

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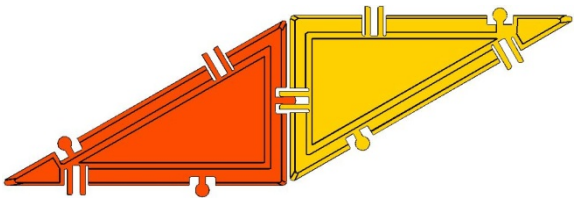

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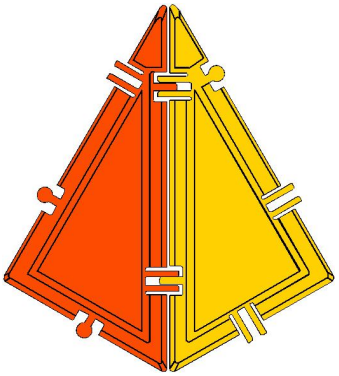

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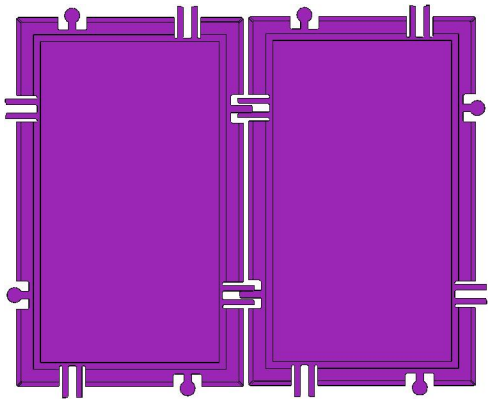

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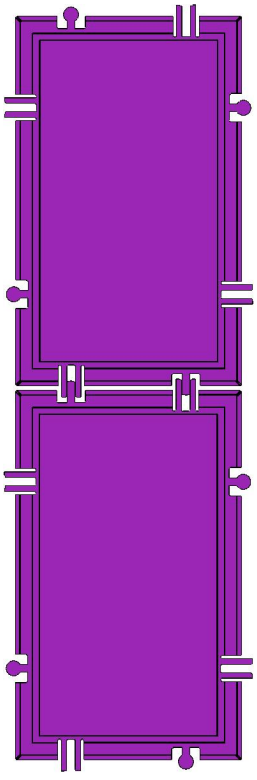

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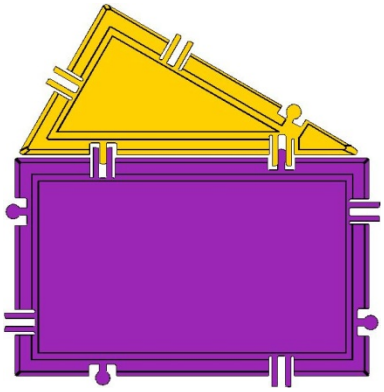

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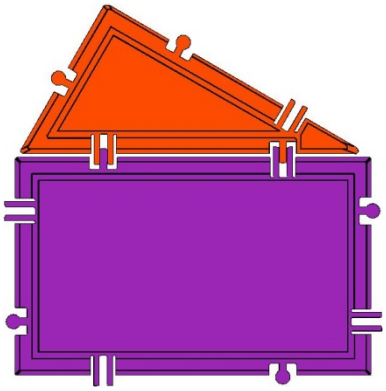

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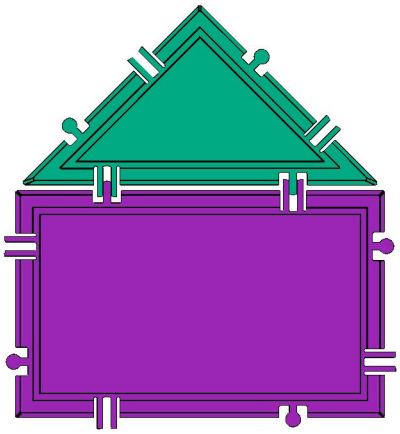

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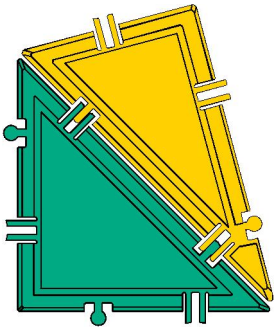

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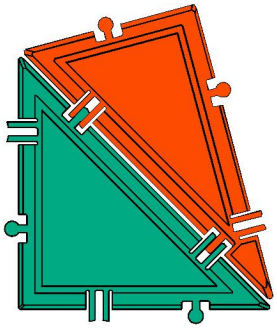

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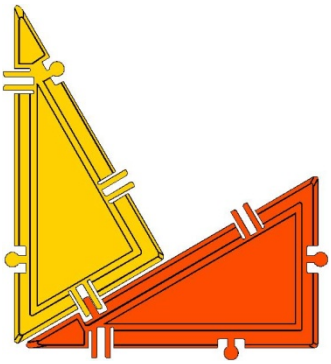

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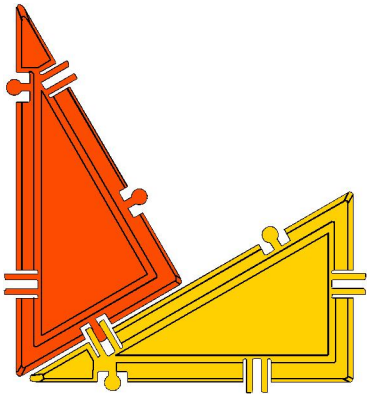

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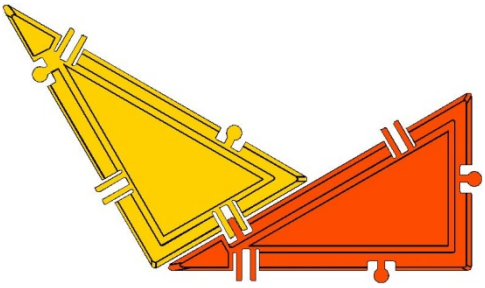
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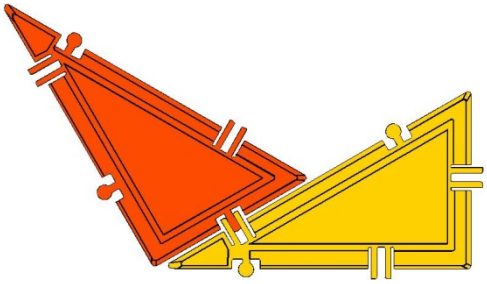

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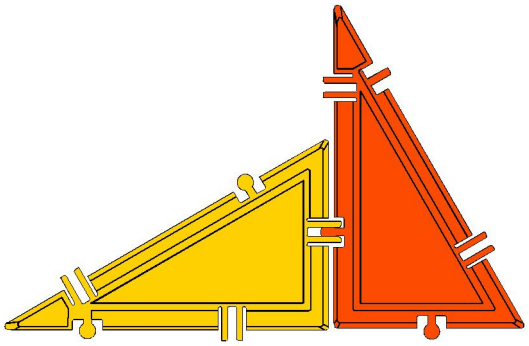
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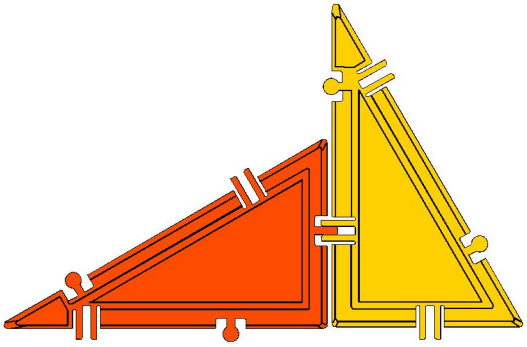

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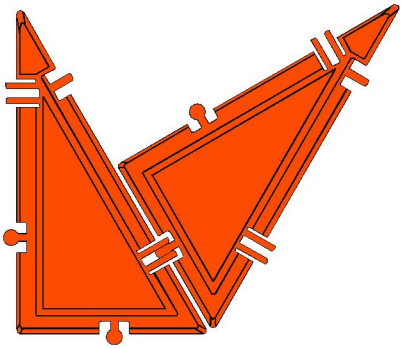
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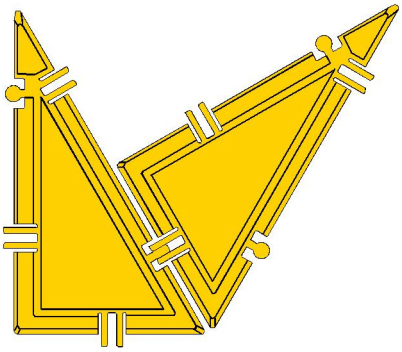

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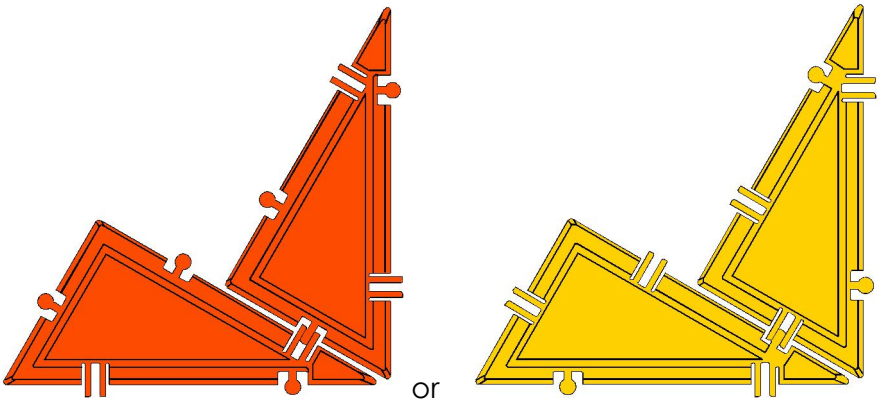

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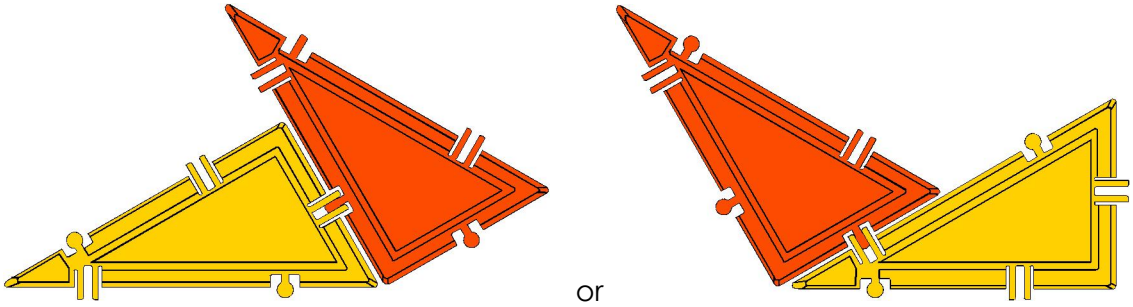

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Group A, Level 2

All puzzles in this group are solved with 3 or 4 pieces.

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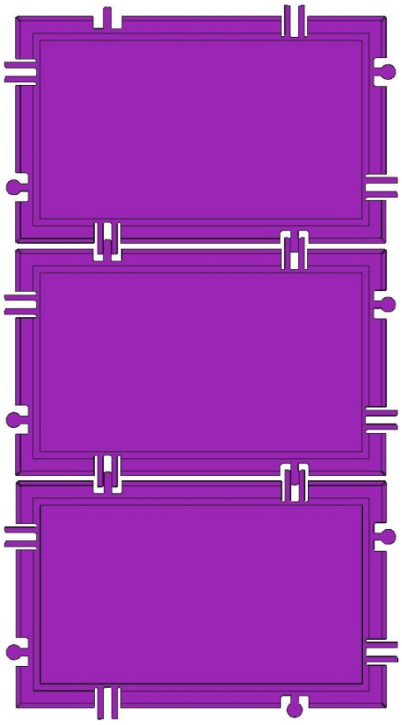

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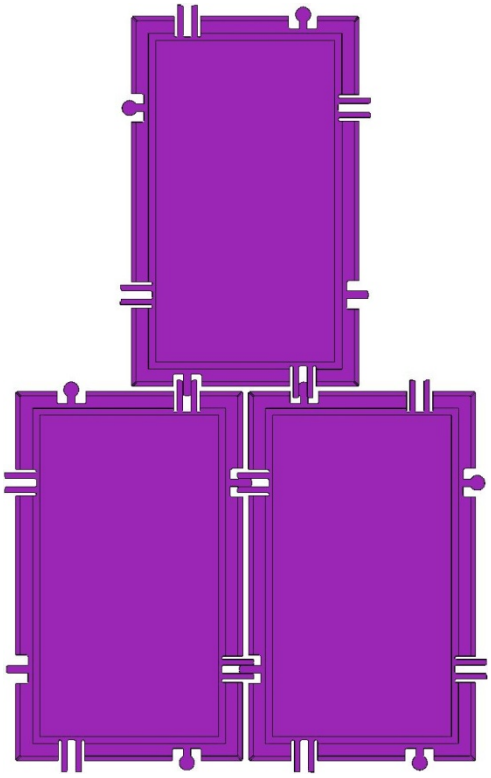

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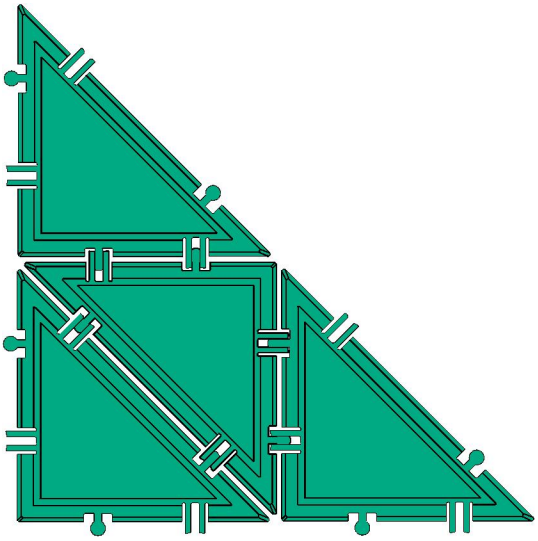

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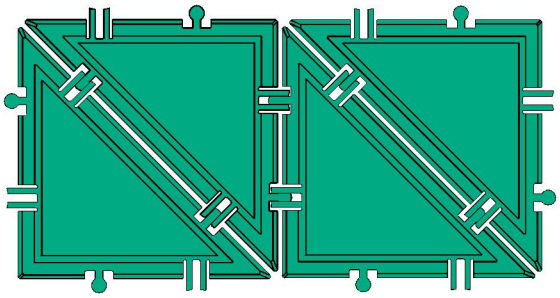

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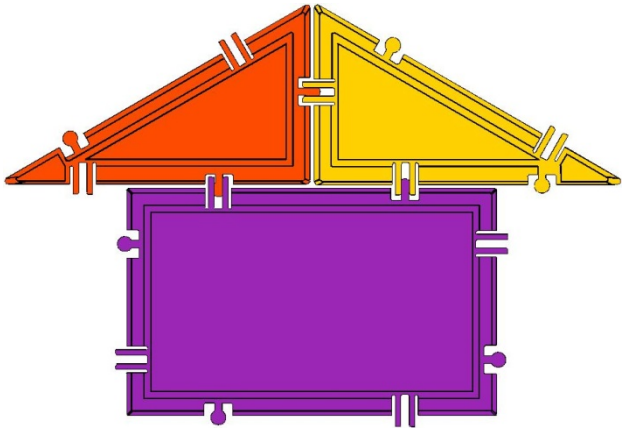

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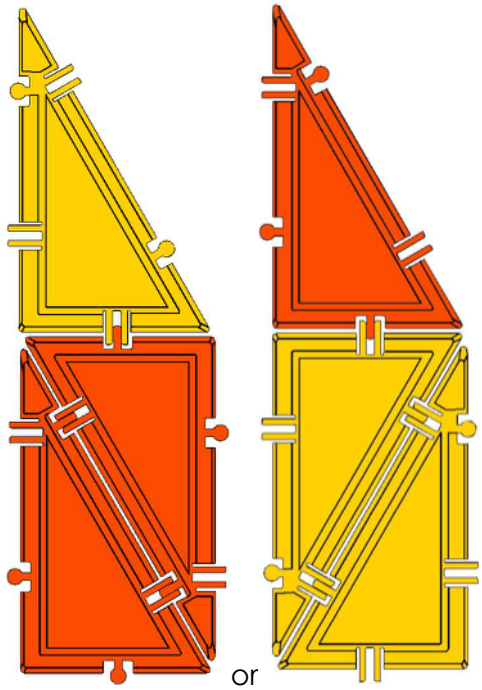
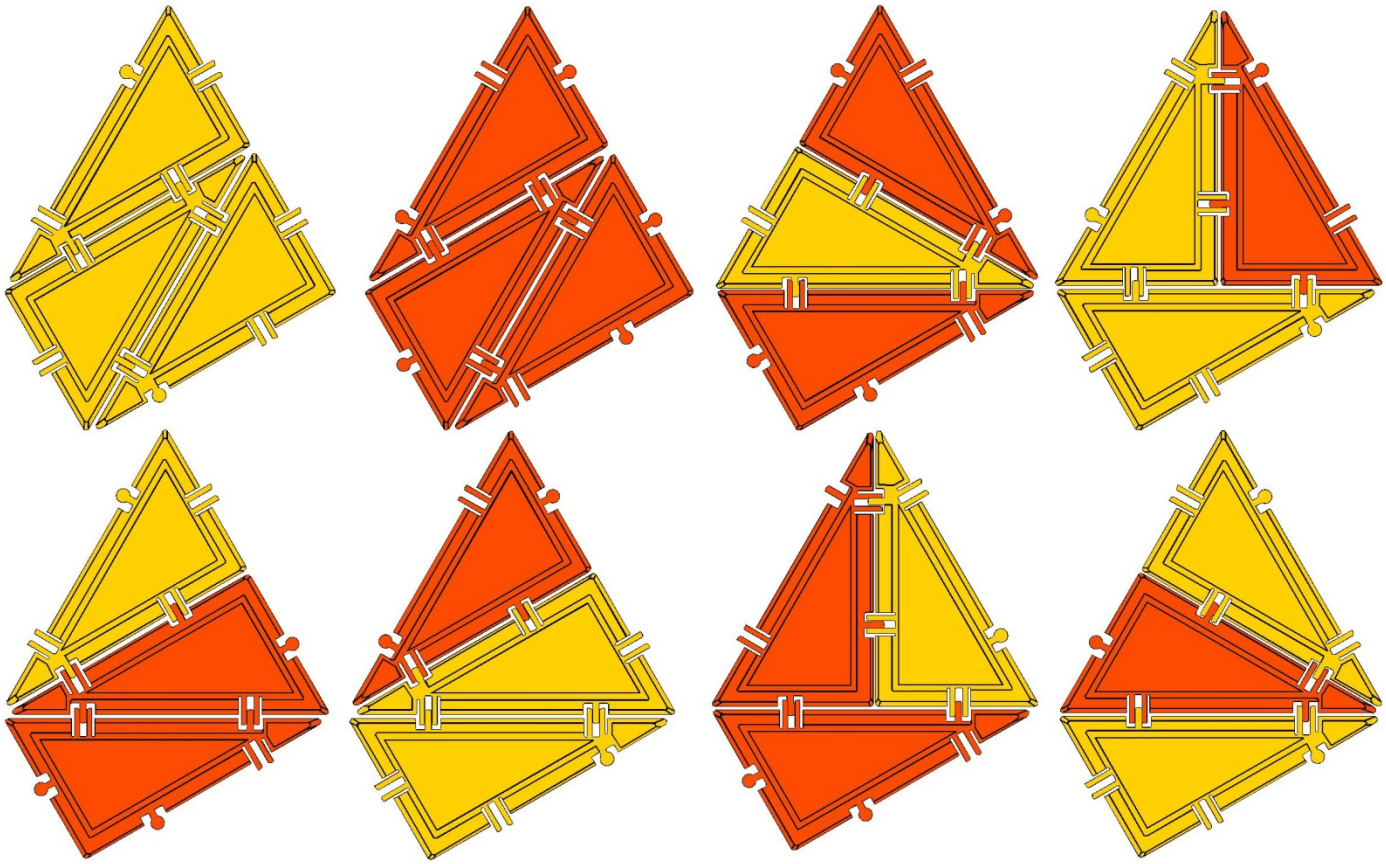


27. 8 possible constructions:



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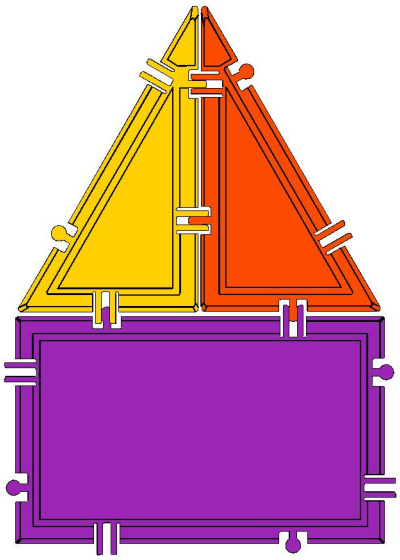
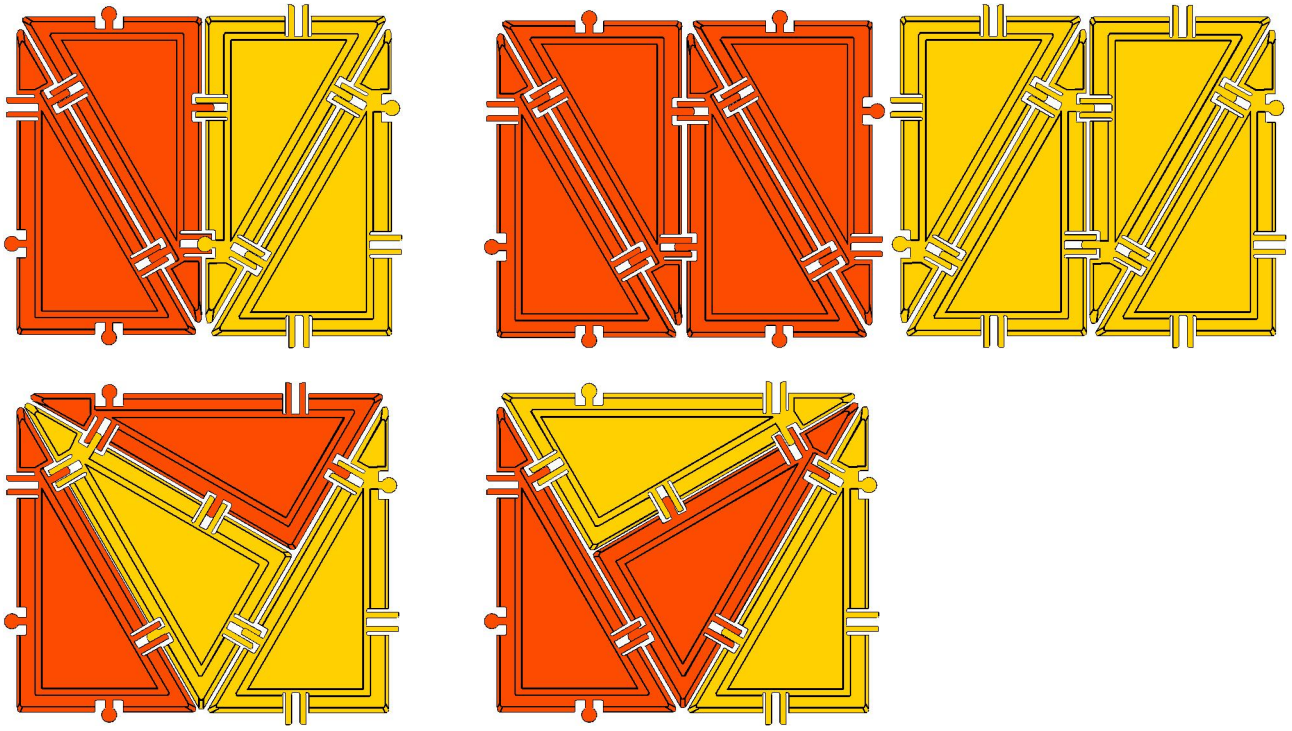


29. 5 possible constructions:



Notice that you can take that all of these can be obtained from the solutions to problem 27 by adding one missing piece that makes a rectangle. In some cases, two different solutions to problem 27 will give you the same rectangle; that's why there are 8 possible solutions to problem 27 and only 5 possible solutions to problem 29.

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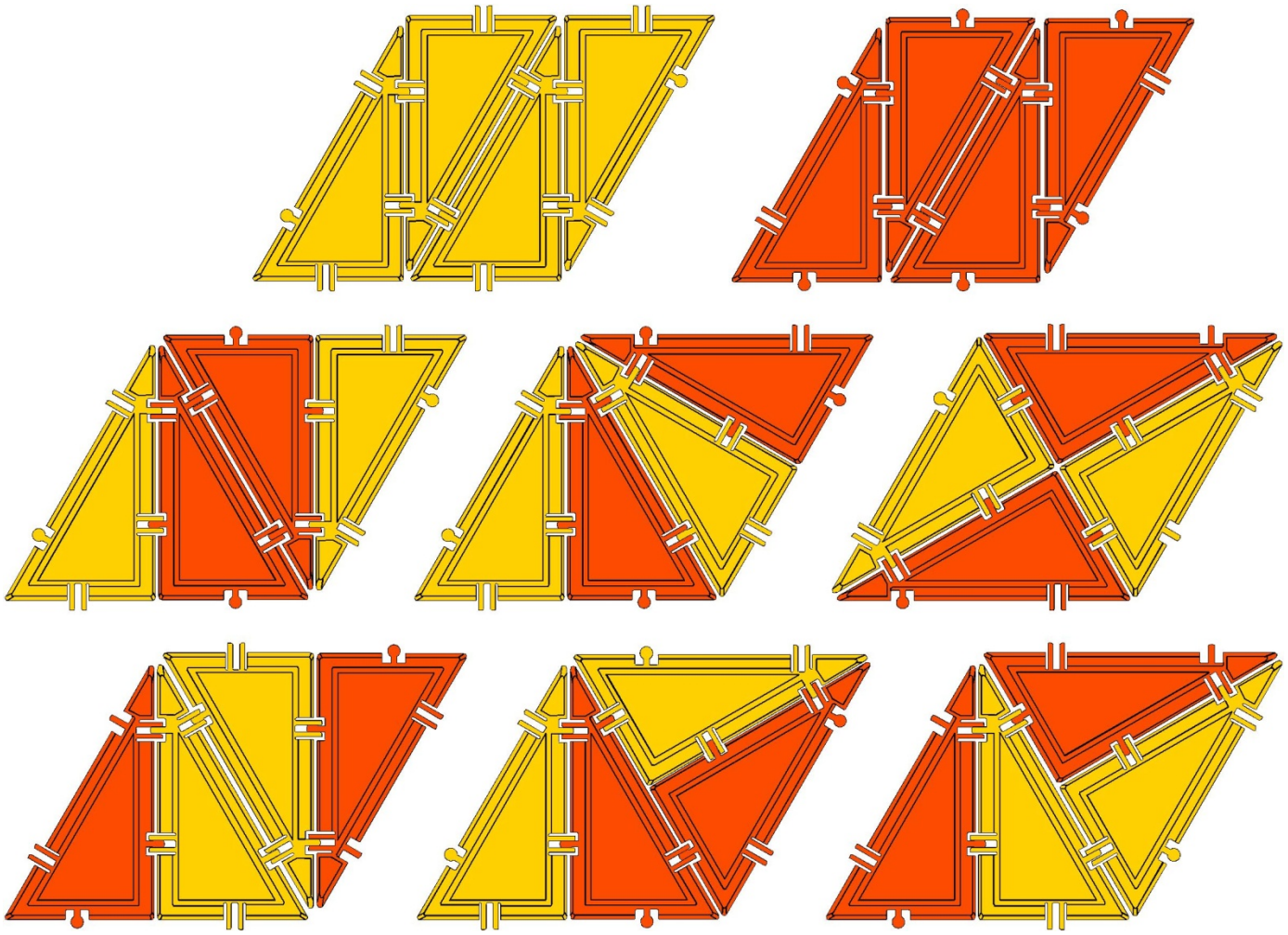
30. 8 possible constructions:



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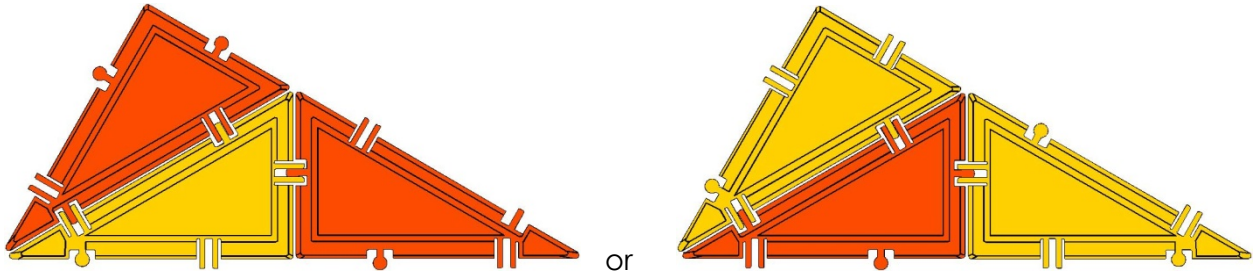

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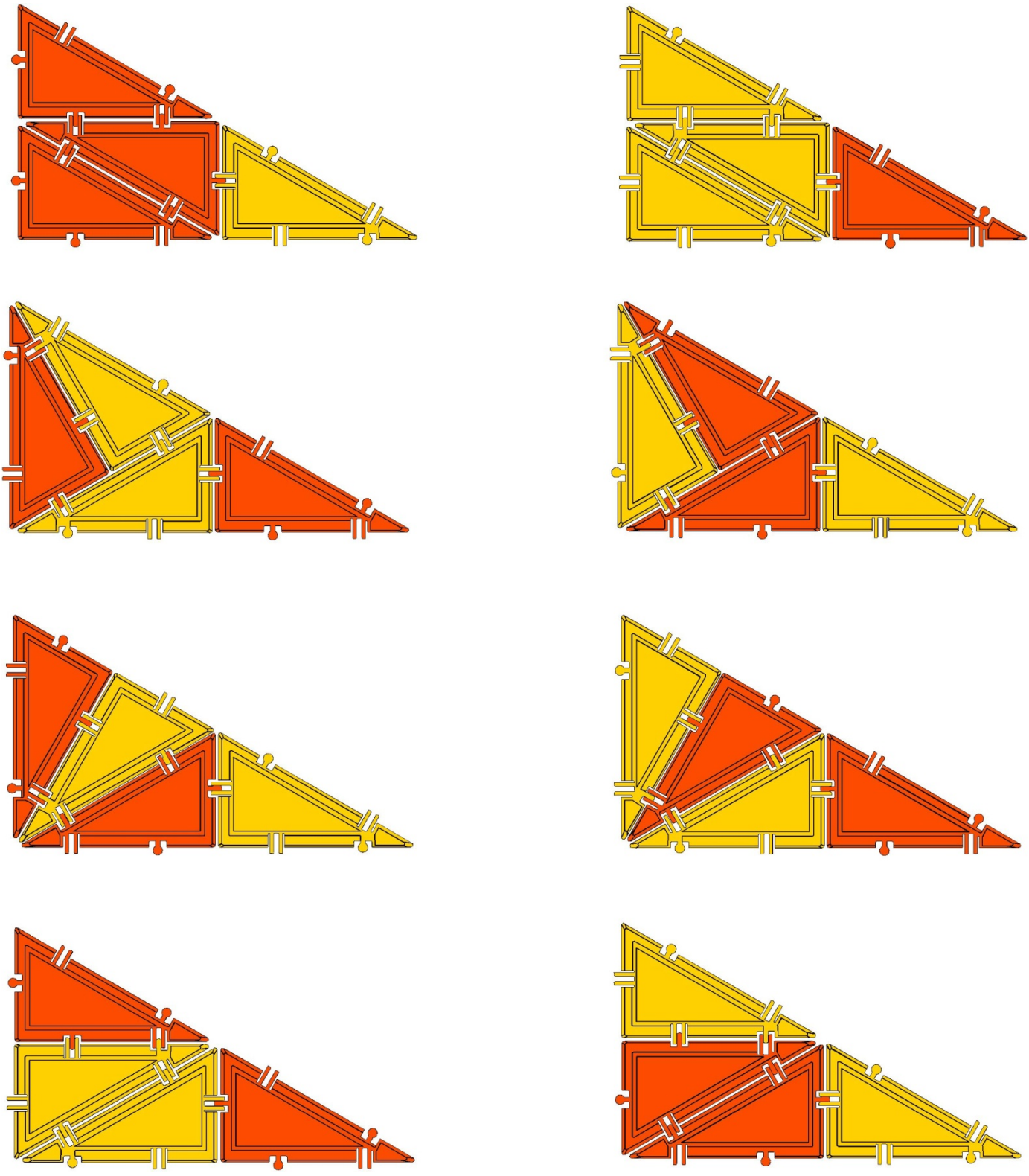
32. 8 possible constructions:



Note that all of these answers can be obtained by adding a single triangle to the answers to problem 27.

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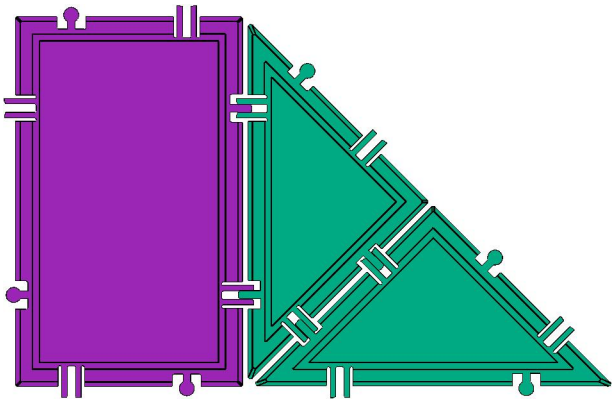
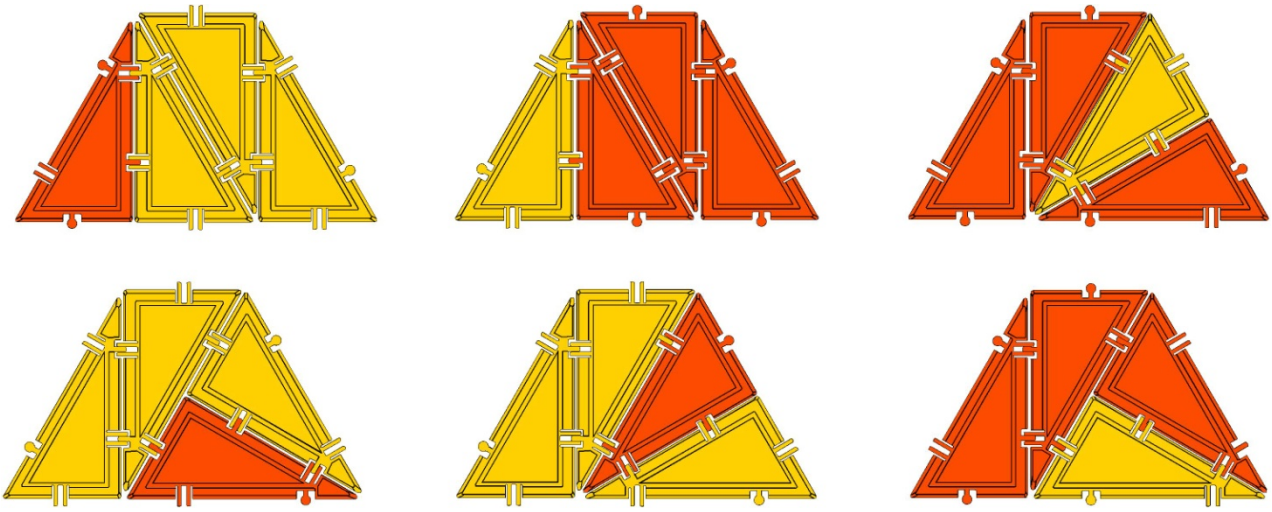
33.



34. 6 possible constructions:



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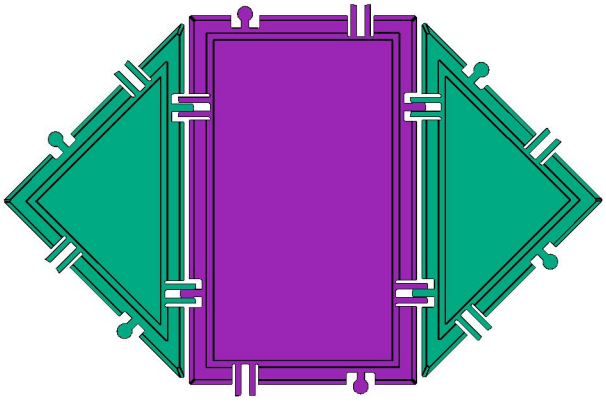

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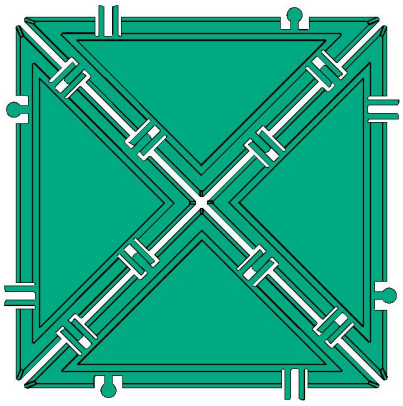

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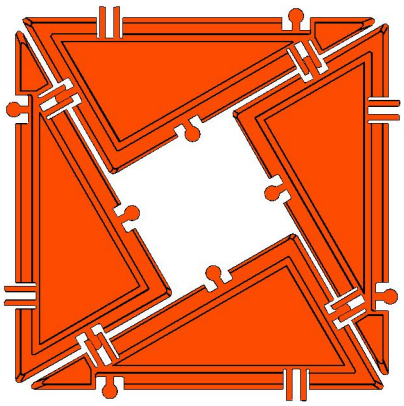

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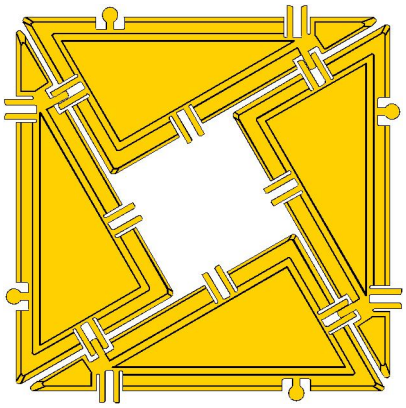

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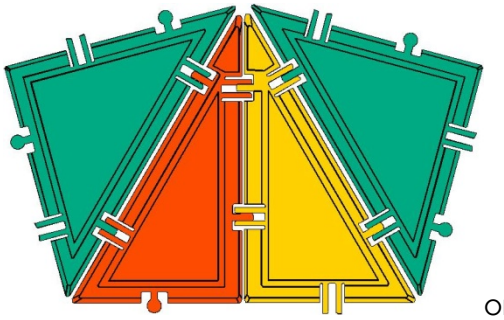

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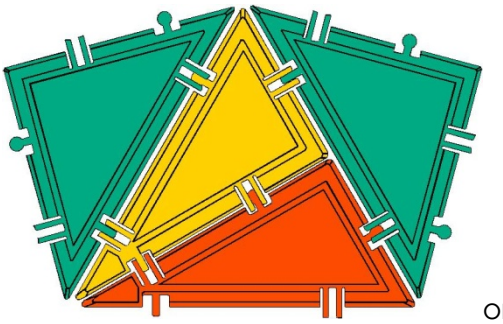

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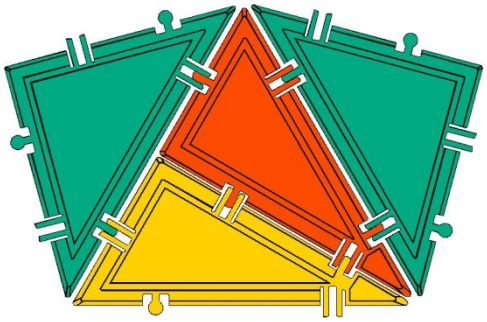

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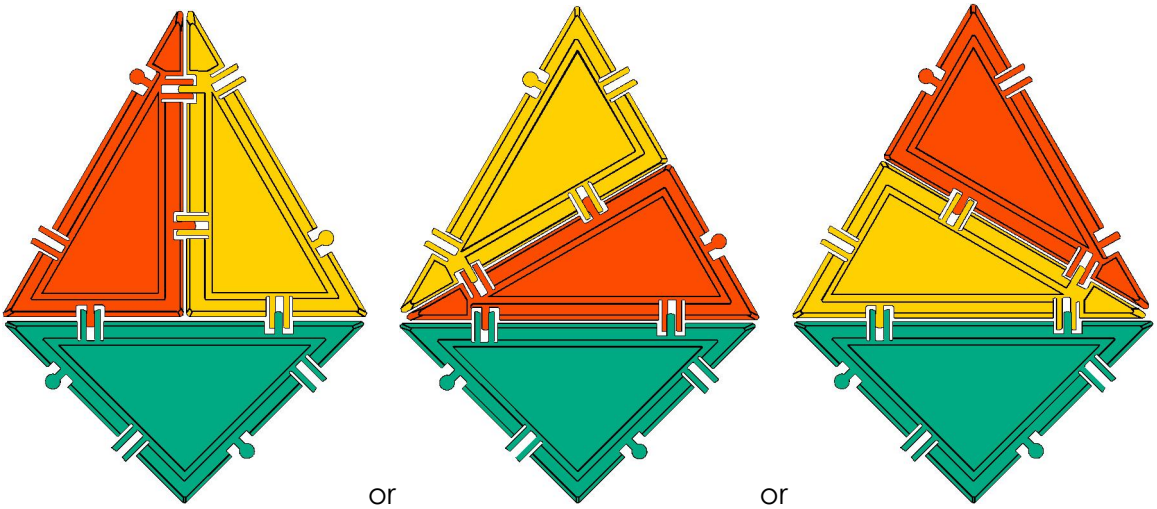

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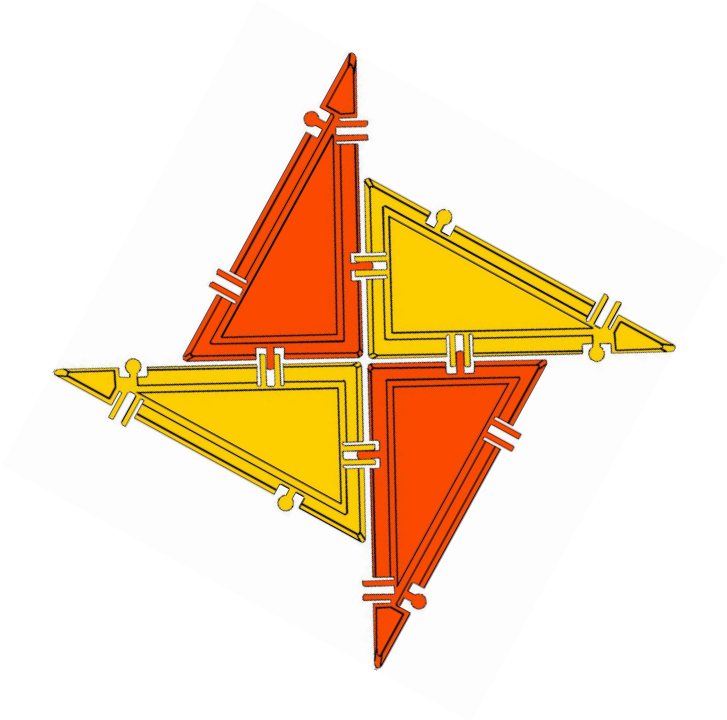

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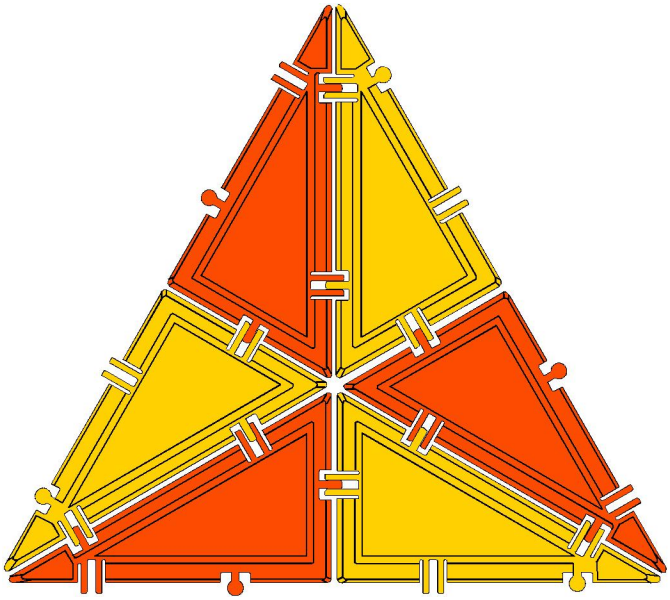
Group A, Level 3

All puzzles in this group are solved with 5 or more pieces.

41.



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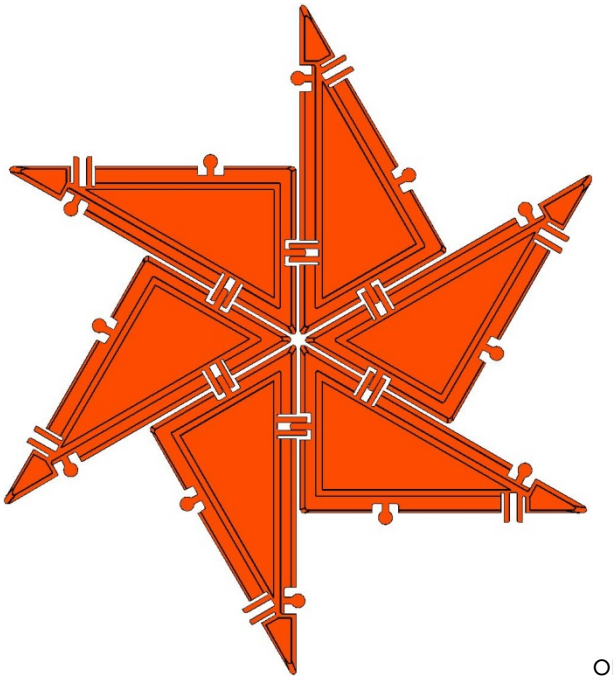

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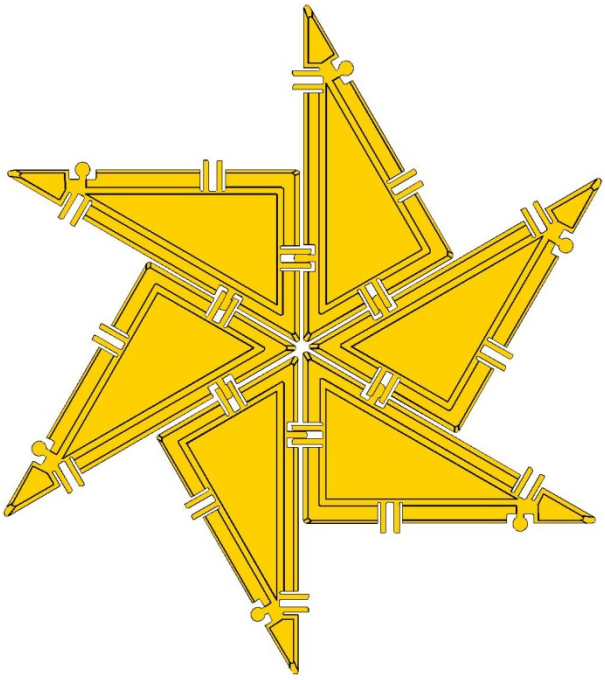

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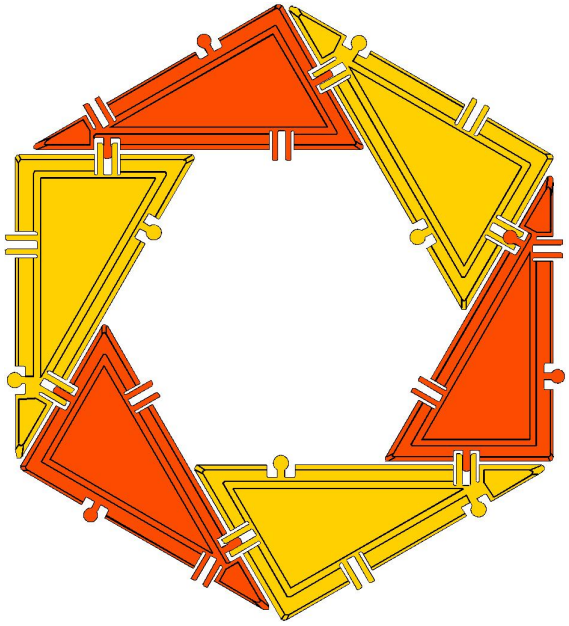
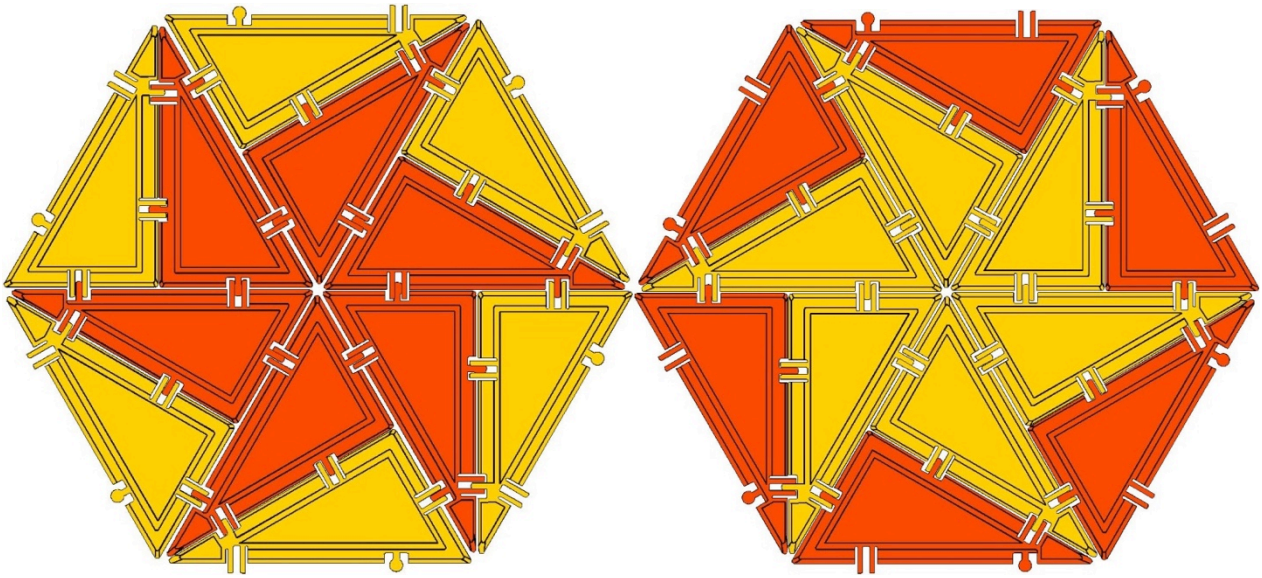


44, There are many possible solutions here; here are some examples:



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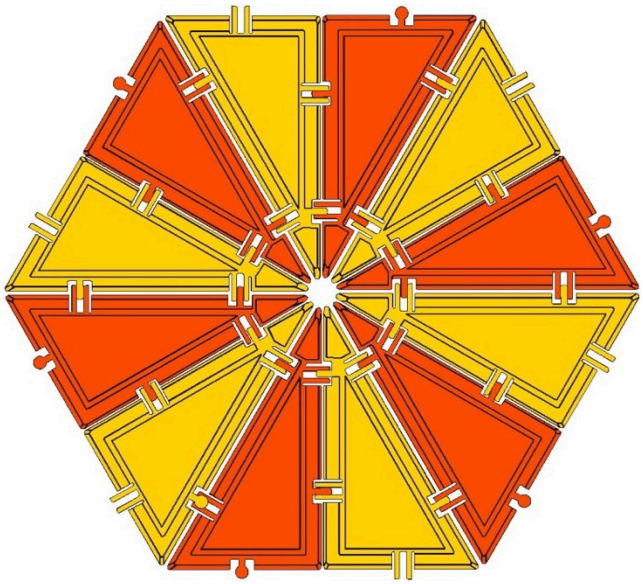

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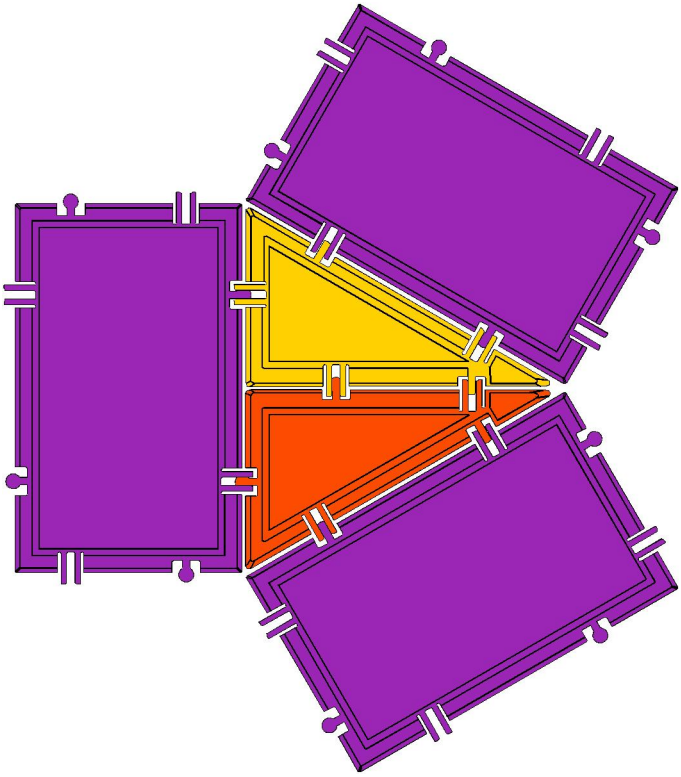

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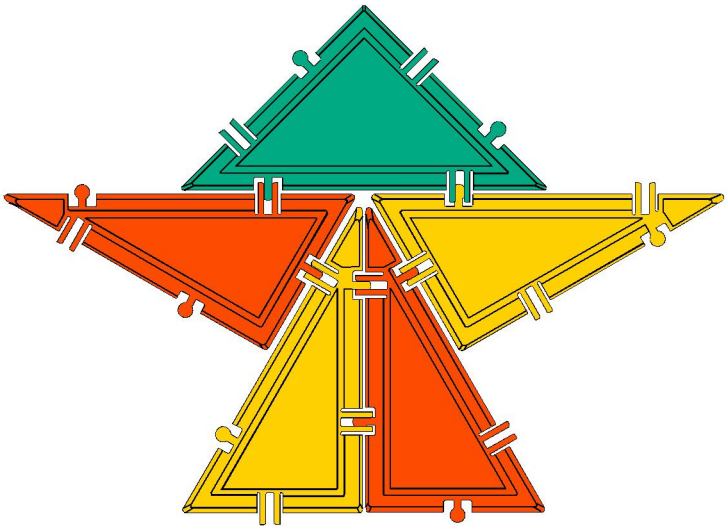

47.

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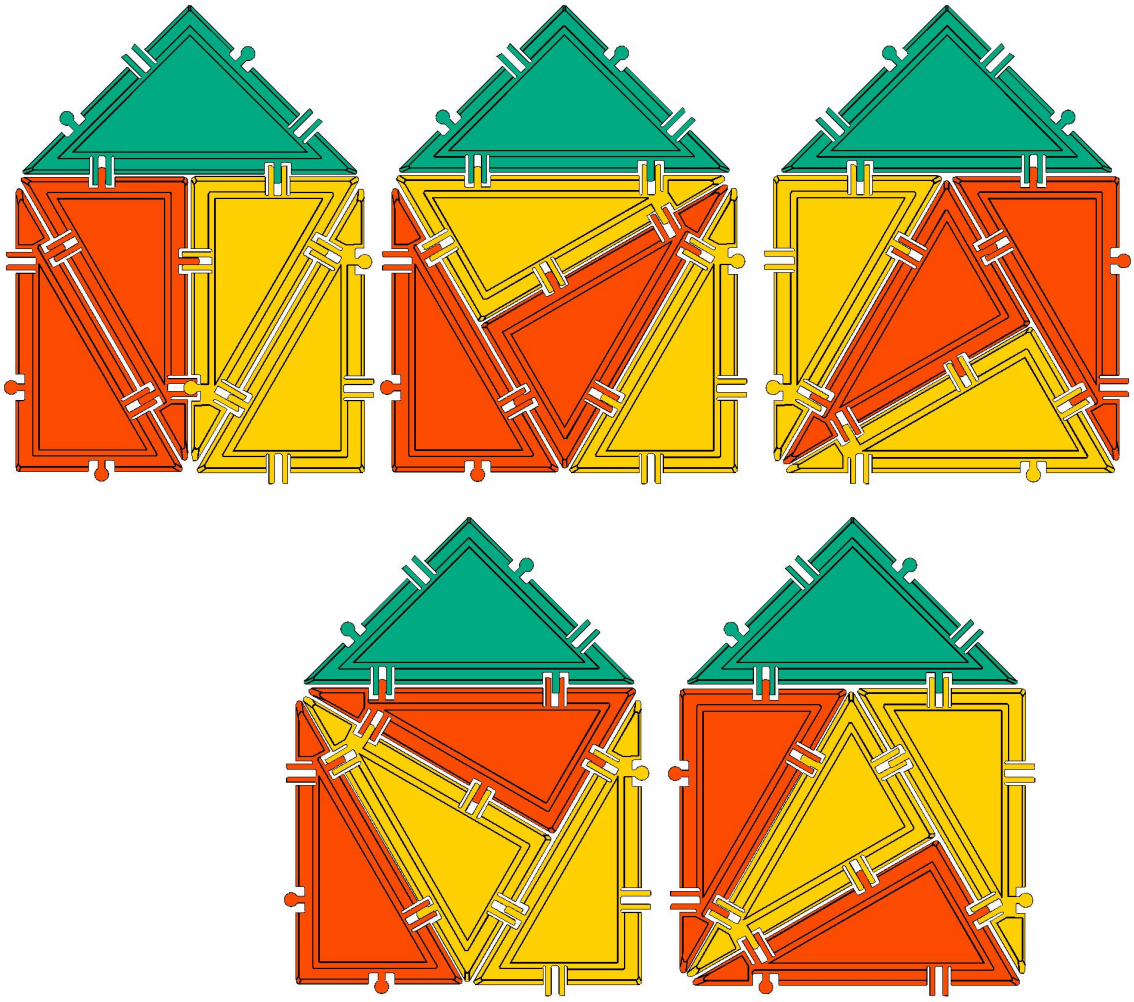
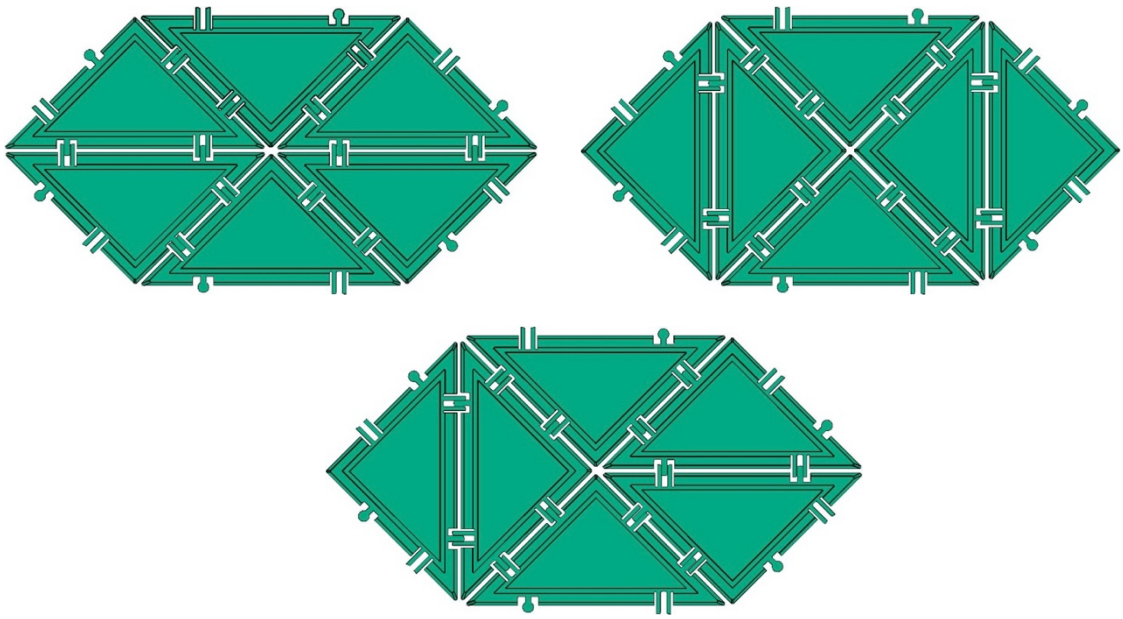
48. 5 possible constructions:



49. 3 possible constructions:



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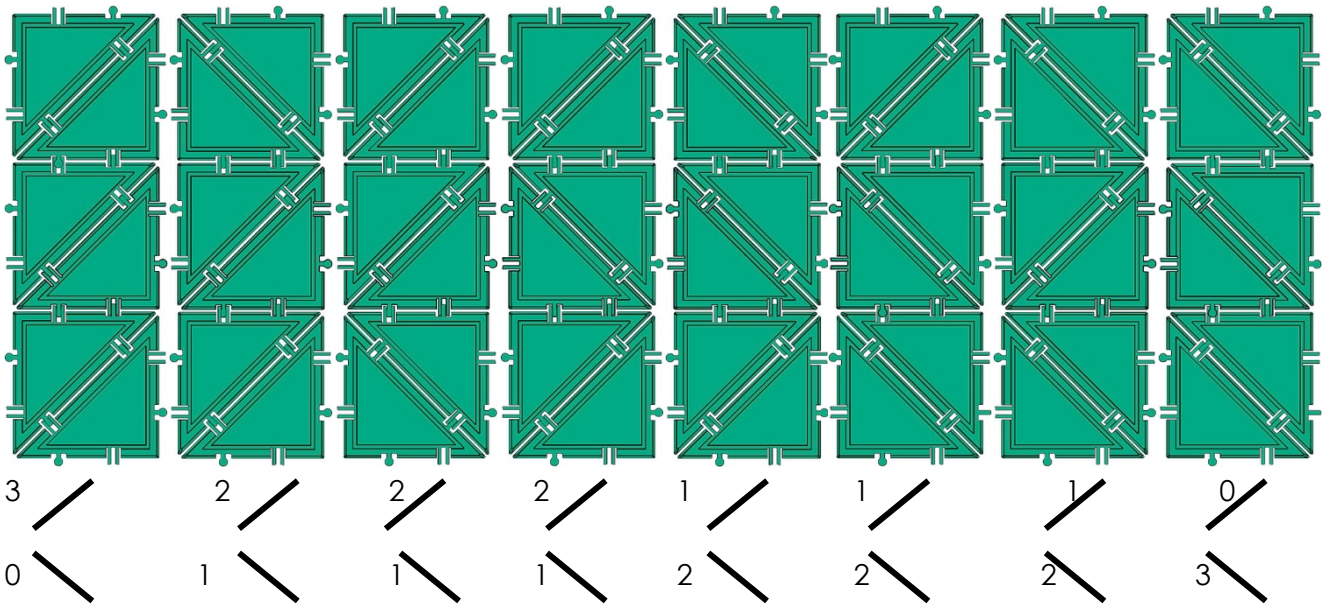
50. Finding all the possible constructions here is quite a challenge. There are a total of 8 possibilities. See if you can help students make a systematic search for the various configurations. One way of thinking about the situation is that for every square made of two isosceles triangles, the diagonal could go either like this:

 or like this:  . There are 3 such squares in the figure, and 2 possibilities for each square. So the total number of possibilities is 2 x 2 x 2=8. We can number the possibilities according to how many
of these two types of diagonals they have:



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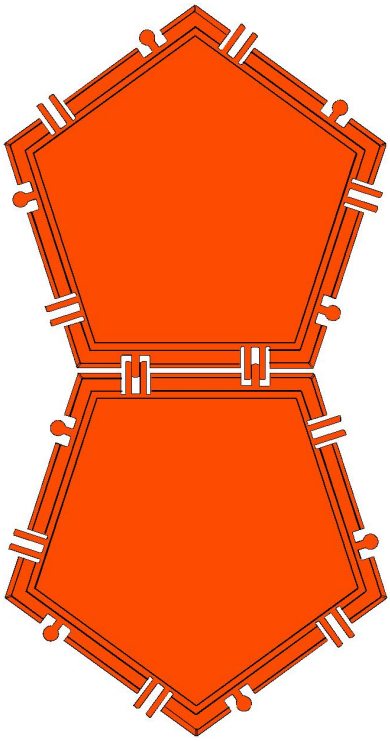
Group B, Level 1

All puzzles in this group are solved with 2 pieces.

1.



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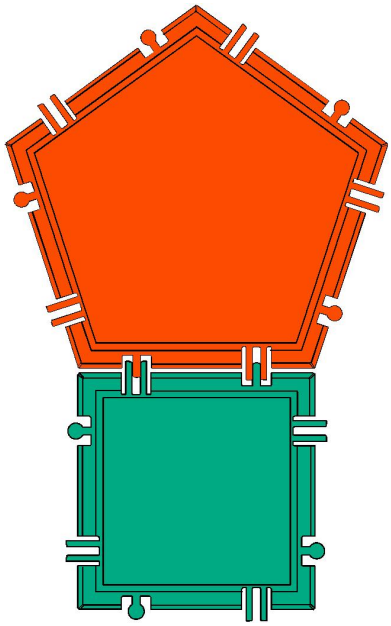

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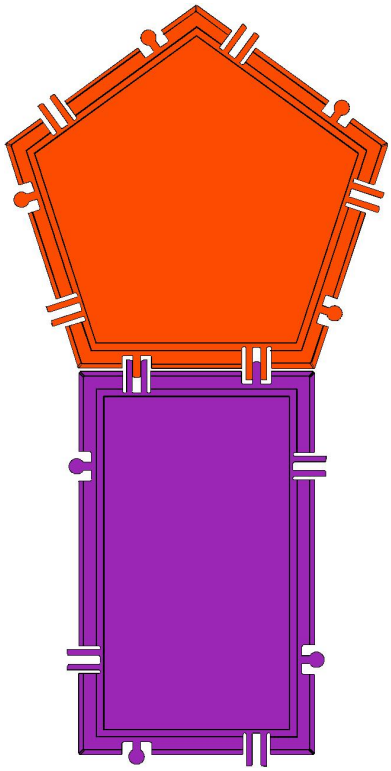

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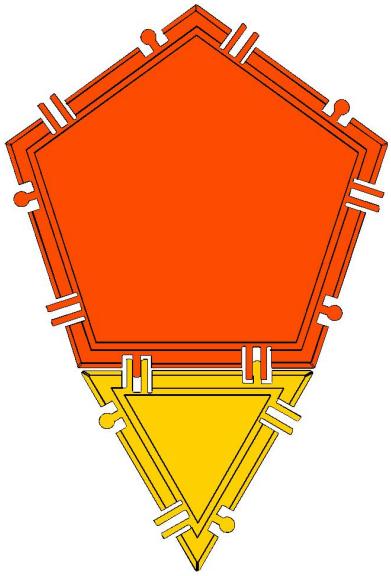

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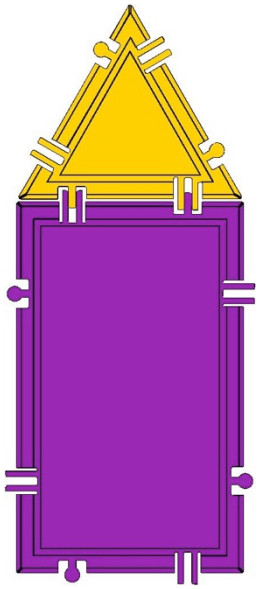

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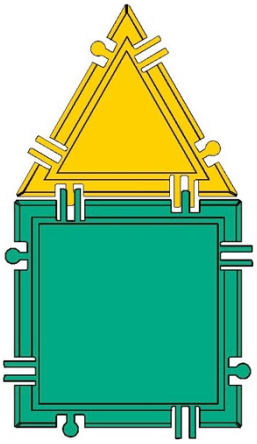

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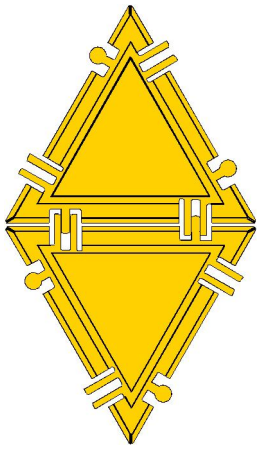

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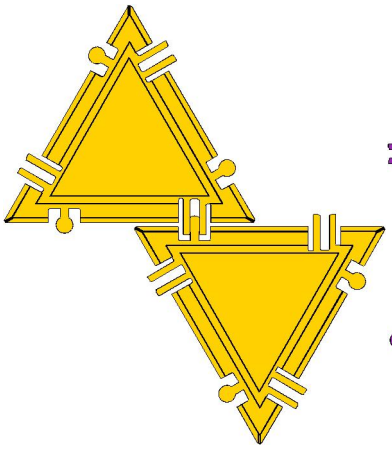

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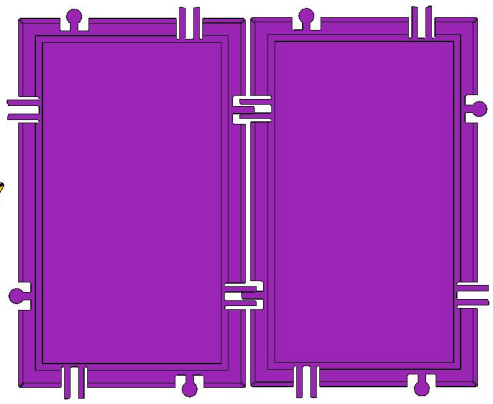

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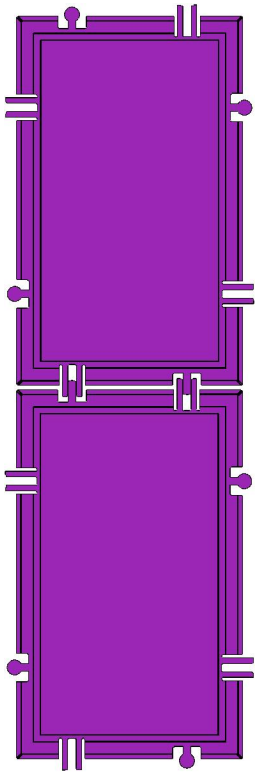

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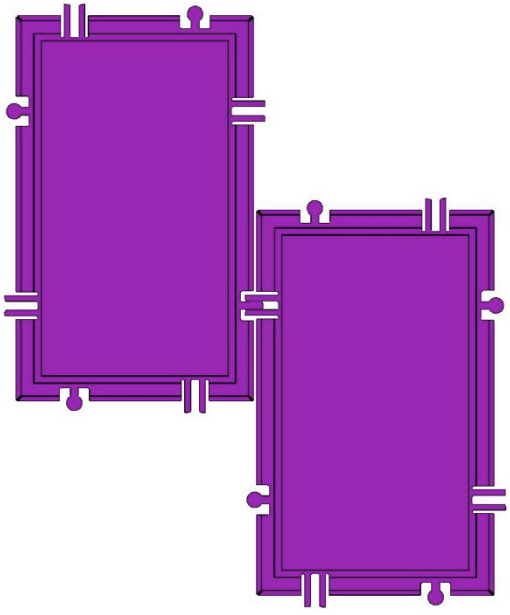

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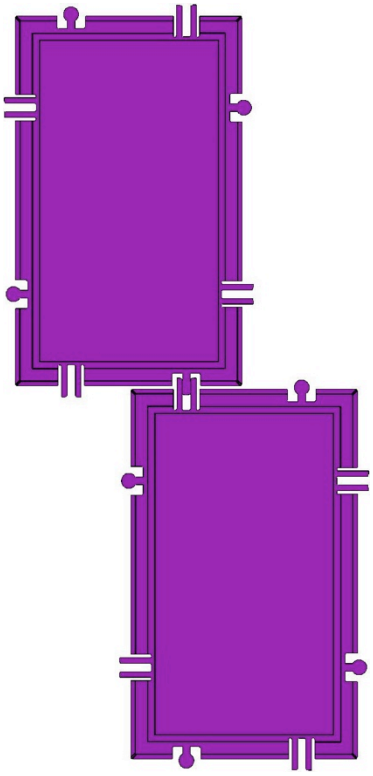

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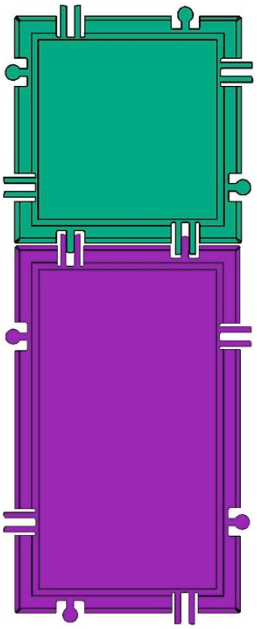

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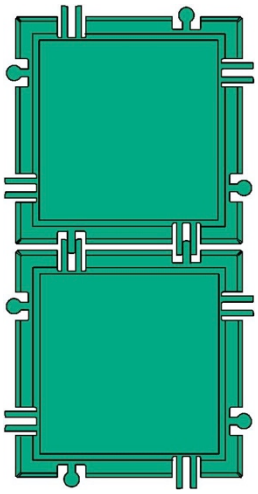

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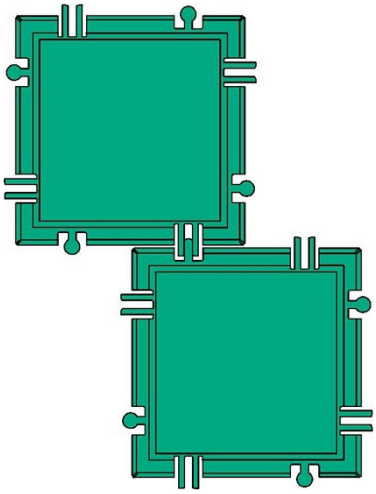

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Group B, Level 2

All puzzles in this group are solved with 3 or 4 pieces.

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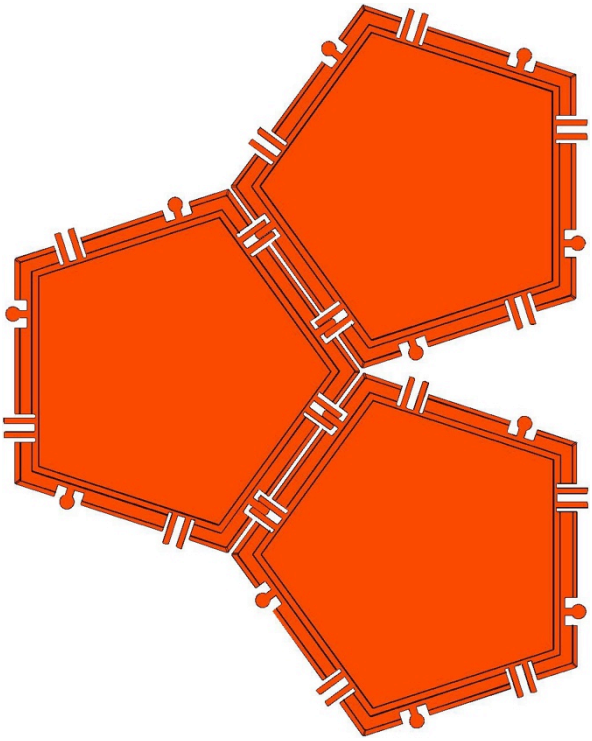

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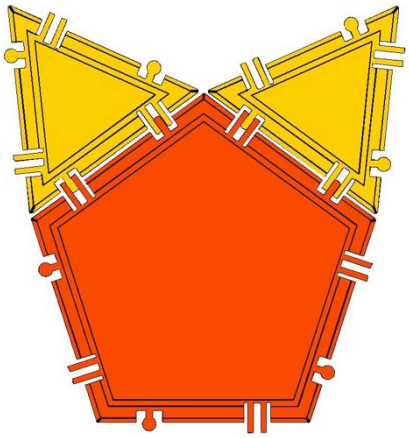

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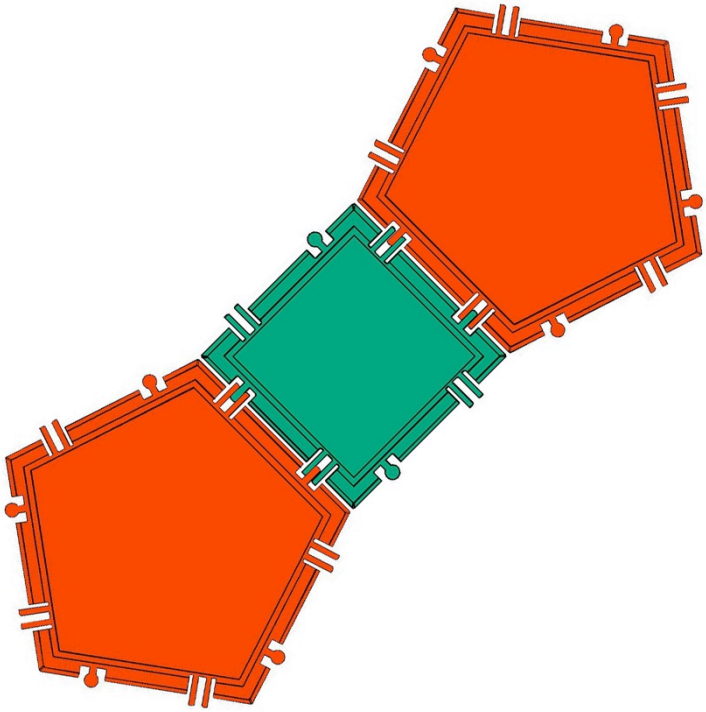

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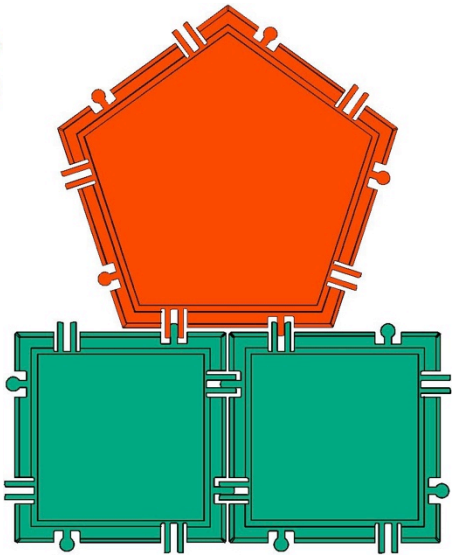

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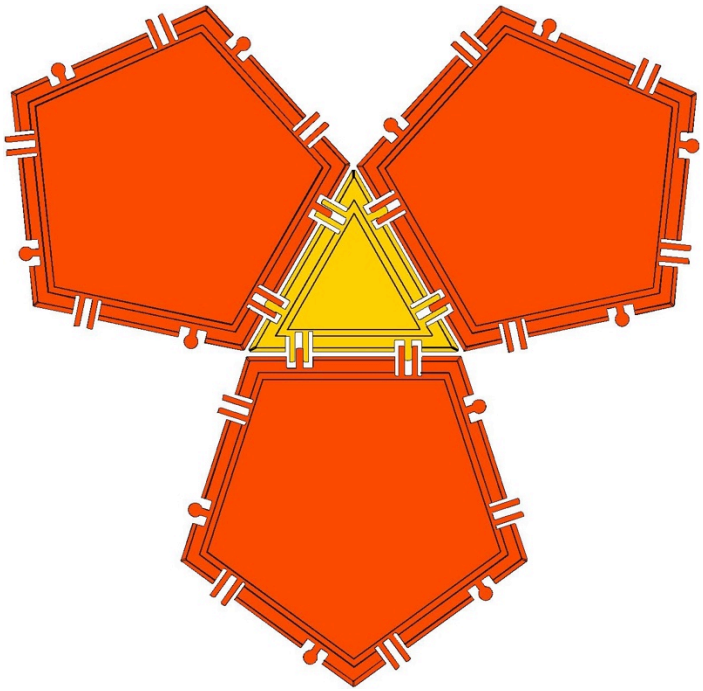

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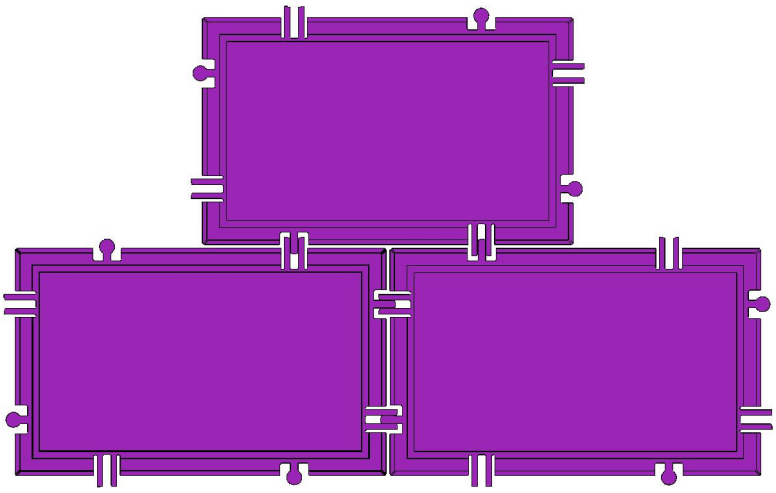

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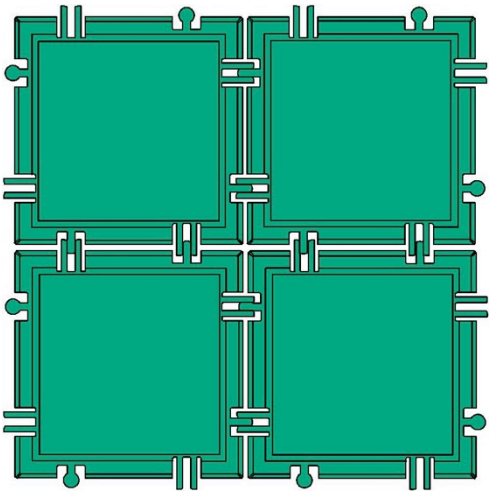

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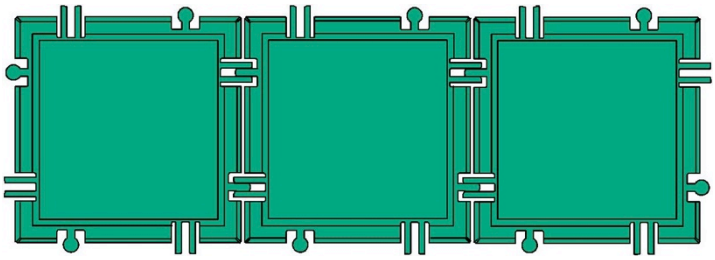

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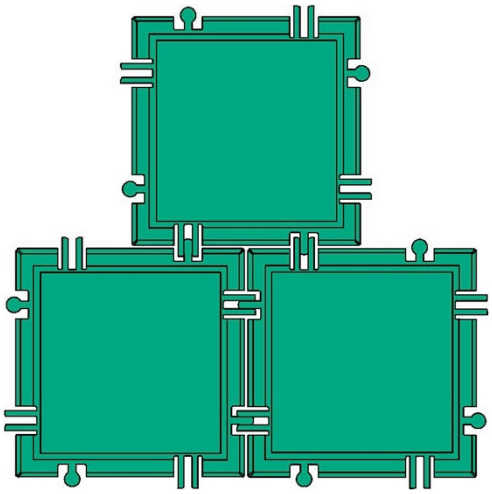

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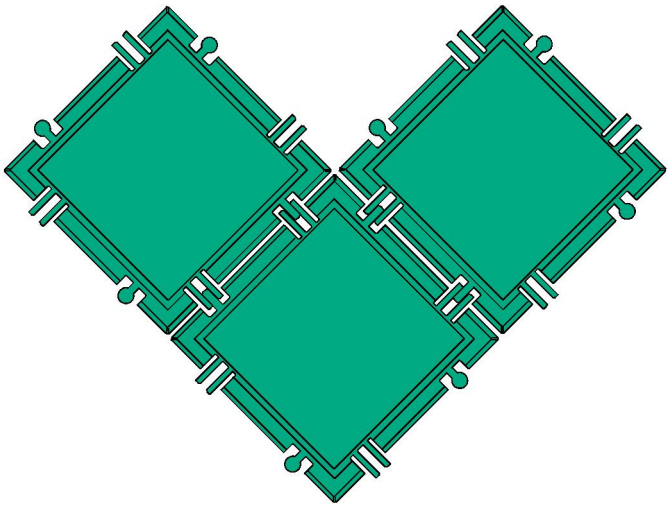

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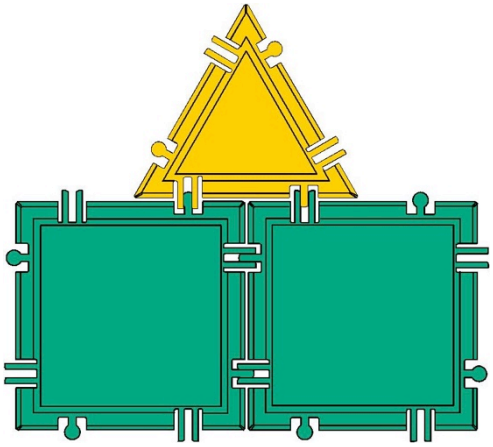

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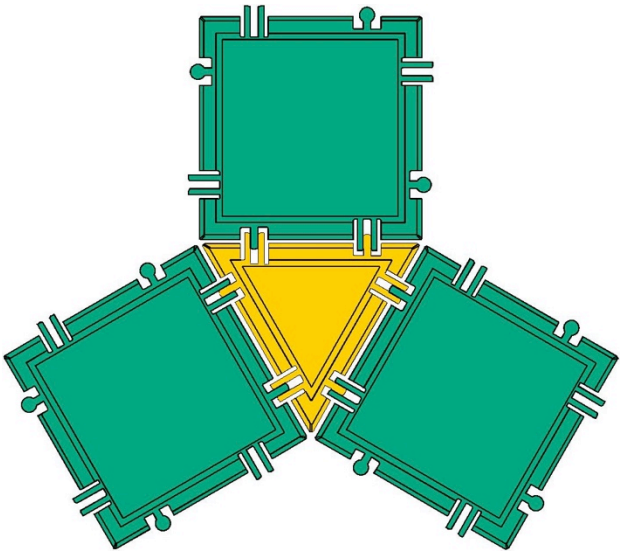

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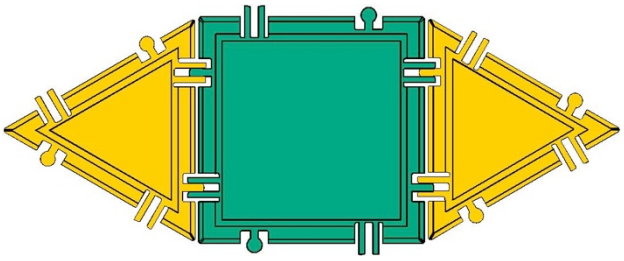

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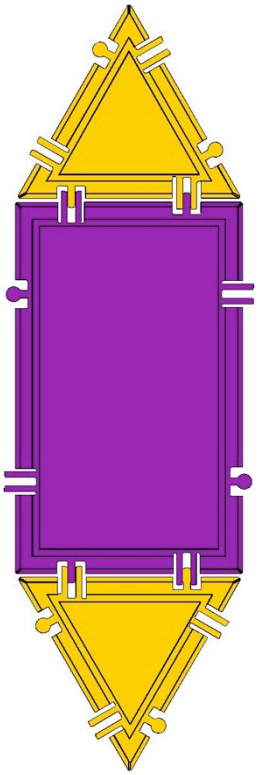

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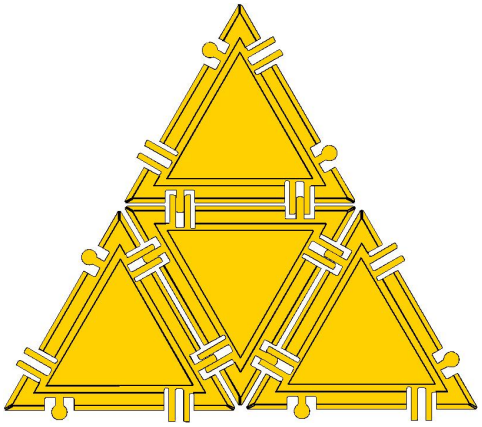

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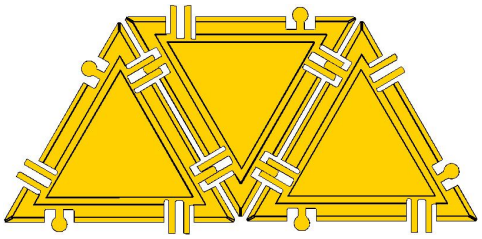

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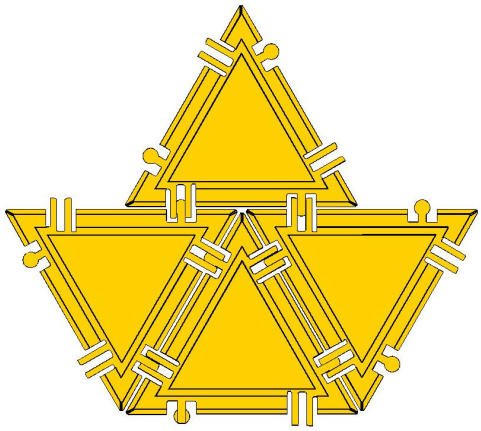

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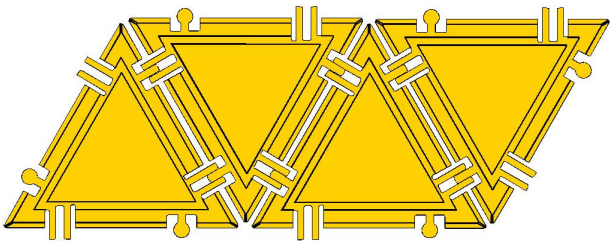

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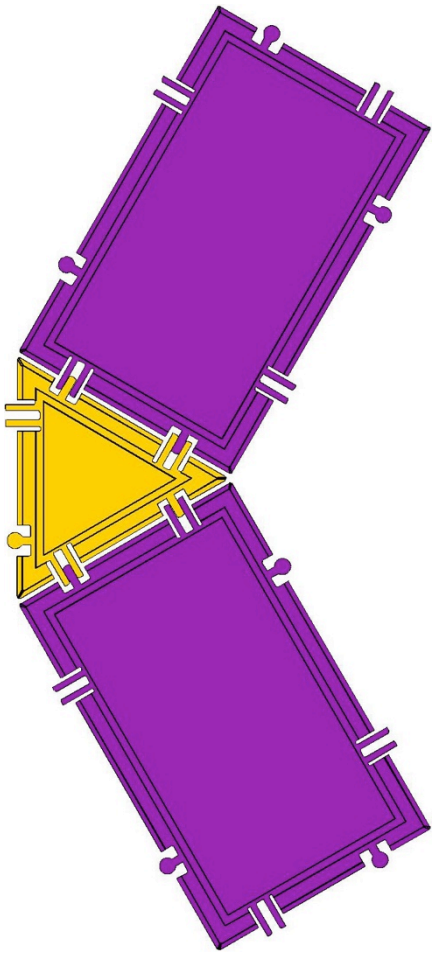

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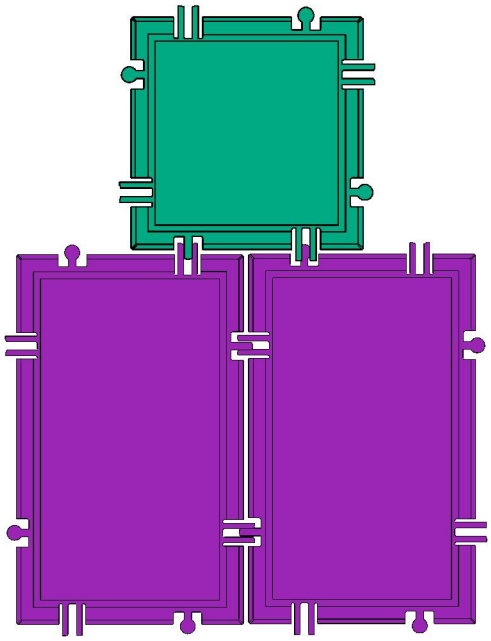

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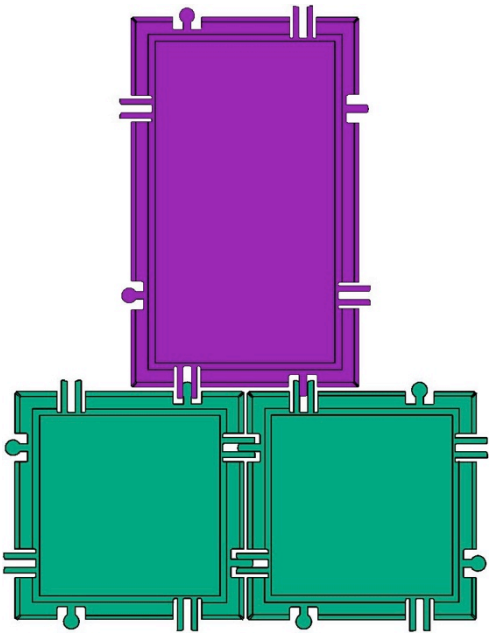

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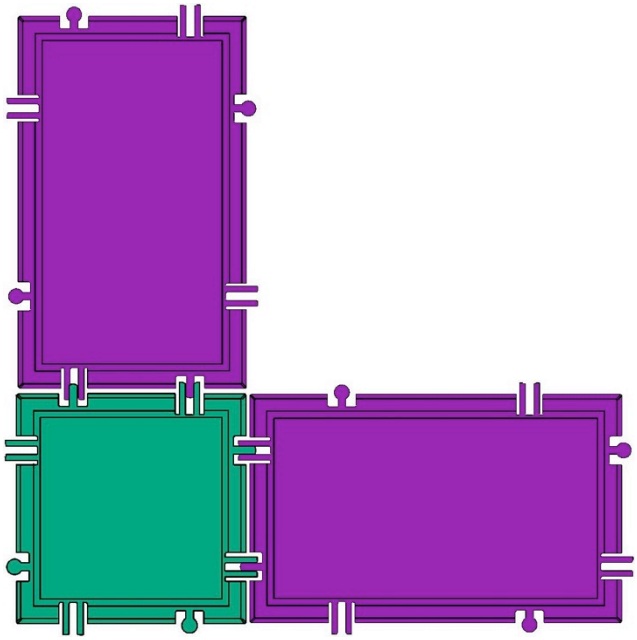

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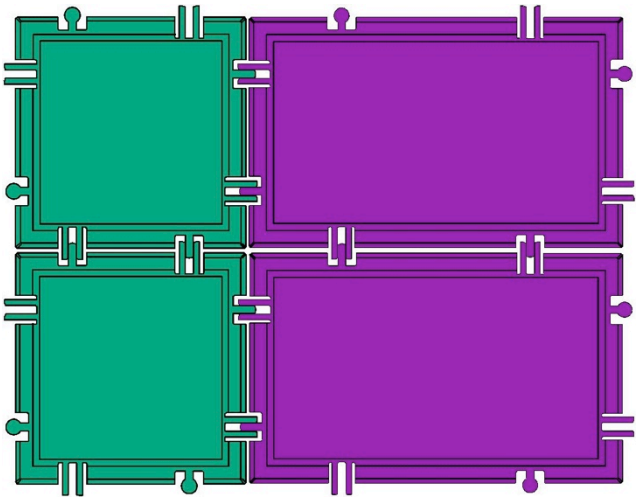

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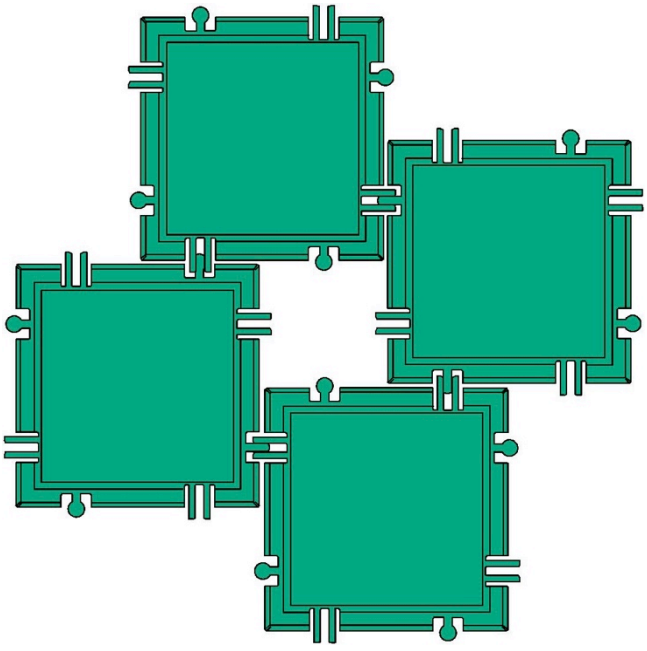

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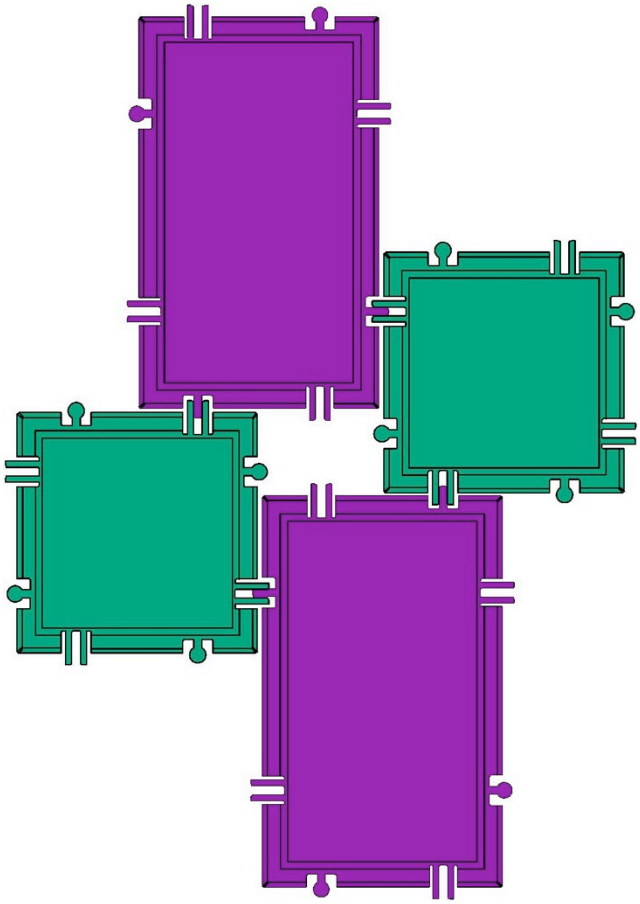

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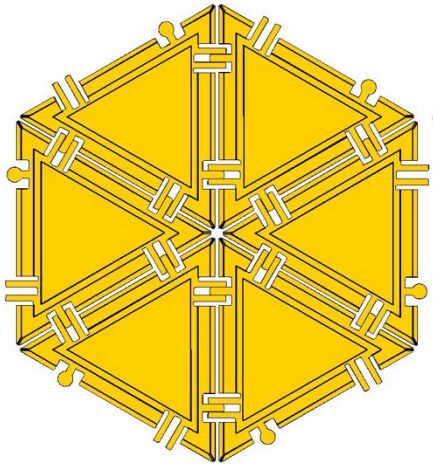
Group B, Level 3

All puzzles in this group are solved with 5 or more pieces.

41.



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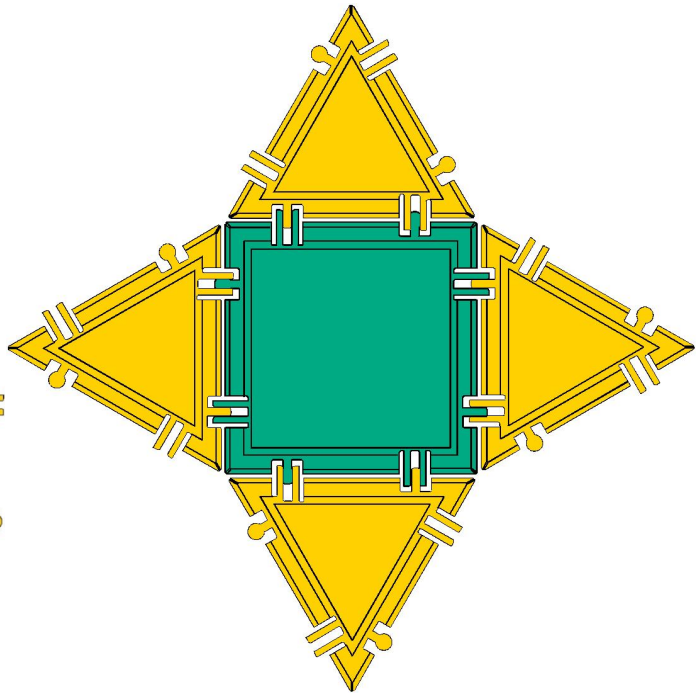

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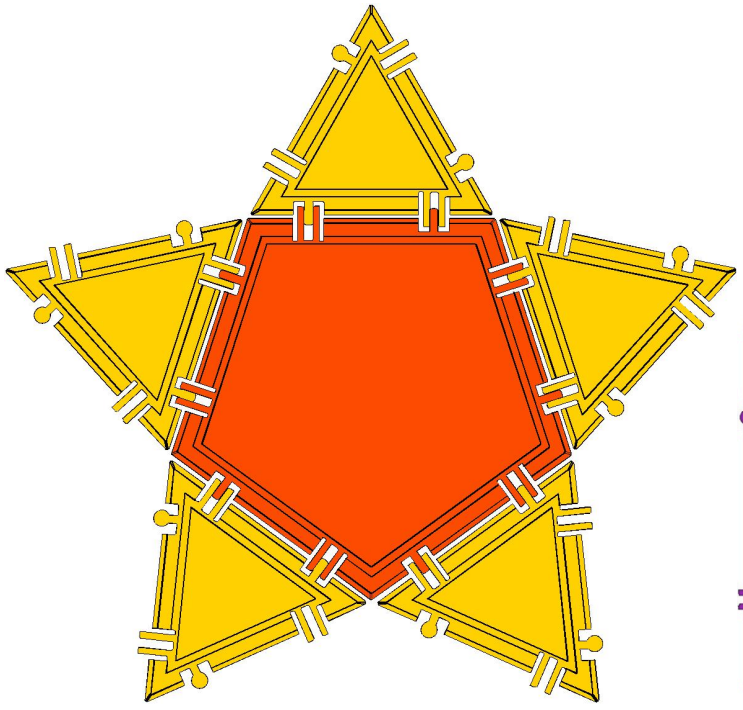

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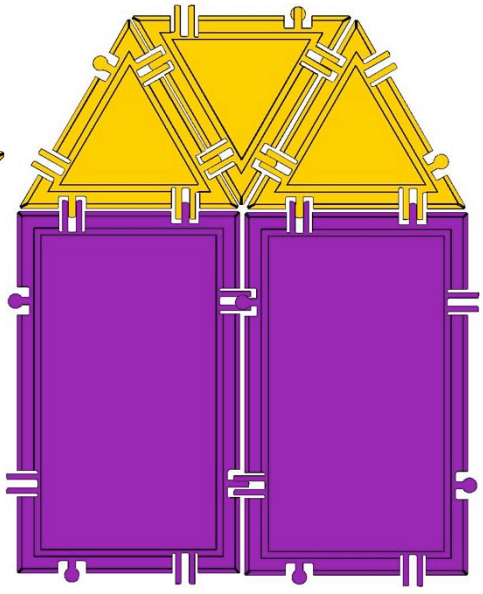

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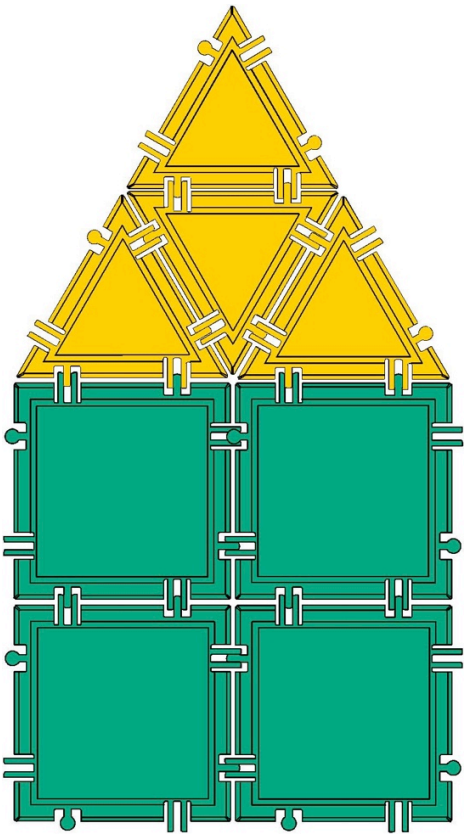

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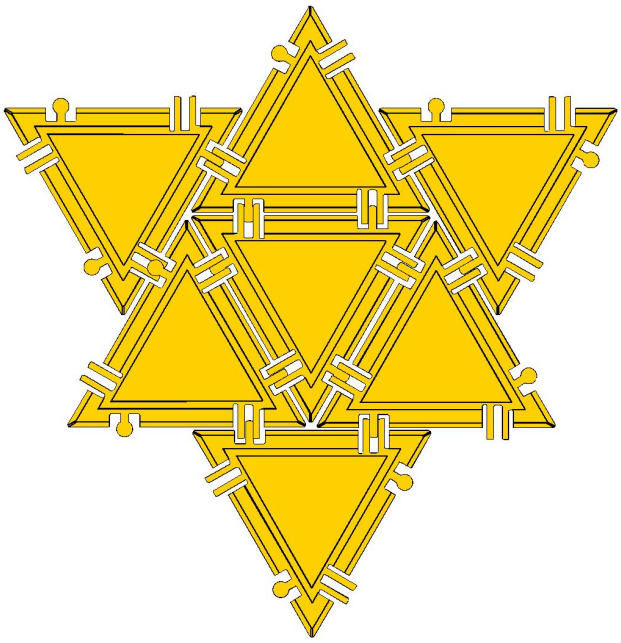

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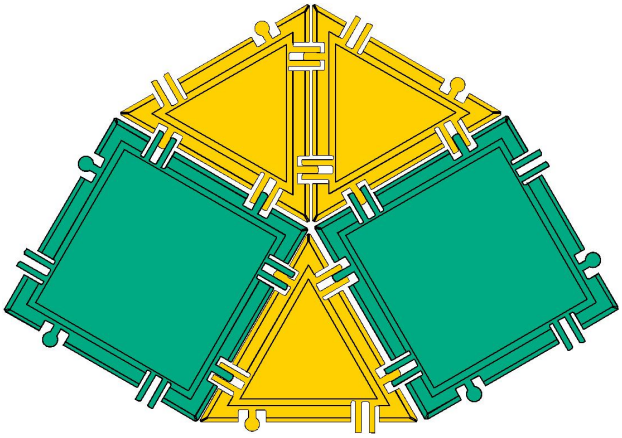

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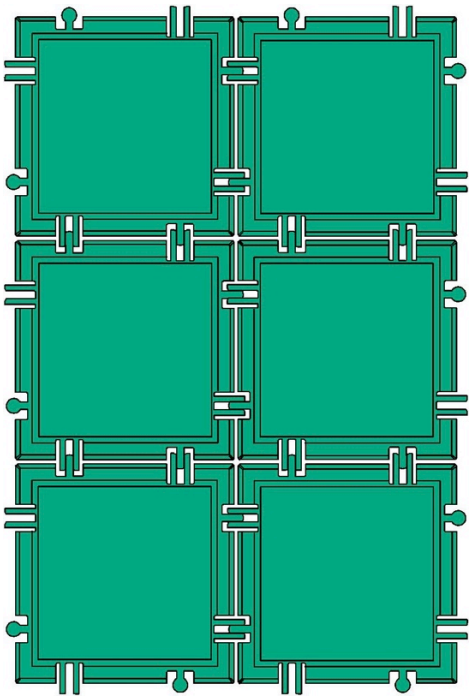

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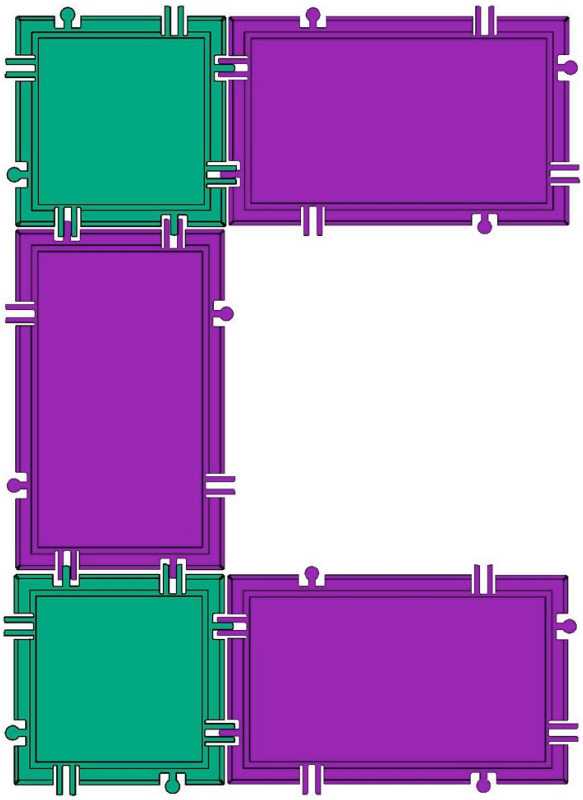

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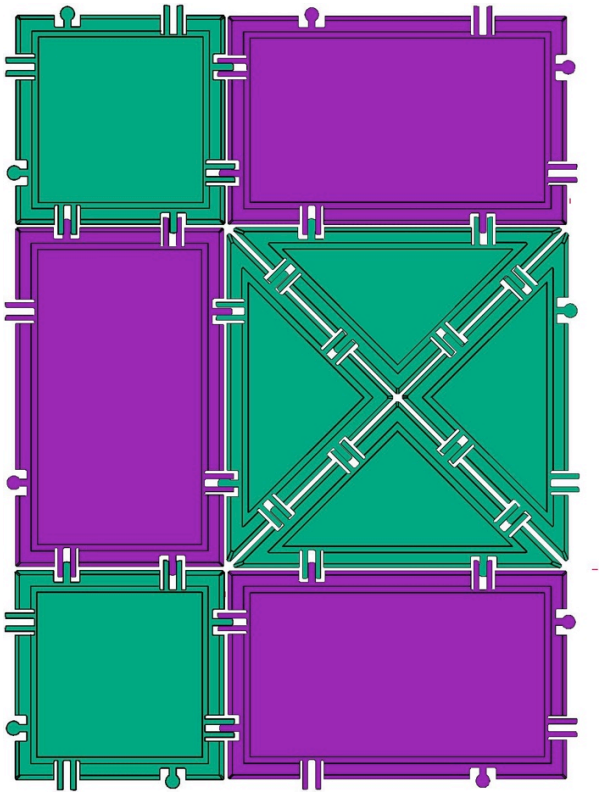

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