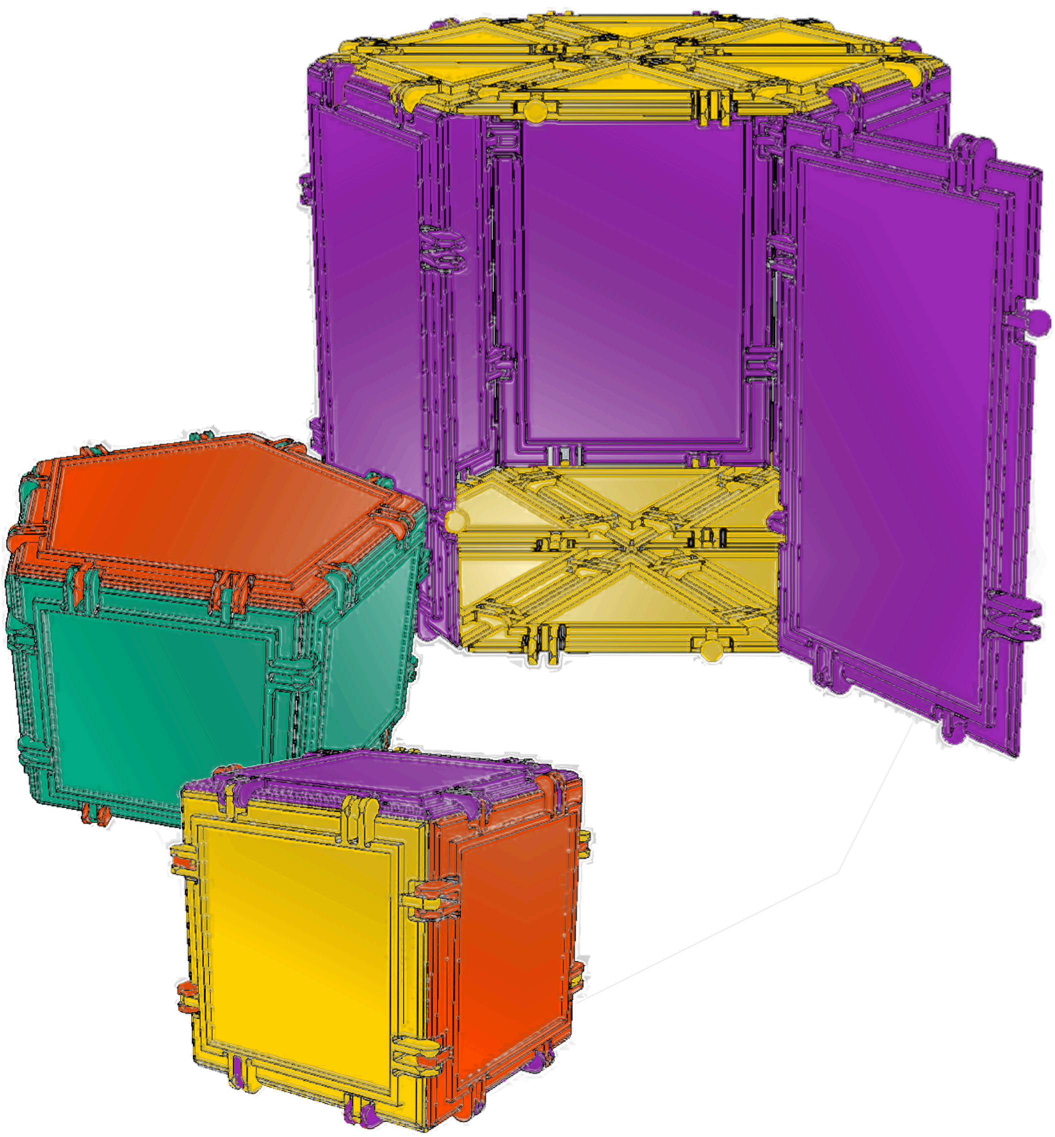
3D SOLID UNDERSTANDING

GEOMETILES™

3D Solid Understanding

For grades 2 and up



GEOMETILES™

Patent Pending

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Introduction

The following exercises are intended to accommodate a wide range of students from grade 2 onwards. It is recommended that students do the exercises in this booklet after completing the exercises in "Shape Challenge". That way students will get familiar with the various 2-dimensional shapes they can build before assembling them into 3-dimensional solids.

Activities in this workbook and the Common Core State Standards (CCSS)

Volume measurement is a large part of 5th grade curriculum. For this reason, it is important that students start building an intuitive understanding of solid structures in the earlier grades. As early as kindergarten, students are expected to analyze 3-dimensional shapes (K.G.B.4). This workbook helps students gain an understanding of solids by having students construct them. We have waited until 2nd grade to introduce this workbook because of the terminology required of the students. There is no reason, however, not to start younger students on building shapes with Geometiles™ without expecting them to construct specific solids.

CCSS for Mathematical Content supported by activities in this workbook

1.G.A.2 *Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.*

CCSS for Mathematical Practice supported by activities in this workbook

MP.1 *Make sense of problems and persevere in solving them.*

Each problem consists of a goal (make a certain shape) and a set of constraints (the tiles students can use). Students are encouraged to try different combinations of shapes until they find one that solves the problem.

MP.3 *Construct viable arguments and critique the reasoning of others.*

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

MP.4 Model with mathematics

Students will observe that most of the solid shapes they make are models for real life objects.

MP.6 Attend to precision

Solving these problems makes it necessary for students to use clear definitions of various polygons and solids in discussion with each other and their teacher.

MP.8 Look for and express regularity in repeated reasoning.

Students will create the 3-dimensional shapes by building upon 2-dimensional shapes they had built earlier.

Learning objectives of this workbook

a. Get students more comfortable with the added complexity of the 3 dimensions compared to 2.
b. Deepen students' understanding of rectangular, triangular, and other prisms.
c. Lay the groundwork for students' future studies of volume.
d. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
e. Learn how to collaborate with one another.

Arguably, point d. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

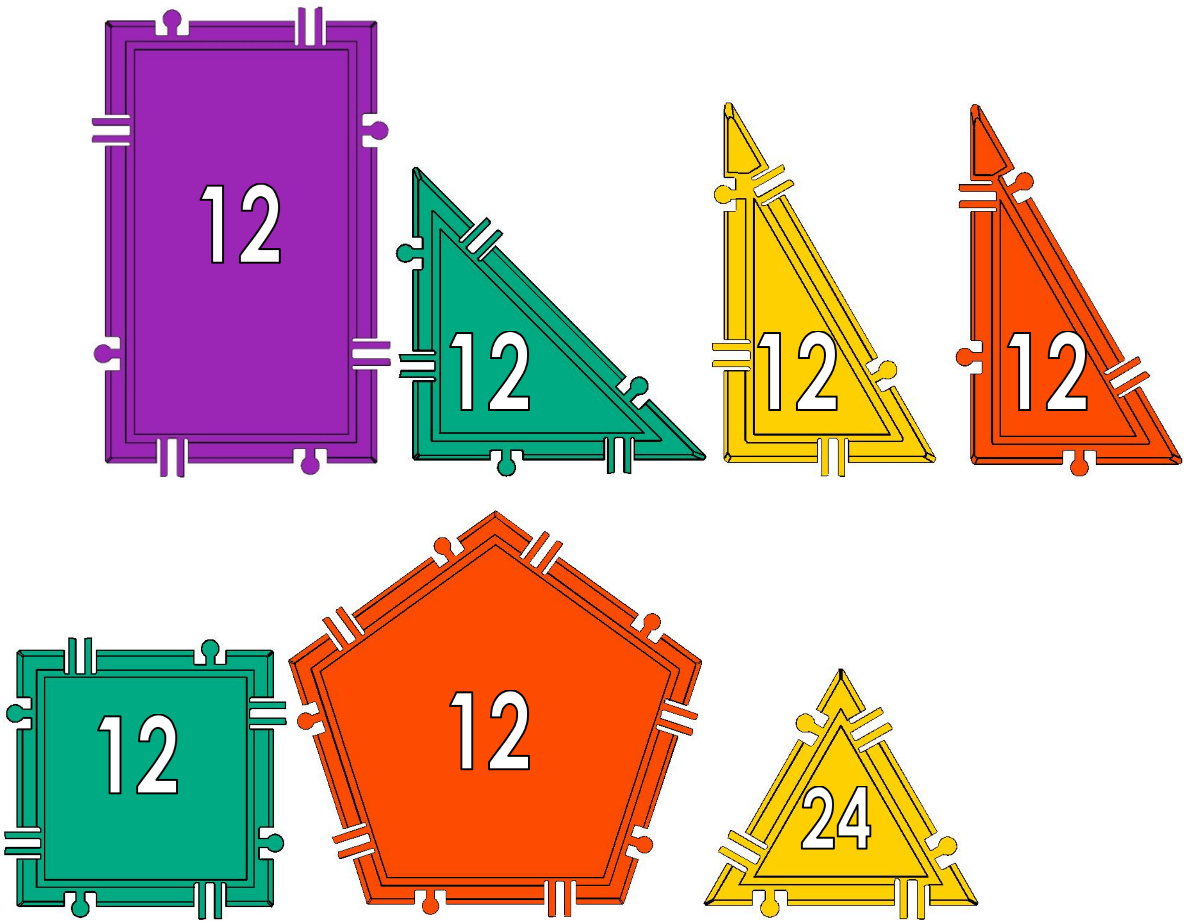
If students are frustrated to the point of disengagement, you can modify the problem so they don't need to construct, say, six different rectangular prsms. They will still get a meaningful learning experience from building two or three.

Hints are provided as necessary to help the students who are having trouble gaining momentum with solving a problem.

Recommended classroom plan

The exercises are designed to work with a set of 96 tiles for a group of 6 to 7 students.

These are the shapes and quantities needed. Colors may vary.



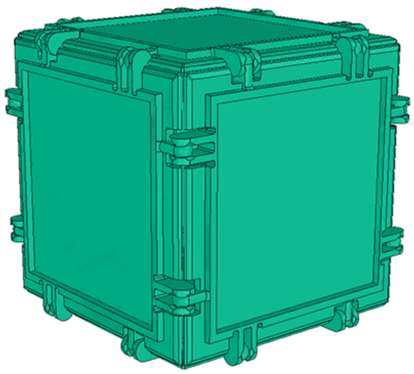
Problems

1. Make a cube using 6 tiles.
2. Now make a larger cube using triangles.
3. Make a box with a square top and bottom. Make sure your box is not a cube.
4. Now make a closed box with all rectangular faces. Make sure that no face is a square.
5. Make as many *different* triangular prisms as you can.
6. Make a prism with a rhombus (diamond) shaped base.
7. Make a pentagonal prism.
8. Make as many different hexagonal prisms as you can.
9. Make a pyramid with a square base.
10. Make a pyramid with a triangular base.

Answers

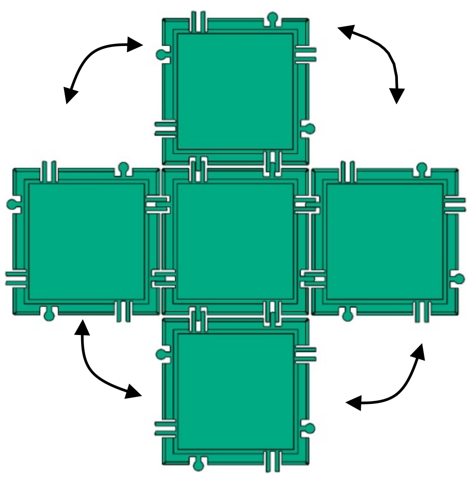
Problem 1

Answer



Hints

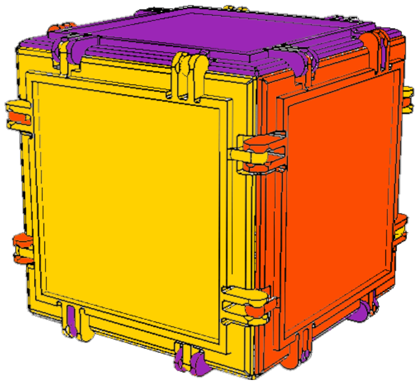
If students have trouble, they can start out with the bottom and sides of a box and connect them:



Further questions:

- How many faces does each cube have? [6]
- How many neighbors does each face have? [4]
- How many faces meet at each corner? [3]

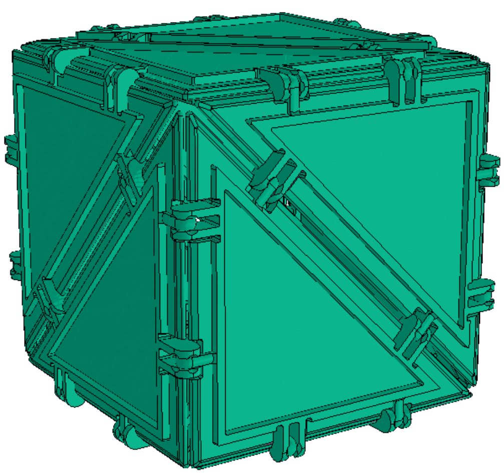
- How many corners does each cube have? [8]
- Use tiles of 3 colors to make a cube, and make sure that no two faces that share an edge have the same color.



  - How many different combinations of 3 colors are there? [4. There is a total of 4 colors, so there are 4 choices of which color to leave out to make a combination of 3 colors. The combinations are: (1) orange, yellow, purple (2) orange, yellow, green (3) orange, green, purple (4) yellow, green, purple.

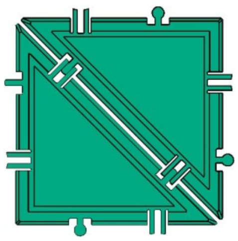
Problem 2

Answer



Hints

The idea here is to make squares out of the isosceles triangles like this:,



and then to make a cube out of these squares.

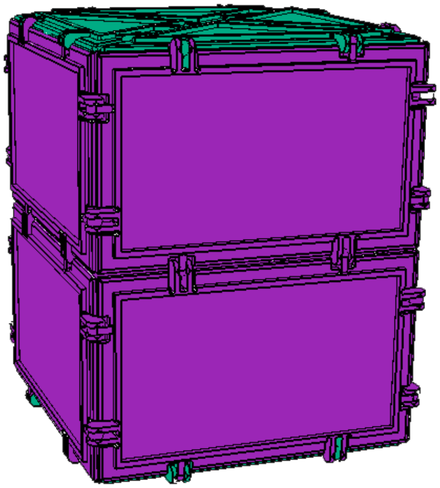
If students are having trouble, you can guide them through the following steps. Omit steps depending on what the students need.

a. What is the shape of each face? [square; they will know this from having built the smaller cube out of squares]
b. How can you make a square out of triangles? [they will know this from having worked through "Shape Challenge"]
c. Make all the faces of a cube.
d. Now connect them. [refer back to the diagram in the solution for Problem 1 if necessary].

Problem 3

Answer

Partial collection of non-cubical boxes with square top and bottom.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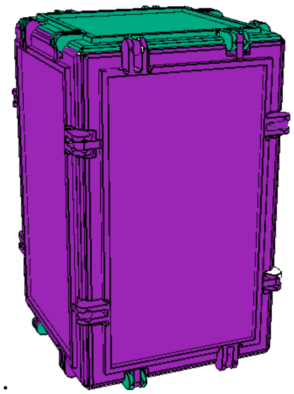
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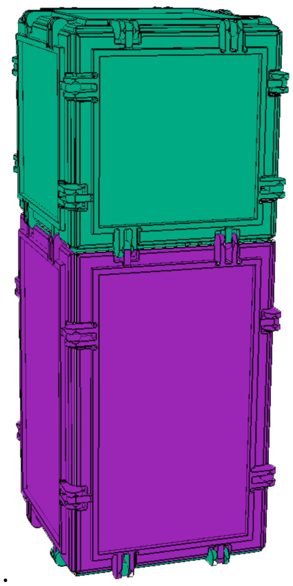
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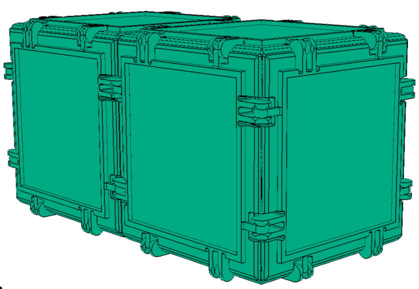
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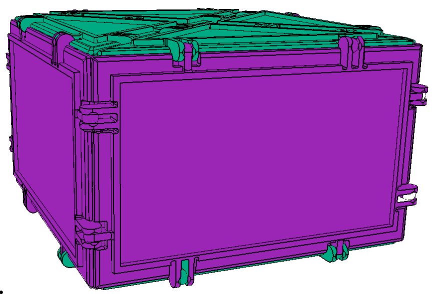
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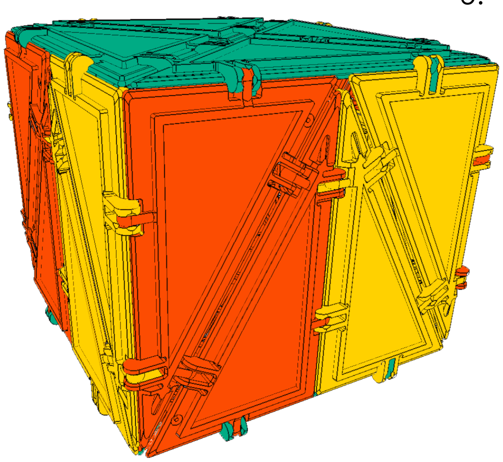

6.

Hints

Keep intact at least one cube from the previous exercise because you may need it for students who may keep trying to make a cube at this point. If you see this happening, you may want to ask them some of the following questions, using the cube for comparison:

- What is going to be the shape of the faces other than the top and the bottom of the box? [rectangle, not square; if they were squares then we would be back to a cube]
- How many faces are we going to have other than the top and bottom? [4]

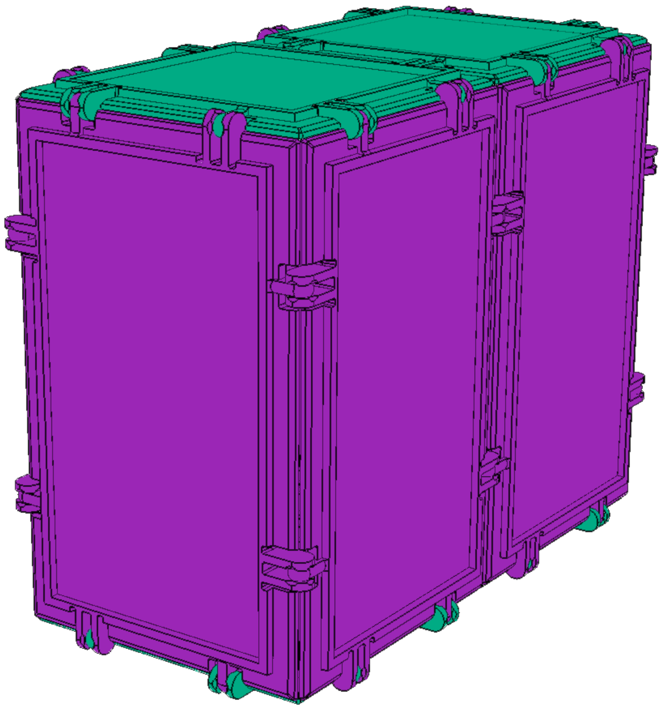
After the students have constructed the various boxes ask them to think about the following questions:

- What attributes of the box make it different from a cube? [rectangular faces instead of square ones]
- What can you say about the 4 rectangles that make up part of the box? [The rectangles are all the same; they have to be, since each one has a side that borders the same square. The square has all equal sides by its definition, so the rectangles must have all equal lengths. They have to have equal widths because they are all connected to each other along their width.]
- How many corners do the rectangular boxes have? [8, same as a cube]
- How many faces meet at each corner? [3, just like in the case of a cube. One face is square and two are rectangular].

As you can see, there are a number of possible solutions to this problem. The students need not make every single one of them, but make sure they make the ones that extend the cube of Problem 1 along one of its dimensions (Solutions 2, 3, and 4). They should observe that by extending the cube they convert it into a non-cubical box. This observation is essential for doing the next problem.

Problem 4

Answer



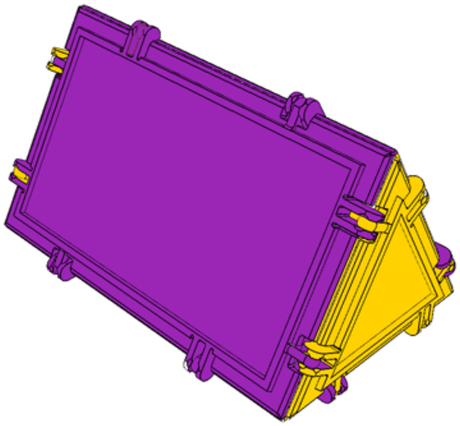
Hints

We have seen in the previous problem that extending a cube along one of its dimensions transforms some of these square faces into rectangles. Now we can extend the cube along a second dimension. We can take Solution 2 of Problem 3 and extend it so that the square face gets doubled. We have to be careful in the way we double the boxes, because if we do it along the wrong direction we may still have a box with a square bottom.

Problem 5

Answer

Partial collection of triangular prisms



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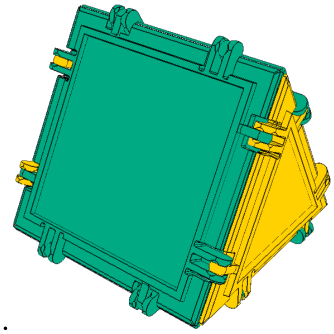

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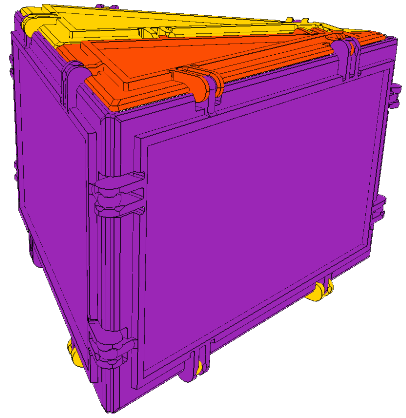

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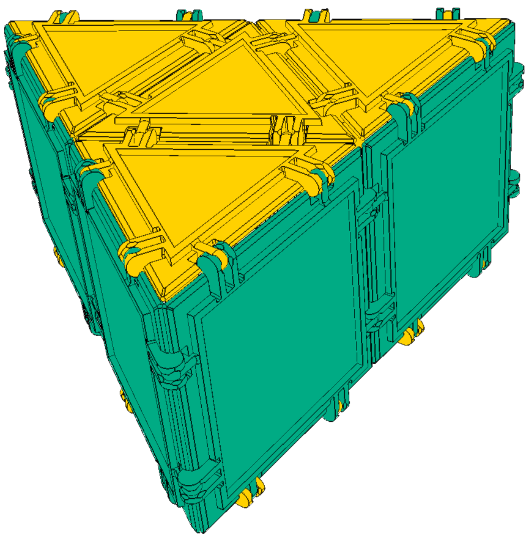

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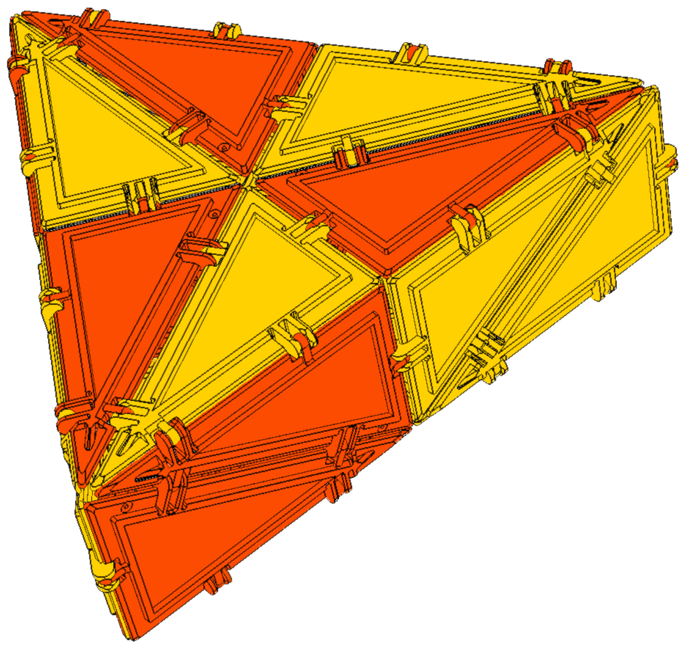
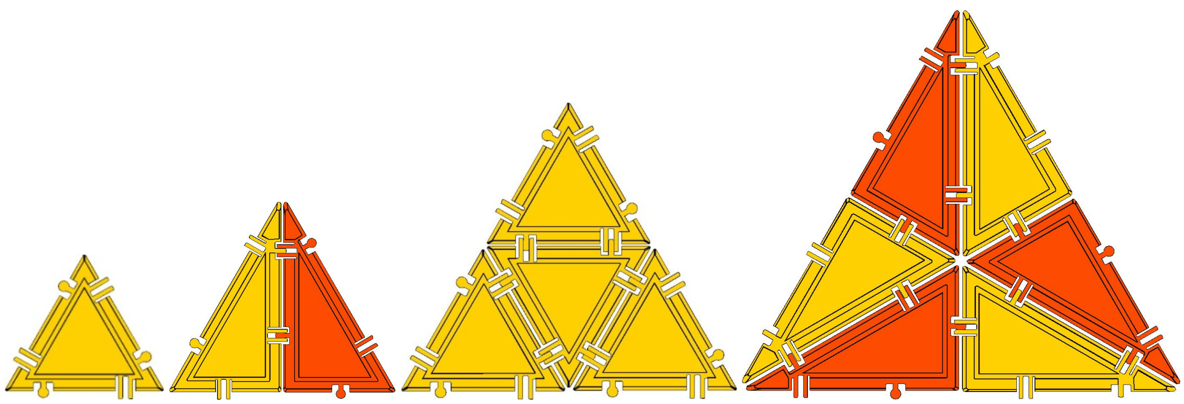


5.

Hints

You can facilitate students' work on this problem by discussing the various characteristics of the prisms:

- How many possibilities are there for the top and bottom of the prism? With Geometiles™, it is possible to make a triangular prism only with equilateral triangles, the focus here is to see how many equilateral triangles the students can make. With this in mind, this part of the exercise is really asking the students to make as many different equilateral triangles as they can. Here are some possibilities:



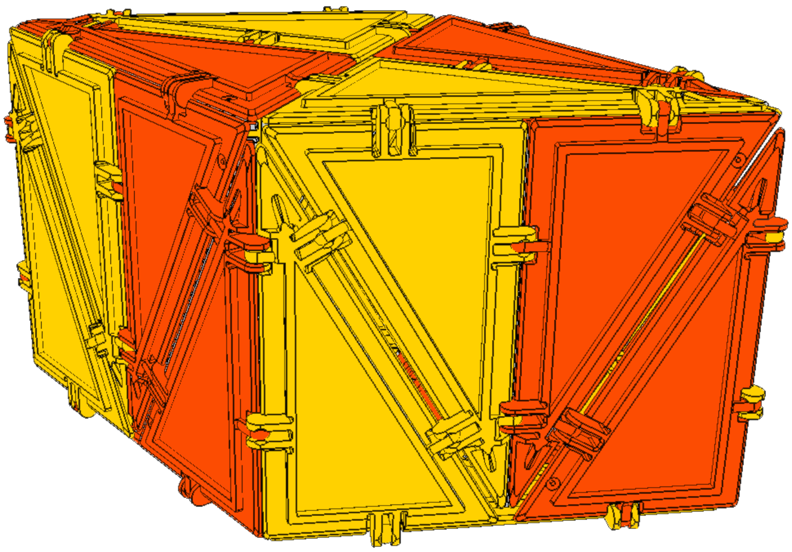
- What is going to be the shape of the **non-triangular** faces of the prism? [rectangles and squares]

Make sure the students include prisms with both rectangular and square faces in their collection. You can, of course, make variations on these by changing the number and height of "floors". You can also make larger equilateral triangles than the ones shown here.

Problem 6

Answer

Here are some of the possibilities. You can make variations by replacing the side rectangles with squares and adding more "floors".



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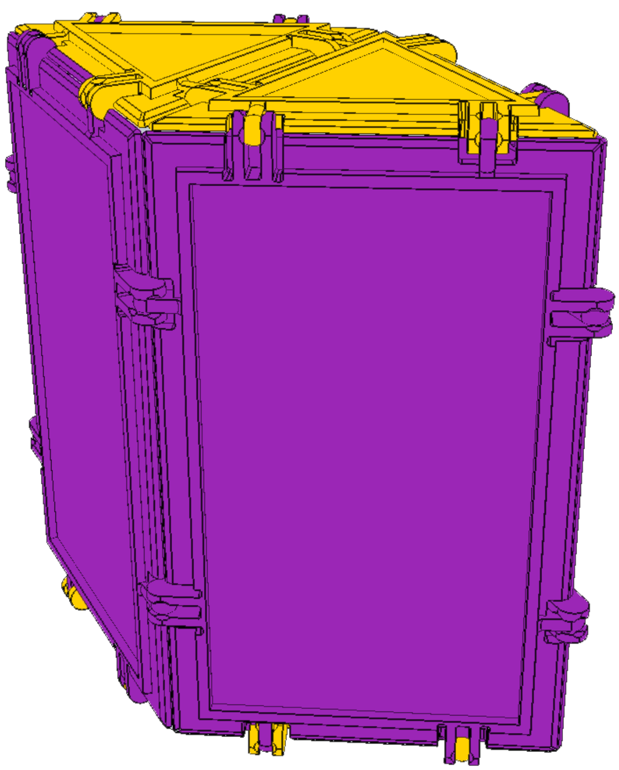

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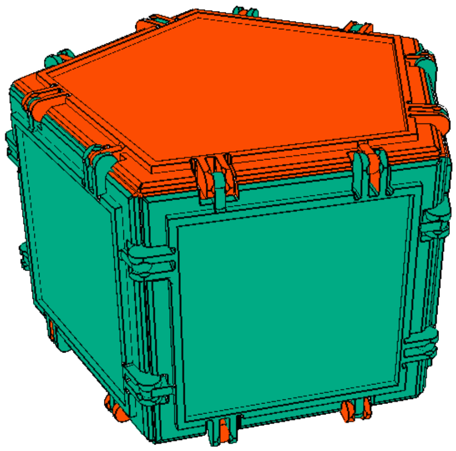
Problem 7

Answer

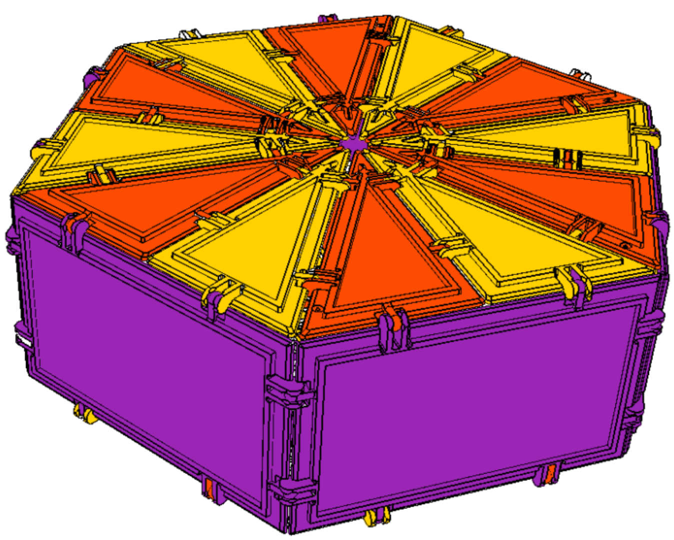


Hints

Again, you can experiment with several "floors" and replacing squares with rectangles.

Problem 8

Answer



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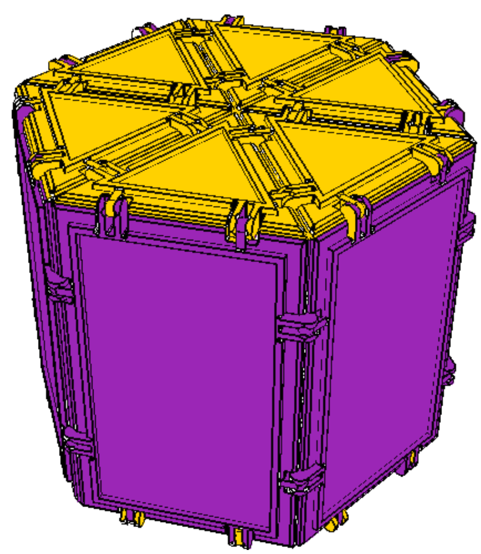


2.

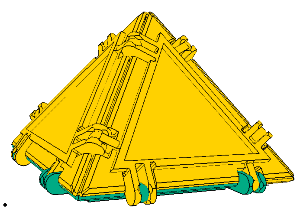
Hints

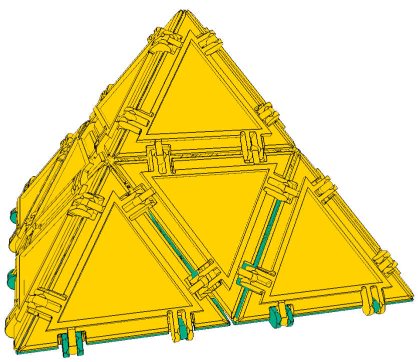
Here by “hexagonal” we mean a regular hexagon. Discuss the ways you can vary hexagonal prisms with the students: you can vary the size of the hexagon (i.e. cross section) and the height.

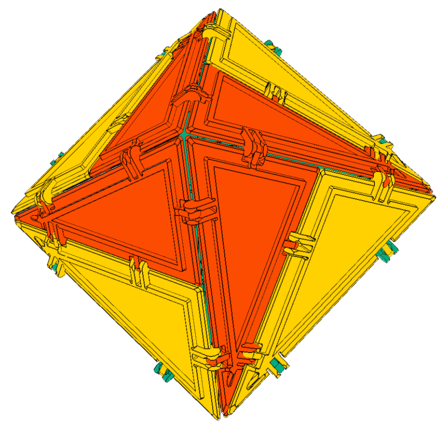
It might be good to round off the discussion on prisms by summarizing what ALL prisms have in common (rectangular or square side faces) and how they are different (different shapes for top and bottom).

Problem 9

Answer

1. 

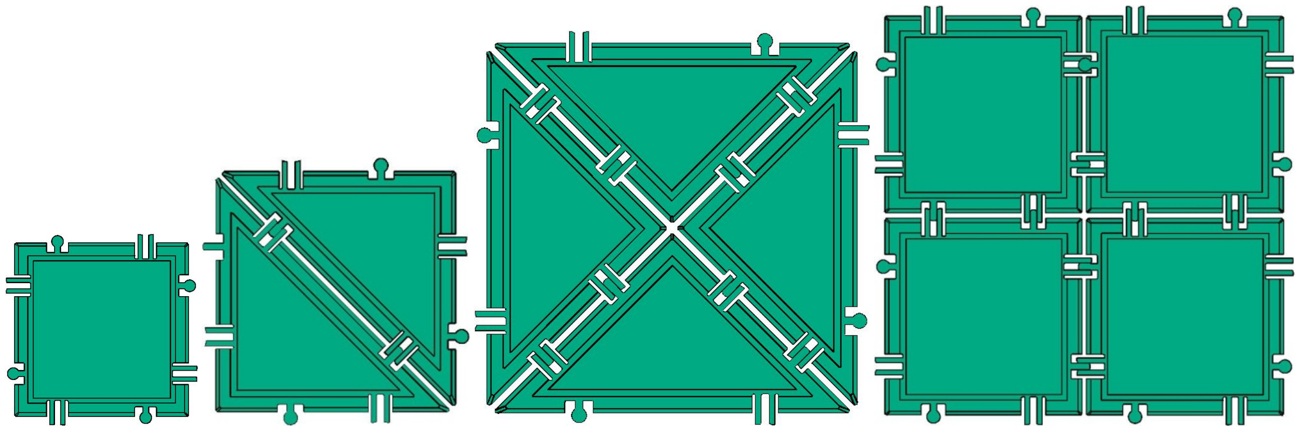
2. 

3. 

Hints

If students are struggling, have them make a square base first.

- What is the shape of the base of a square pyramid? A square.
- We have made squares earlier. Let's make them again.



(a) (b) (c) (d)

To make a pyramid, we need triangles that have a base equal to the side of the square. Do we have such triangles?

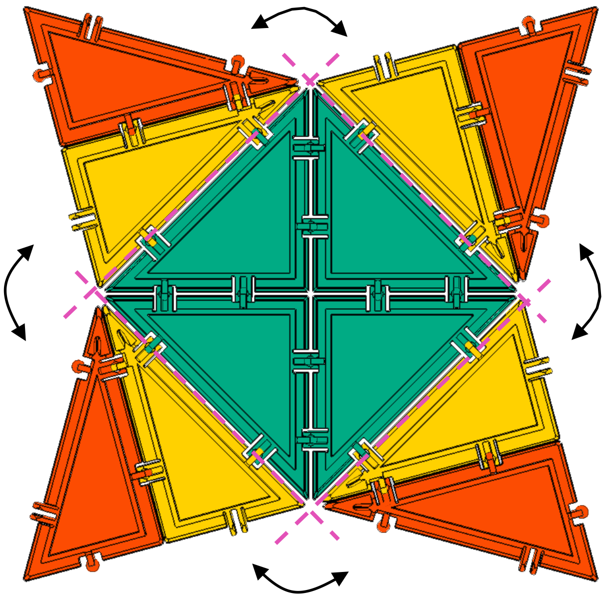
Solution 1 shows a pyramid using square (a) for a base.

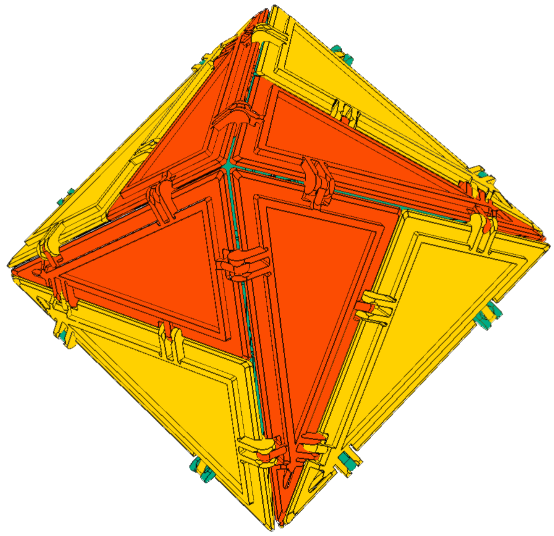
Note that square (d) is quadruple the size of square (a). If we also quadruple the triangular side walls, we will get a larger version of the the pyramid of Solution 1, which is Solution 2.

We <u>cannot</u> make a pyramid using square (b) for a base because there is no isosceles triangle in the set whose base is equal in length to the side of square (b).

Here is how you can build the pyramid of Solution 3 that with square (c) as its base:

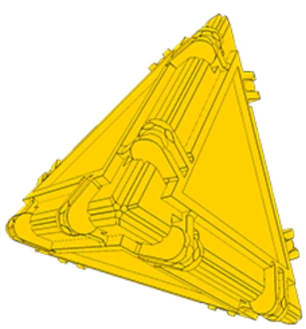
Fold up along dashed lines (valley fold in origami terminology) and snap together edges as shown by arrows. The 4 corners will join at the top of the pyramid.





Problem 10

Answer



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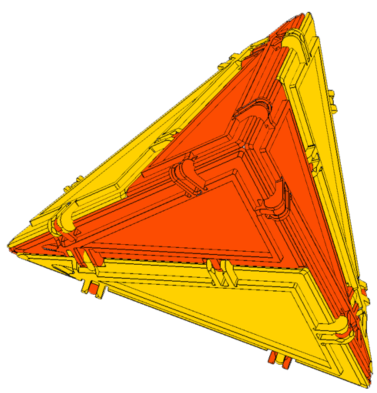

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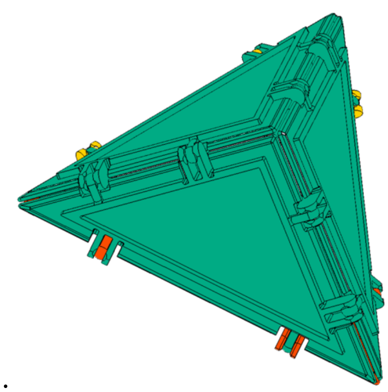
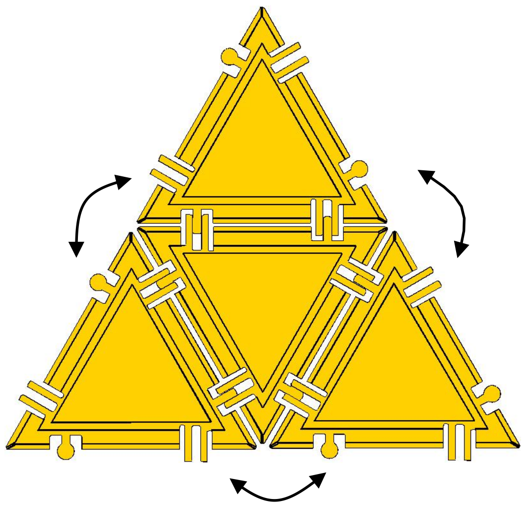


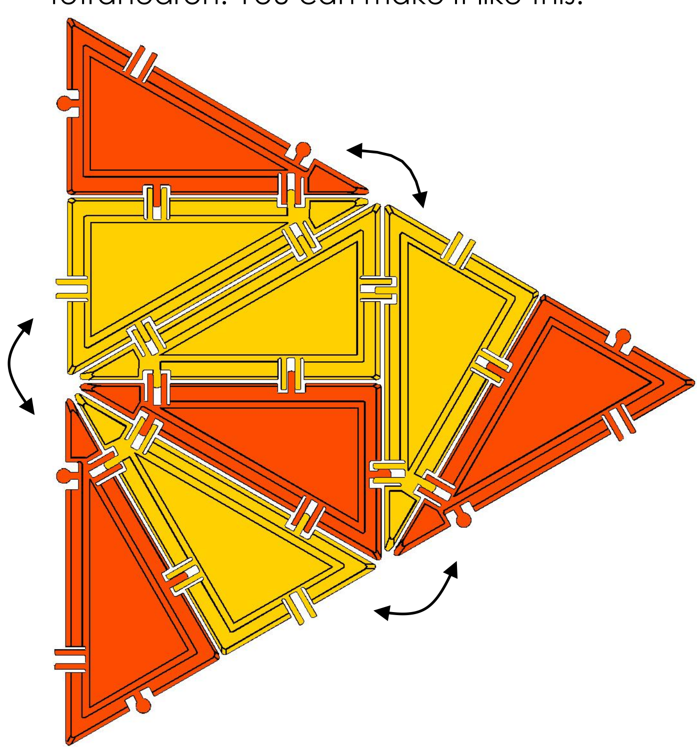
3.

Hints

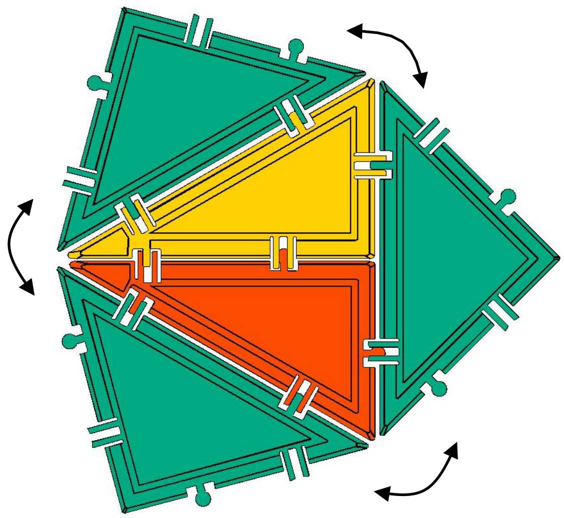
Solution 1 one can be made from a triangle we have already seen:



Just as you did with the square pyramid, bring the outside triangles up and snap together as shown by arrows. This shape is called a *regular tetrahedron*, and it is made of 4 identical equilateral triangles. You can make it in many sizes, depending on the size of your equilateral triangle. For example, Solution 2 is a larger regular tetrahedron. You can make it like this:



Solution 3 shows a tetrahedron that is NOT regular. While its base is an equilateral triangle, its side faces are isosceles triangles. Here is how you can make it:



A follow up would be to ask students to try to describe the difference in the shapes of the tetrahedra of Solutions 2 and 3.

Q: How are these tetrahedra similar?
A: They both have 4 corners, 4 faces, and 6 edges. They have the same base, which is an equilateral triangle.

Q: How are these tetrahedra different?
A: One of them is “taller” than the other. The “shorter” one has a less sharp peak.

