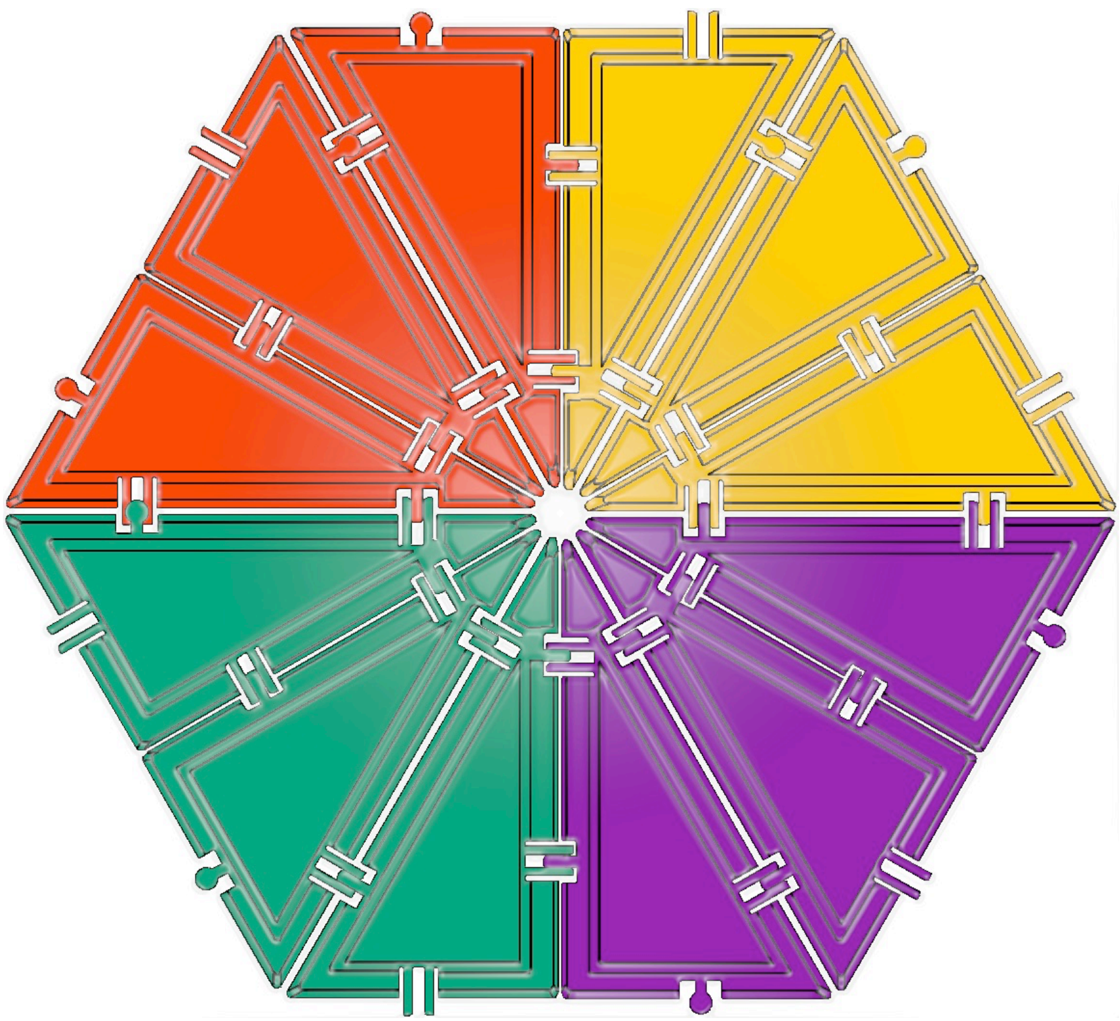
FRACTIONS

GEOMETILES™

Fractions

For grade 1 and up



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Version: 10/22/15

Introduction

This workbook takes the student from the very basics of fractions (Grade 1) all the way to multiplication and division of fractions (Grade 6). About half of the exercises are similar to those in the "Tangrams" workbook, but with the added fractions component. The rest of the exercises are divided between building 2D and 3D shapes, word problems, and basic calculations, all incorporating fractions skills. The kinesthetic aspect helps make the exercises more enjoyable and solidify students' understanding of fractions. It is recommended that you do the "Tangrams" and "Shape Challenge" workbooks before this one in order to give students a chance to familiarize themselves with the shapes before adding fractions concepts.

Activities in this workbook and the Common Core State Standards (CCSS)

CCSS for Mathematical Content supported by activities in this workbook

1.G.A.3 *Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.*

2.G.A.2 *Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.*

2.G.A.3 *Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.*

3.NF.A.1 *Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.*

3.NF.A.3 *Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.*

4.NF.A.1 *Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though* *the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.*

4.NF.A.2 *Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

4.NF.B.3 *Understand a fraction a/b with a > 1 as a sum of fractions 1/b.*

4.NF.B.4 *Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.*

5.NF *Use equivalent fractions as a strategy to add and subtract fractions. Apply and extend previous understandings of multiplication and division.*

6.NS.A.1 *Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.*

CCSS for Mathematical Practice supported by activities in this workbook

MP.1 *Make sense of problems and persevere in solving them.*

Each problem consists of a goal (make a certain shape expressing a given fraction) and a set of constraints (the tiles students can use). Students are encouraged to try different combinations of shapes until they find one that solves the problem.

MP.2 *Reason abstractly and quantitatively.*

Students solve word problems by representing quantities in the problems as fractions.

MP.3 *Construct viable arguments and critique the reasoning of others.*

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

MP.4 Model with mathematics.

Students model real life situations involving dividing pizza, pie, etc. using the tiles.

MP.8 *Look for and express regularity in repeated reasoning.*

For example, students model multiplication of two fractions by building upon their previous work in modeling multiplication of a fraction by an integer.

Learning objectives of this workbook

a. Work several math skills at a time: solve tangrams, construct 2-dimensional and 3-dimensional shapes and exercise fractions skills.
b. Understand operations on fractions using visual fractions models, as described in the CCSS.
c. Model real life problems with fractions.
d. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
e. Learn how to collaborate with one another.

Arguably, point c. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

Recommended classroom plan

The Level 1 exercises are designed so that two groups of students works with a set of 96 tiles. The set will be split up into 2 sets with 42 pieces in each: SET A and SET B. Have a group of students work with SET A, and another group work with SET B. Then switch the sets so that the SET A students will get a chance to work with SET B and vice versa. It might be good to separate tiles into sets and put the sets in clear plastic bags before the lesson starts

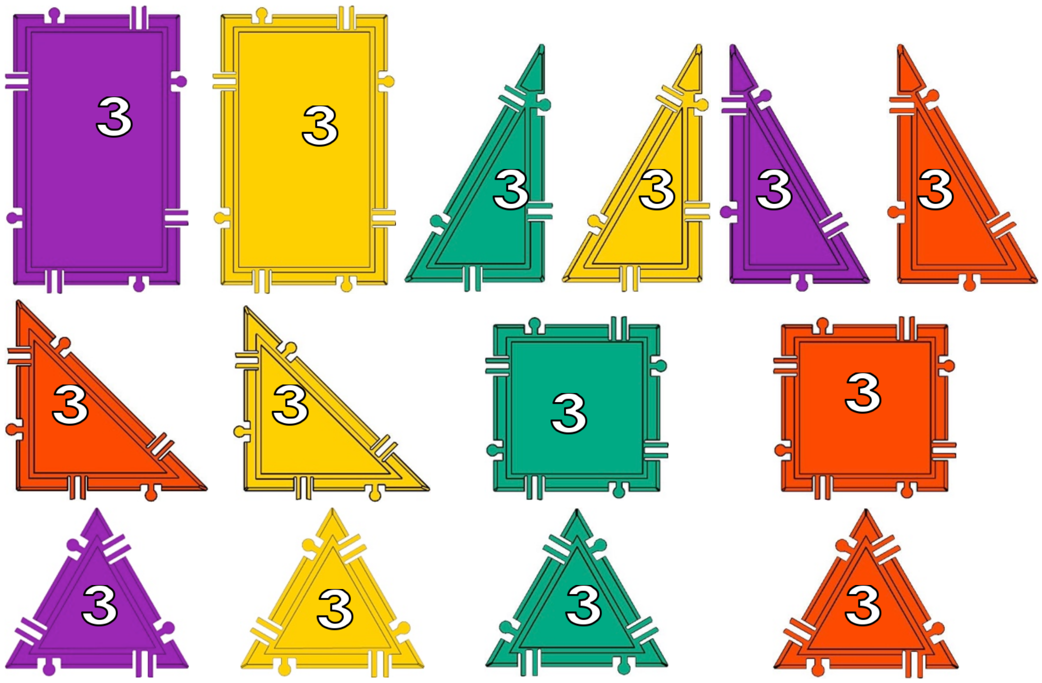
Each group can consist of 2-3 students.

The Level 2 exercises are more involved and require one group of students work with all the triangles and all the squares in a set of 96 tiles. Pentagons are not used in this workbook.

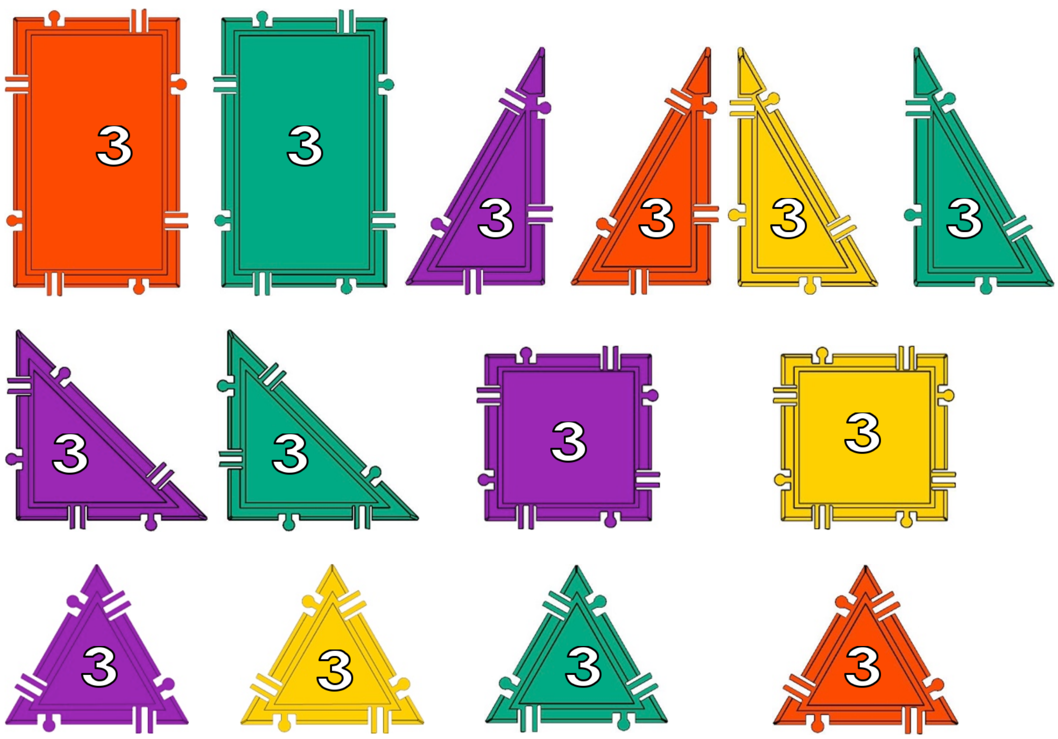
Level 1

Pieces need for each group doing Level 1 exercises are shown below.

SET A, Group A



SET B, Group B



Tangrams and Fractions

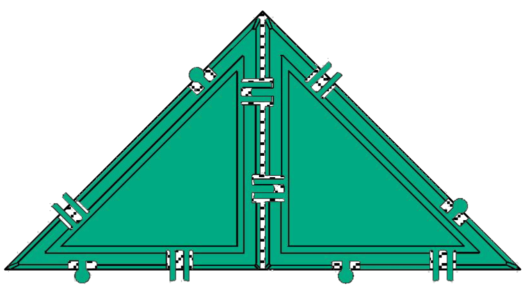
The Rules

Explain to the students that they are supposed cover the shaded figures with the tiles precisely.

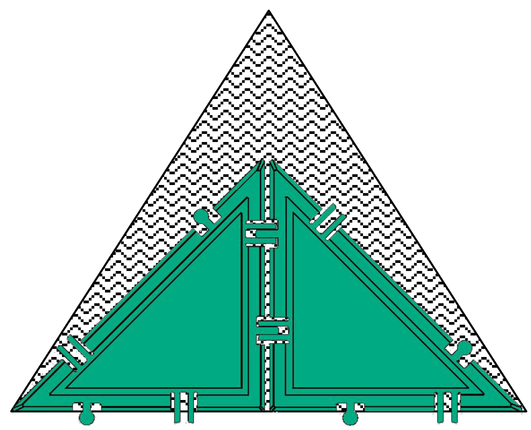
The rules are:

1. Any shapes that are next to each other have to snap together.
2. The edges of the shape should be exactly on top of the edge of the shaded figure. They should not be inside the shaded figure or outside its border. See example below.

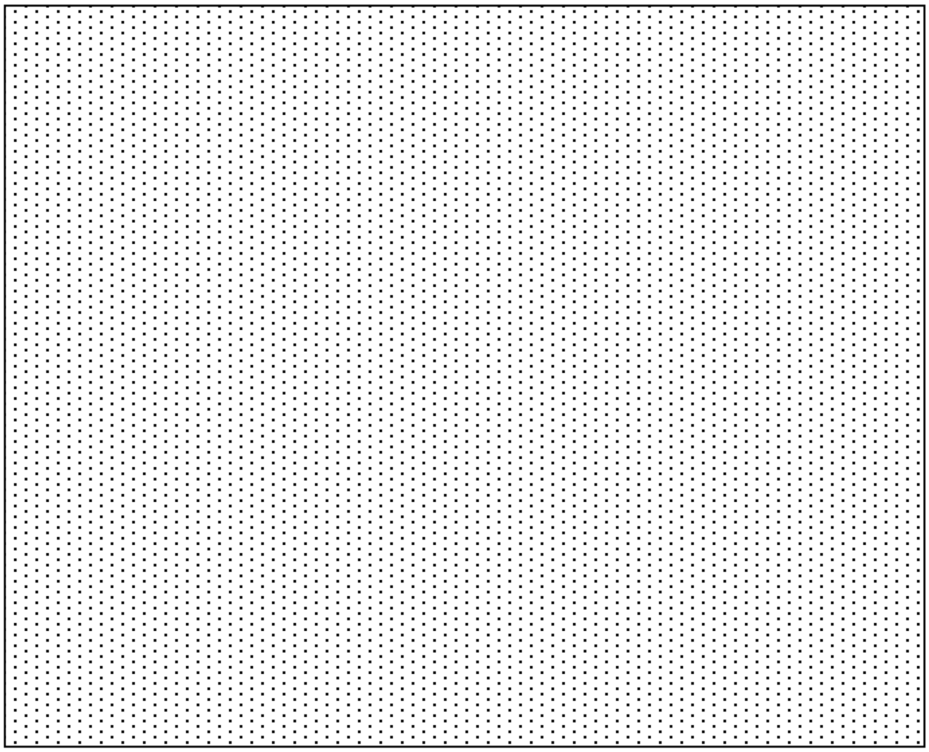
RIGHT:



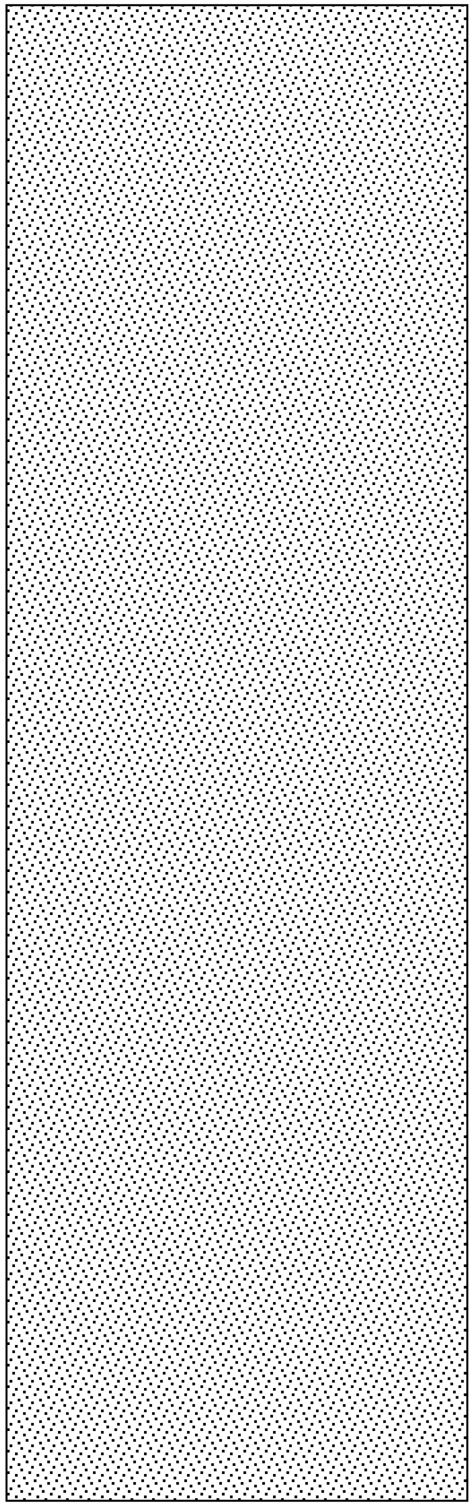
WRONG:



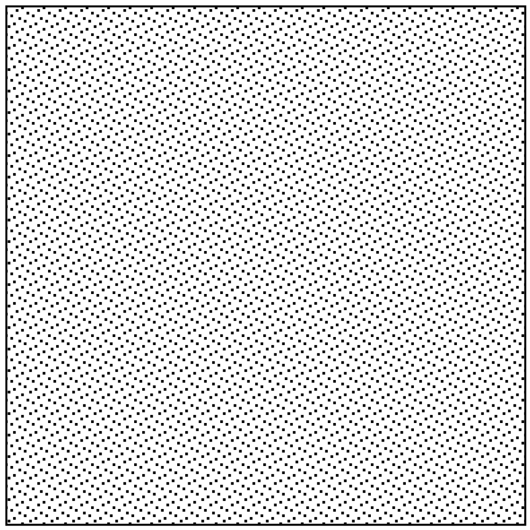
1. Make the shape so that that **half of it is purple** and **half of it is yellow**.



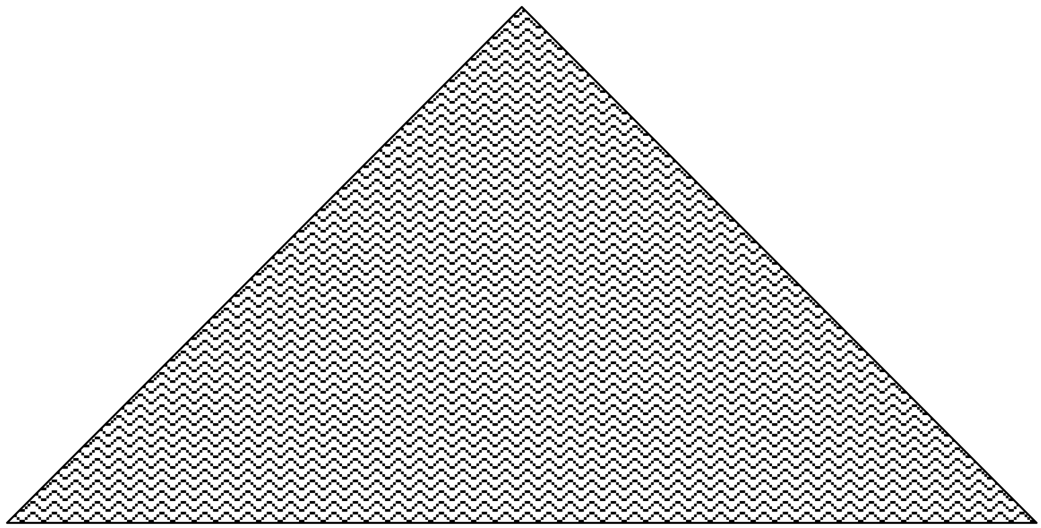
2. Make the shape so that that **half of it is purple** and **half of it is yellow**.



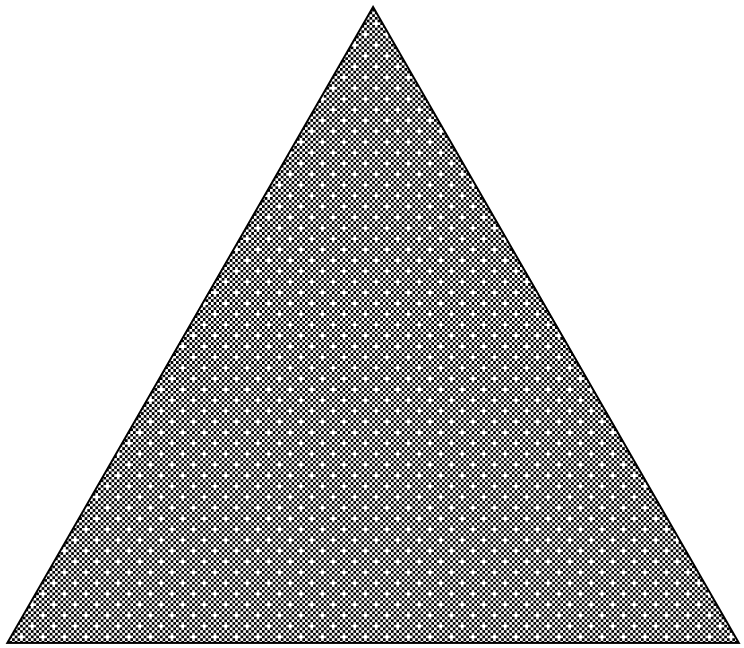
3. Make the shape so that that **half of it is orange** and **half of it is yellow**.



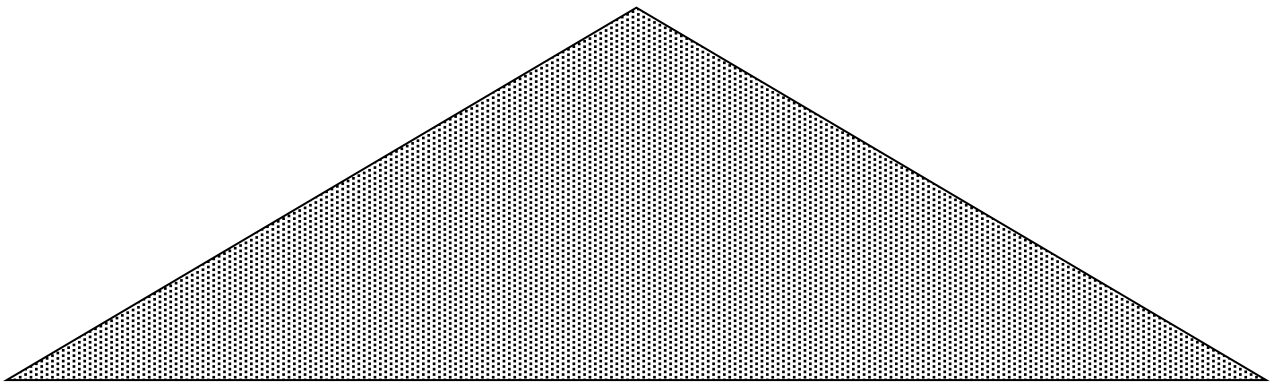
4. Make the shape so that that **half of it is orange** and **half of it is yellow**.



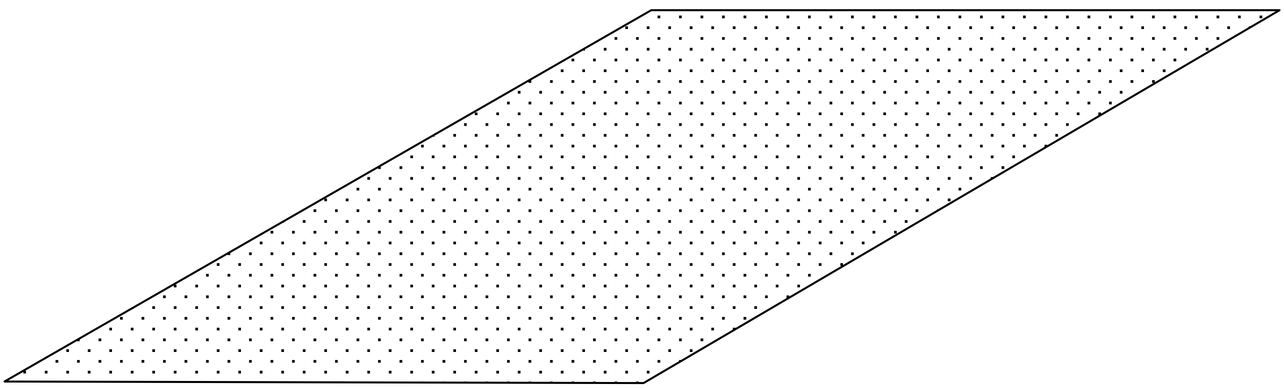
5. Make the shape so that **half of it is purple** and **half of it is green**.



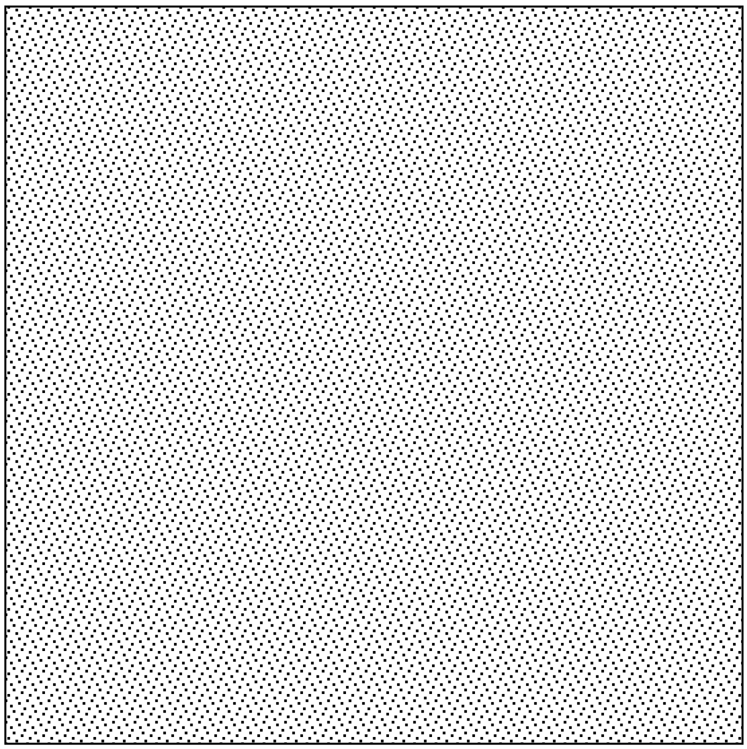
6. Make the shape so that **half of it is purple** and **half of it is green**.



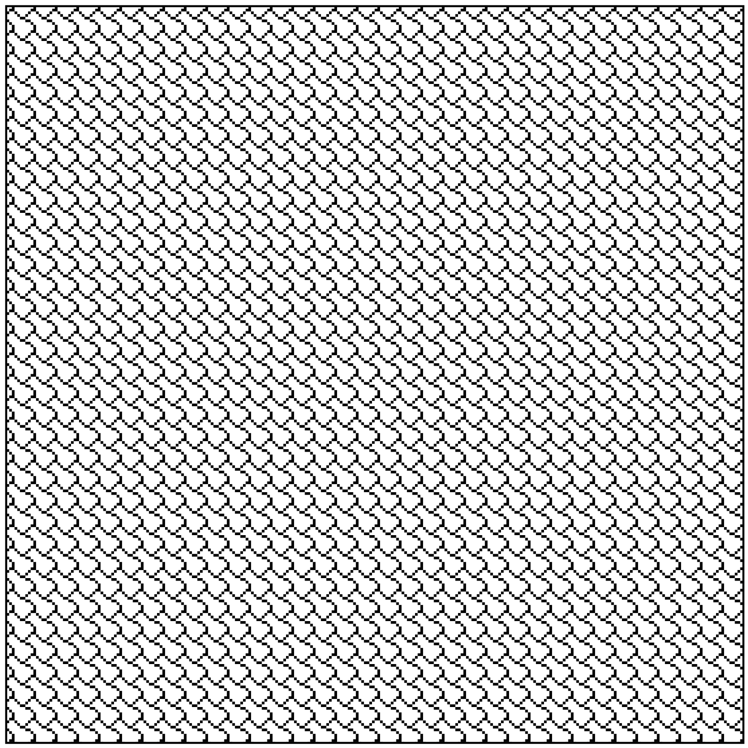
7. Make the shape so that **half of it is purple** and **half of it is green**.



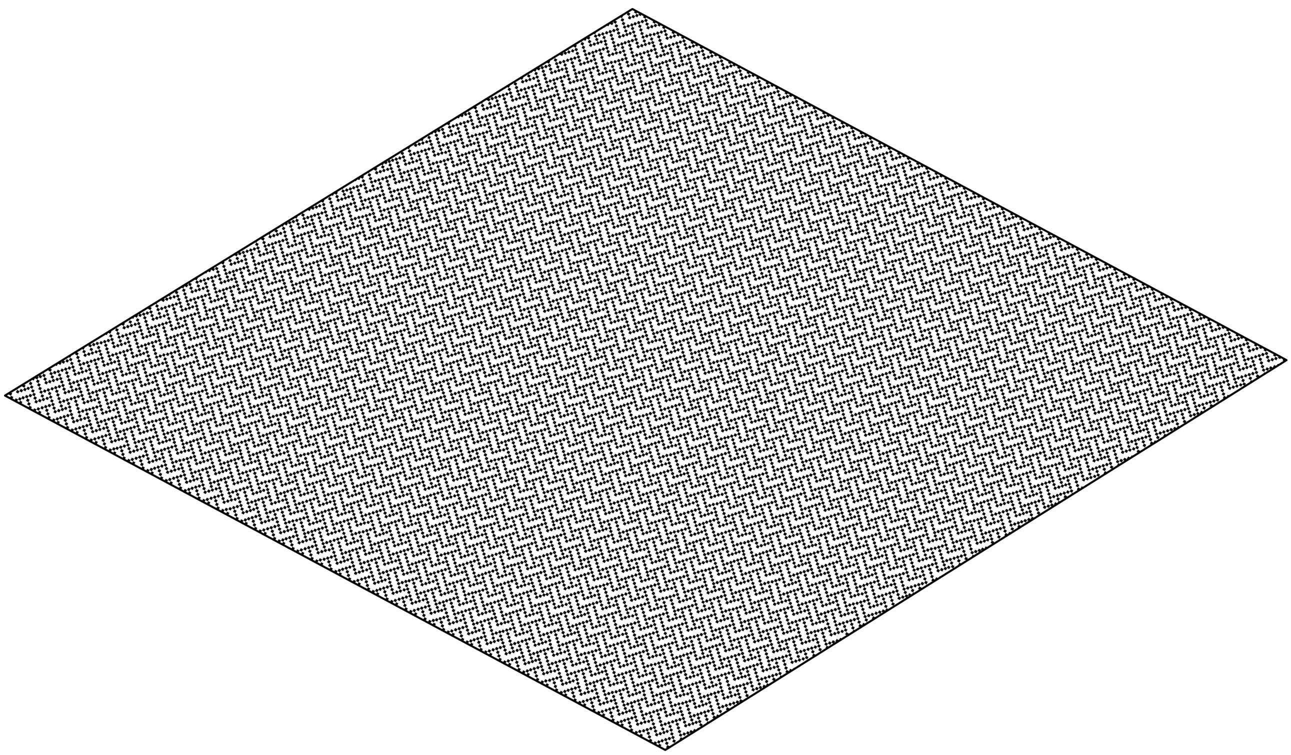
8. Make the shape so that half **of it is yellow and half of it is orange**. Try to construct an answer in 2 different ways.



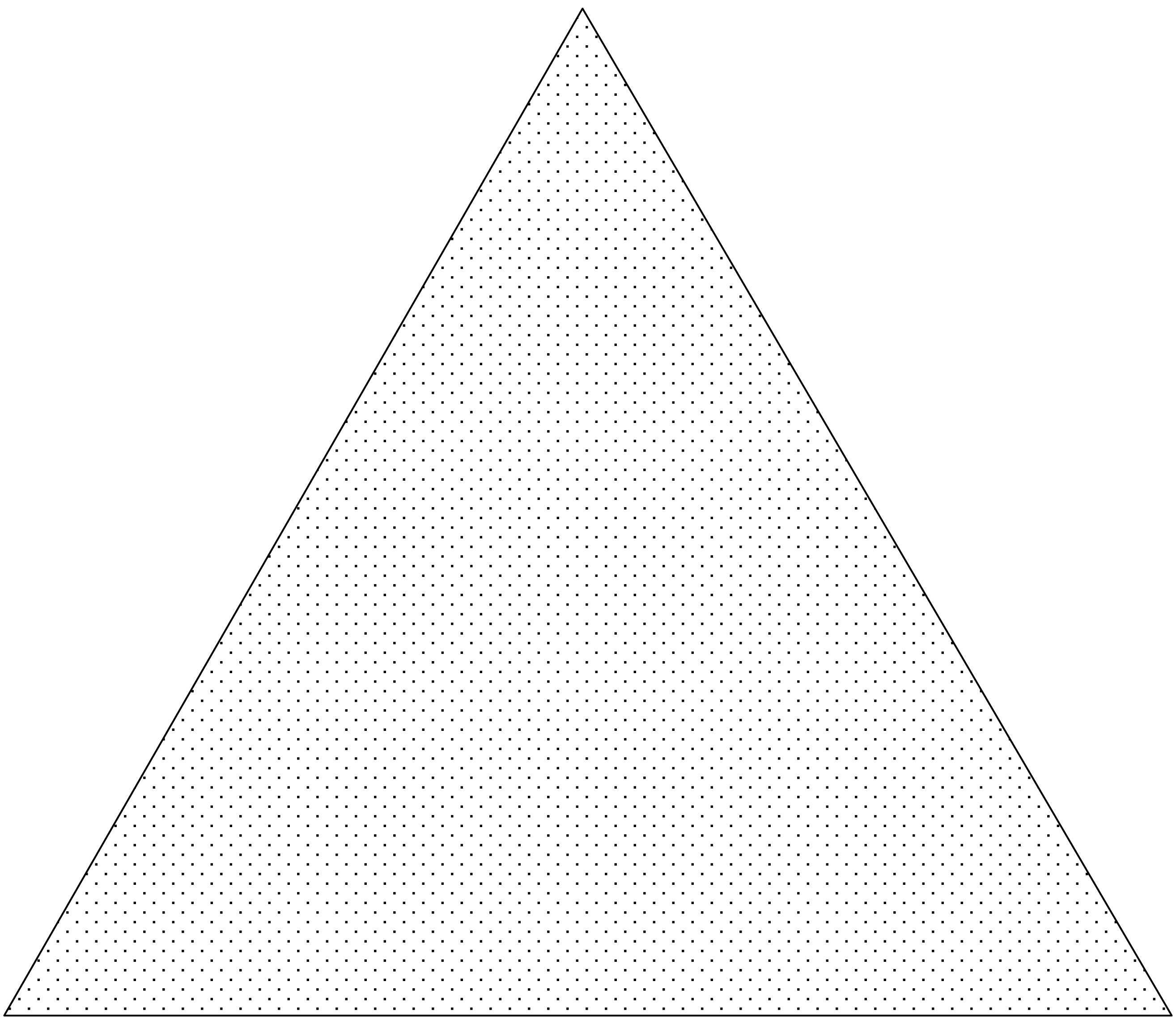
9. Make this shape so that **one fourth of it is yellow and three fourths of it is orange**.



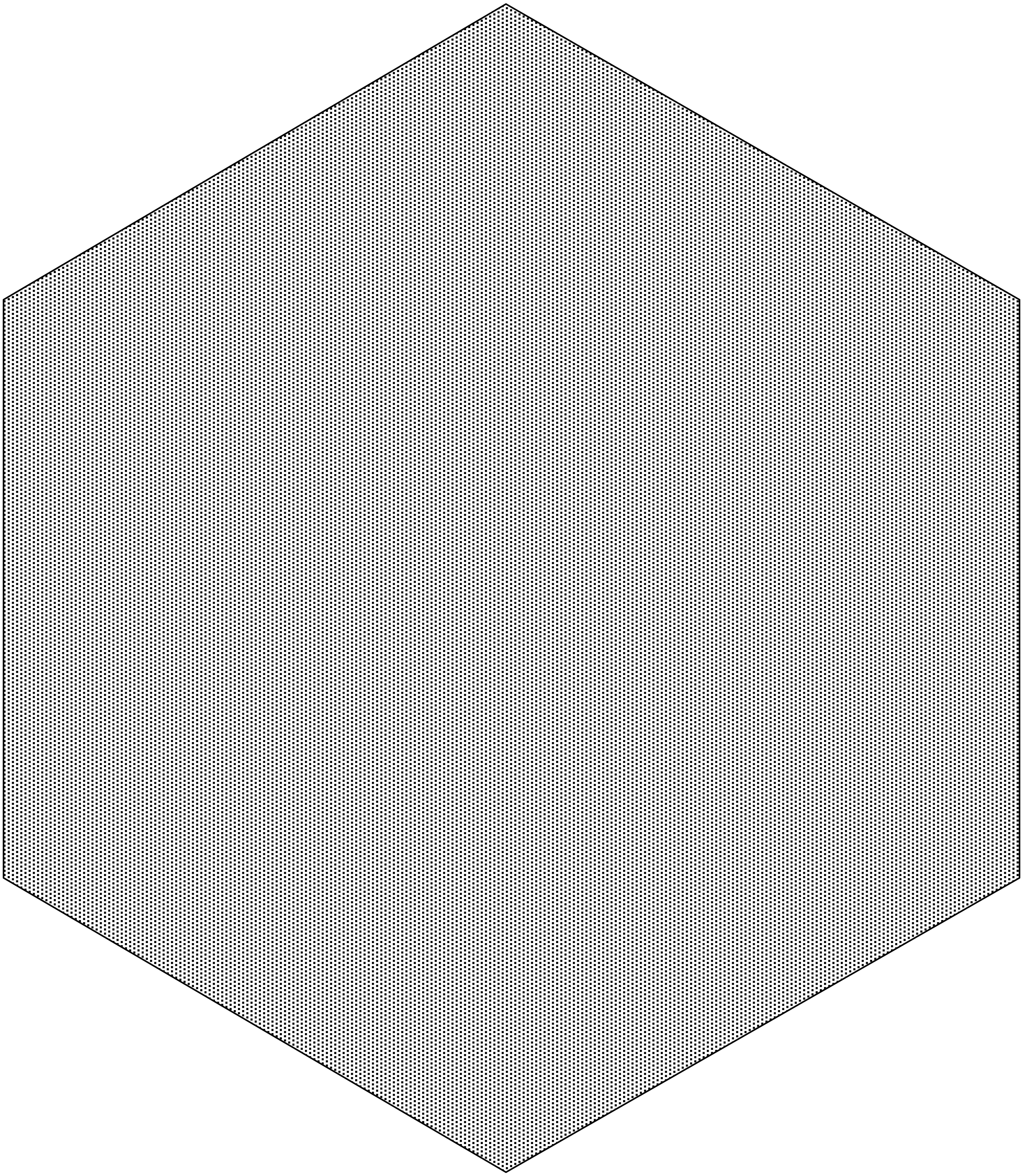
10. Make this shape so that **half of it is green** and **half of it is orange**.



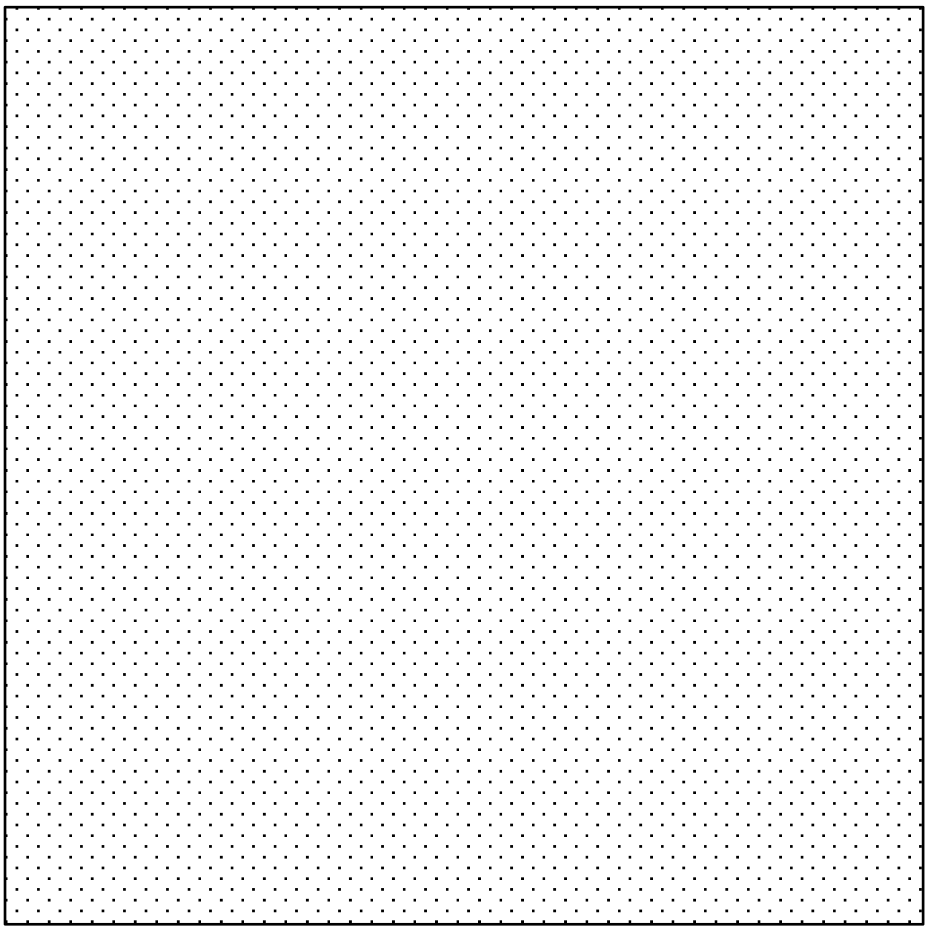
11. Make the shape so that **half of it is yellow** and **half of it is orange**.



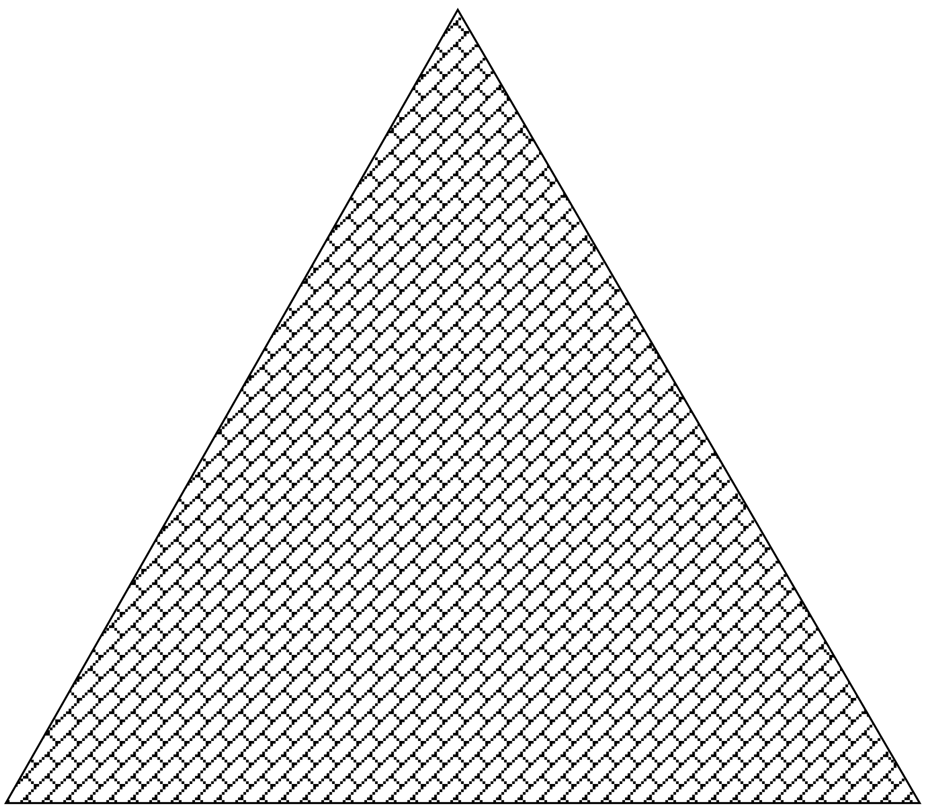
12. Make the shape so that **a quarter of it is purple**, **a quarter of it is green, a quarter of it is yellow**, and **a quarter of it is orange**.



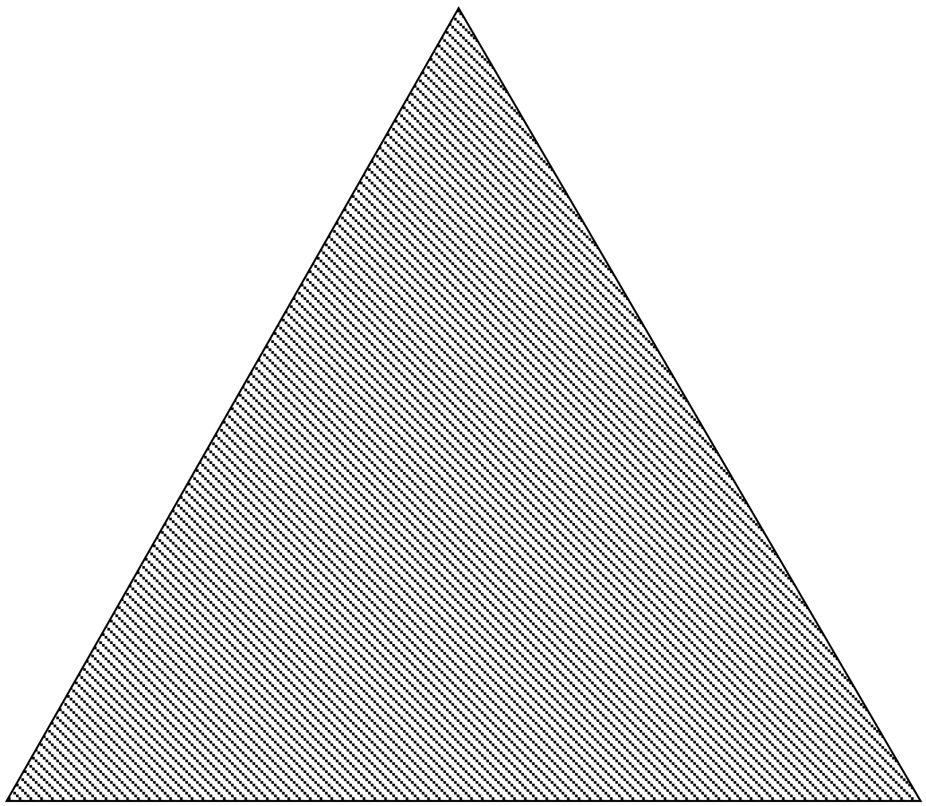
13. Make the shape so **a quarter of it is orange, and the rest is green**. What fraction of it is green?



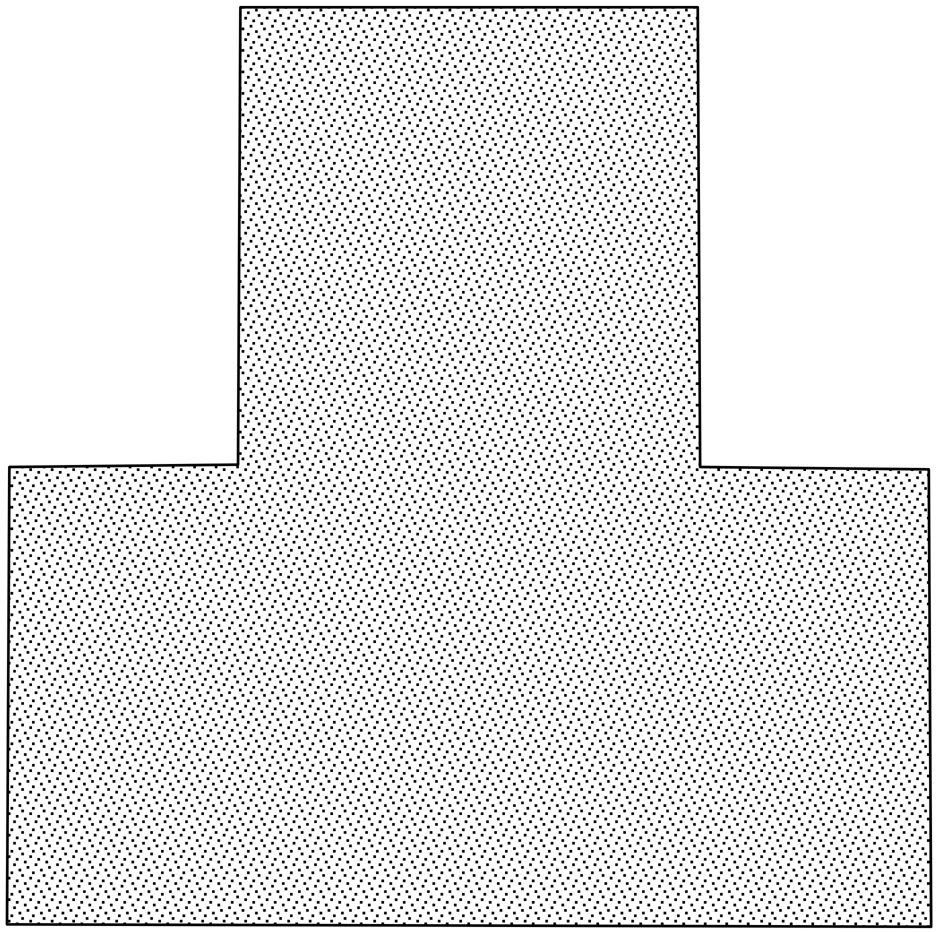
14. Make the shape so that **a quarter of it is orange, a quarter of it is yellow**, **a quarter of it is purple**, and **a quarter of it is green**.



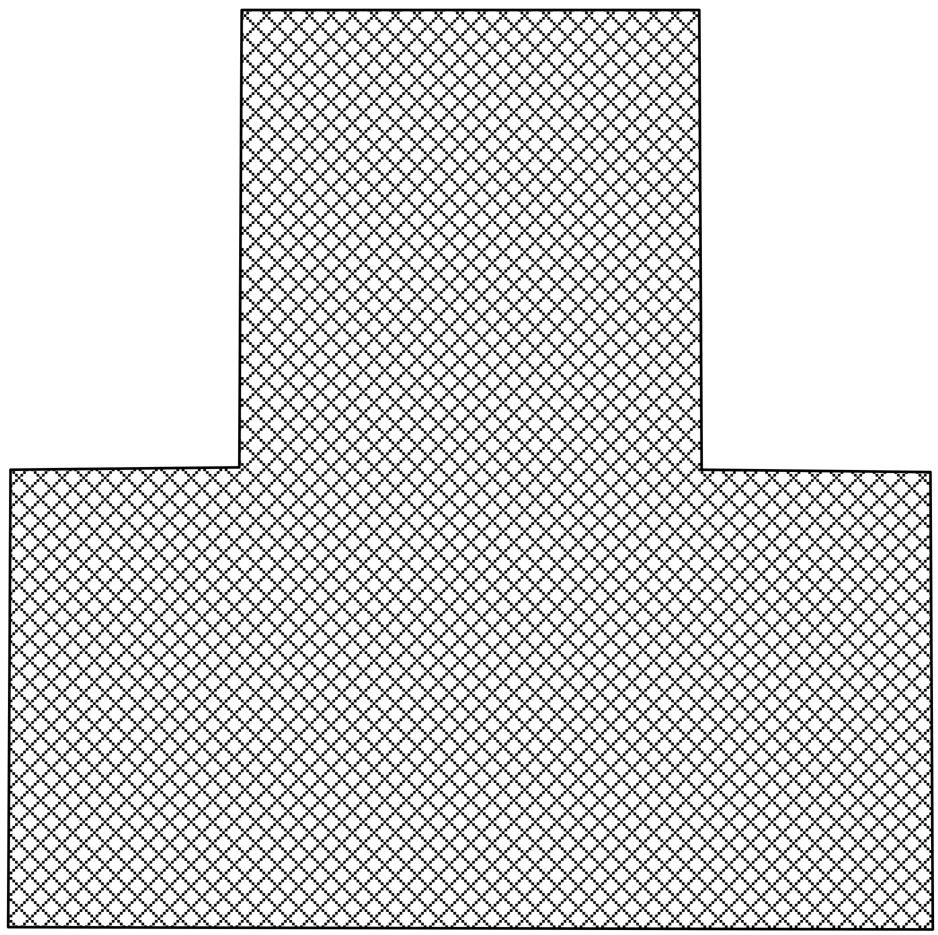
15. Make the shape so that a **half of it is orange, a quarter of it is yellow**, and **a quarter of it is purple**.



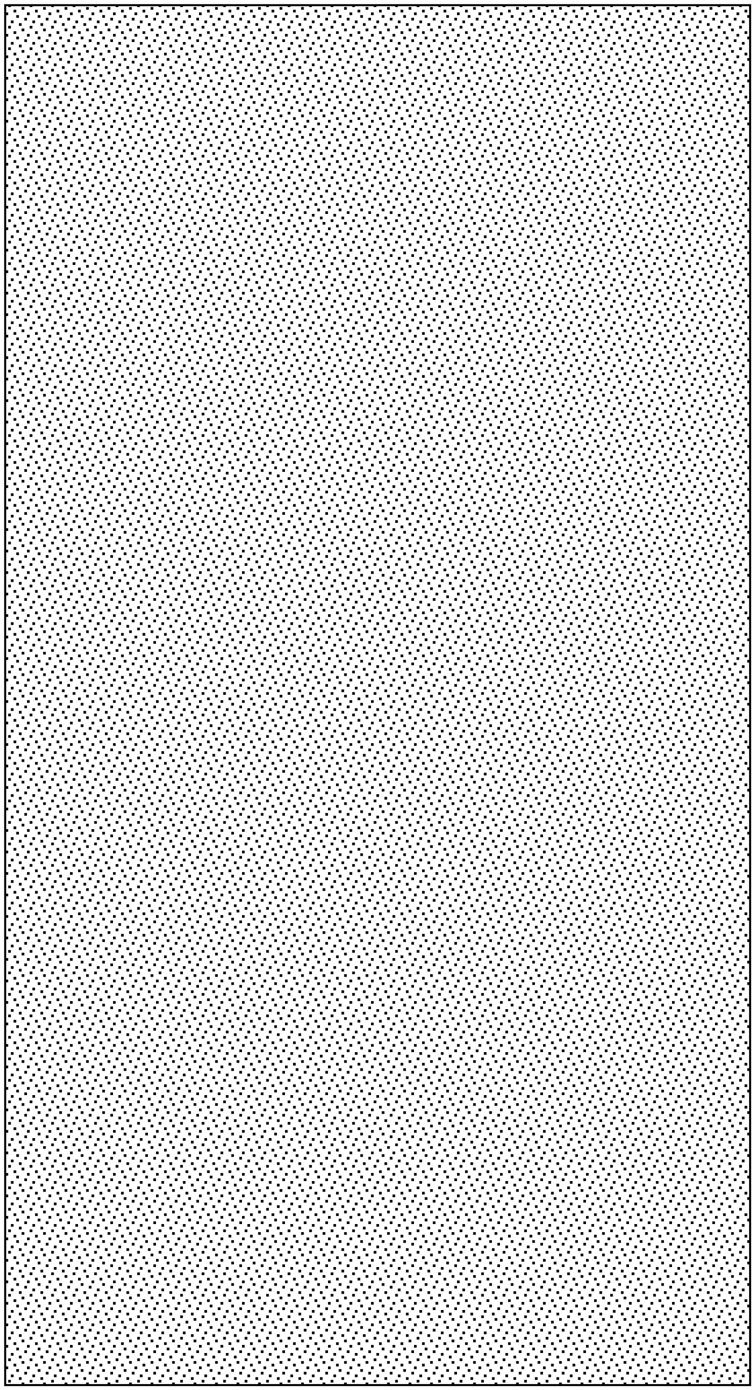
16. Make the shape so that **three thirds of it is green**.



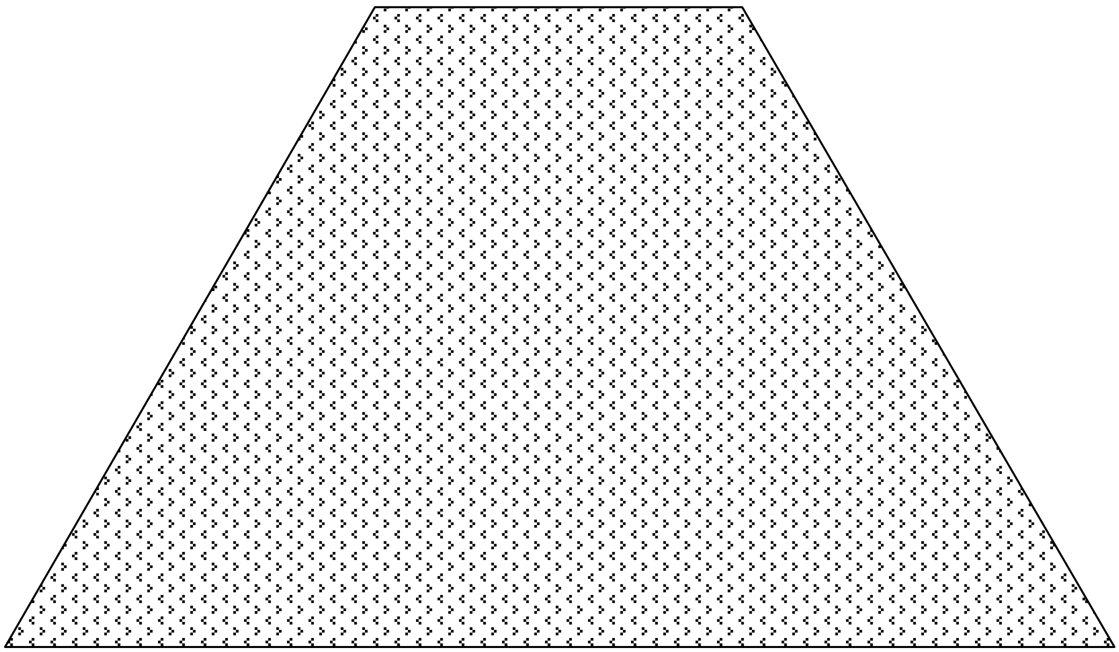
17. Make the shape so that **one third of it is orange, and the rest of it is green**. Try to find all the different ways to arrange the tiles.



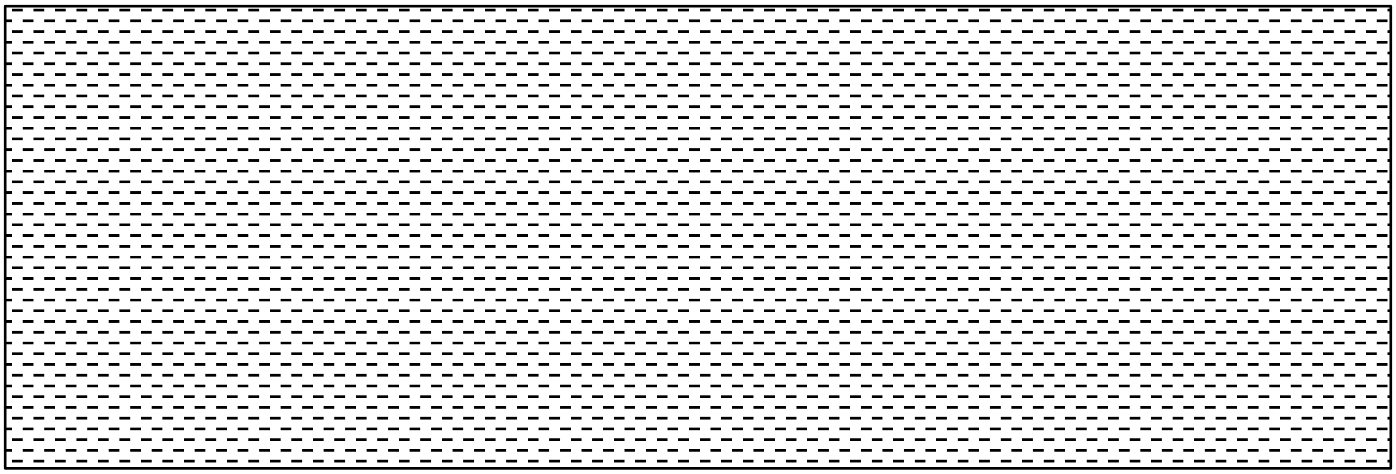
18. Make the shape so that **one third of it is yellow, and two thirds of it is purple**. Try to find all the different ways to arrange the tiles.



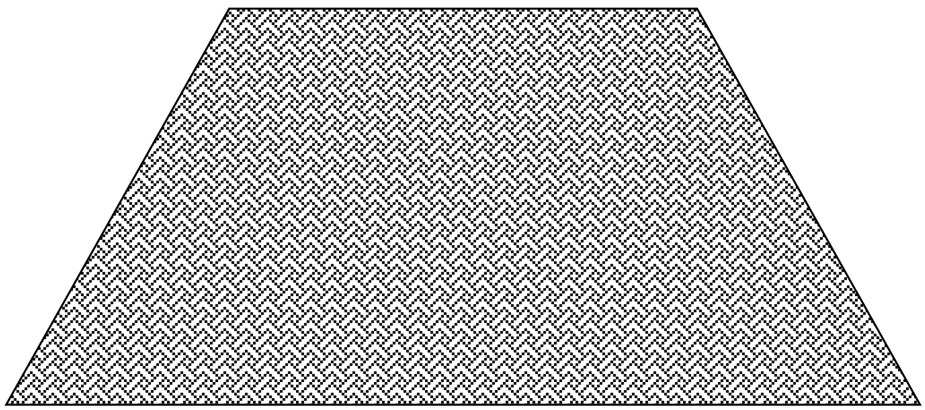
19. Make the shape so that **one quarter of it is yellow, and the rest of it is orange**. What fraction of it is orange? Try to find all the different ways to arrange the tiles.



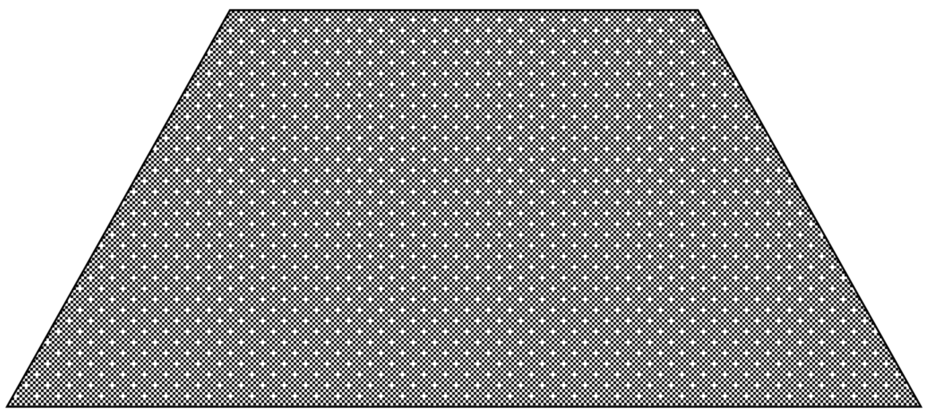
20. Make the shape so that **three thirds of it is orange**.



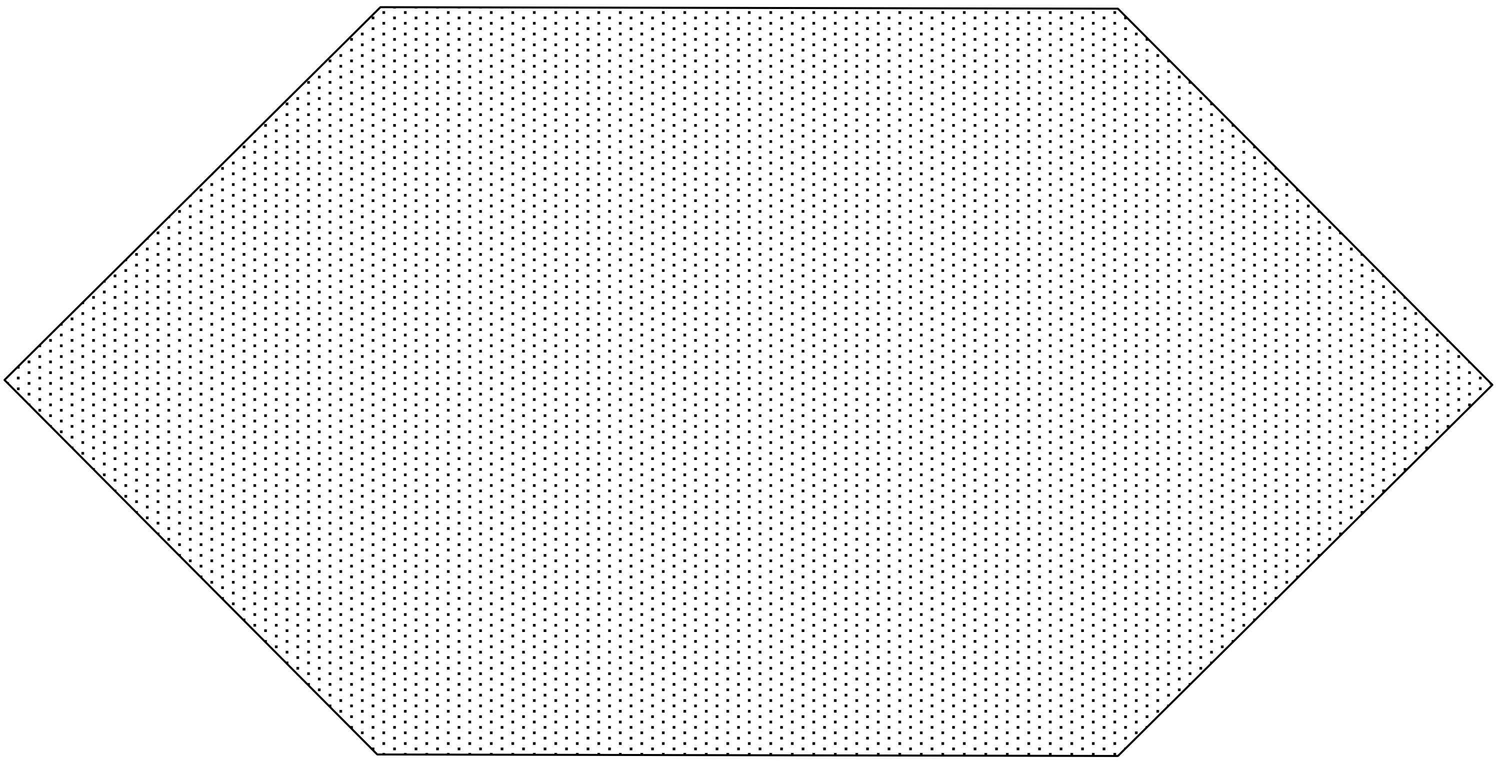
21. Make the shape so that **one third of it is purple and two thirds of it is orange**.



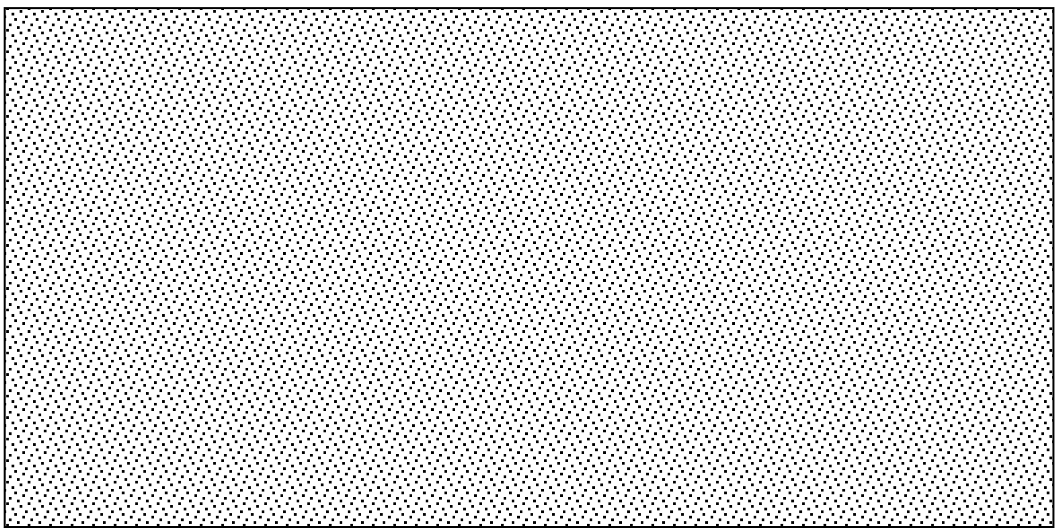
22. Make the shape so that **three thirds of it is yellow**.



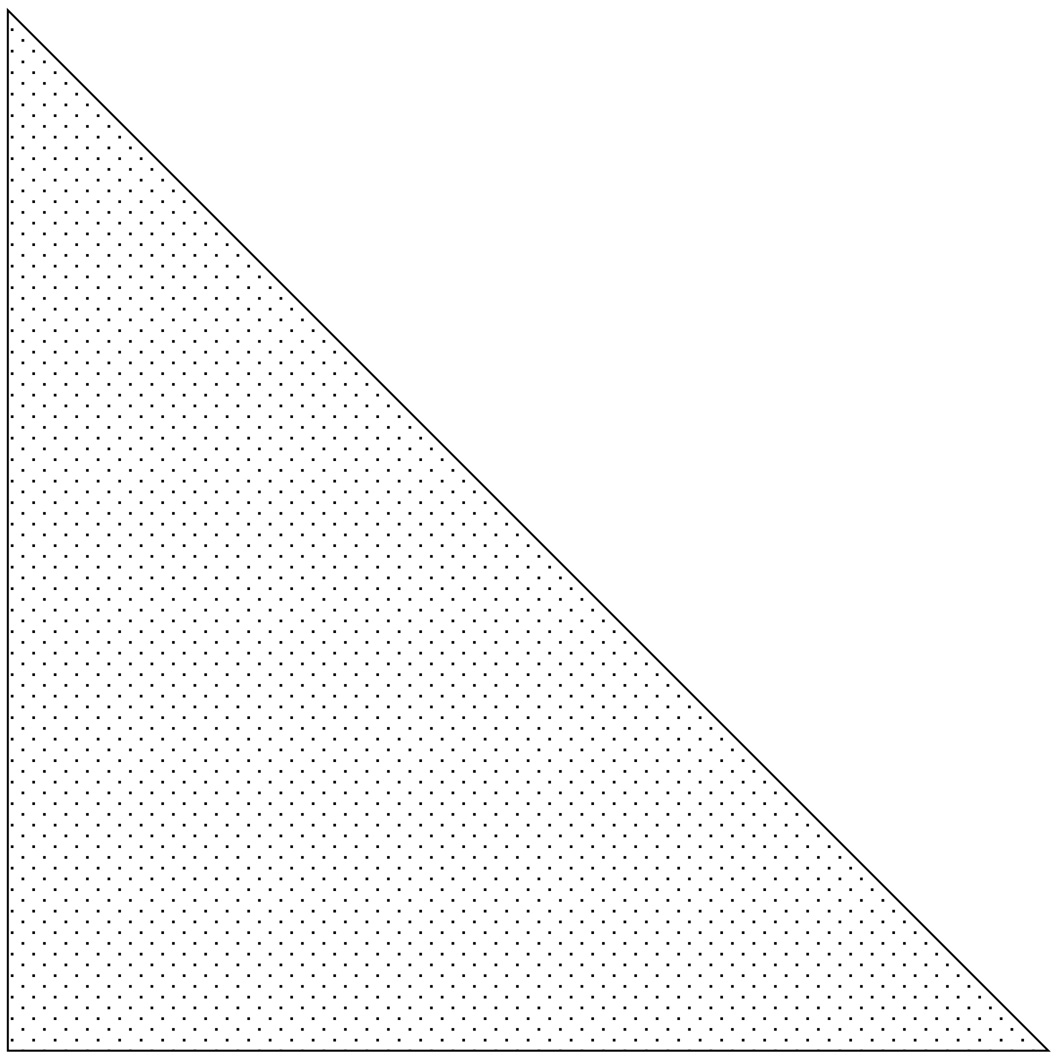
23. Make the shape so that that **half of it is purple** and **half of it is green**. Try to arrange the pieces in at least two different ways.



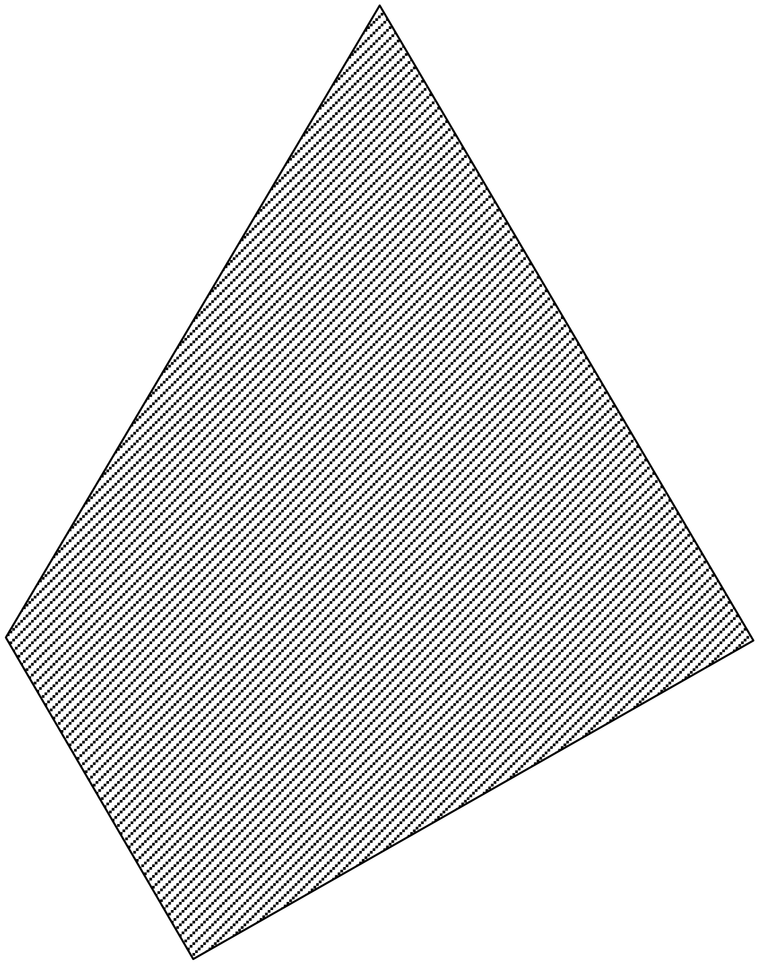
24. Make the shape so that that **half of it purple** and **half of it is green**. Try to arrange the pieces in at least two different ways.



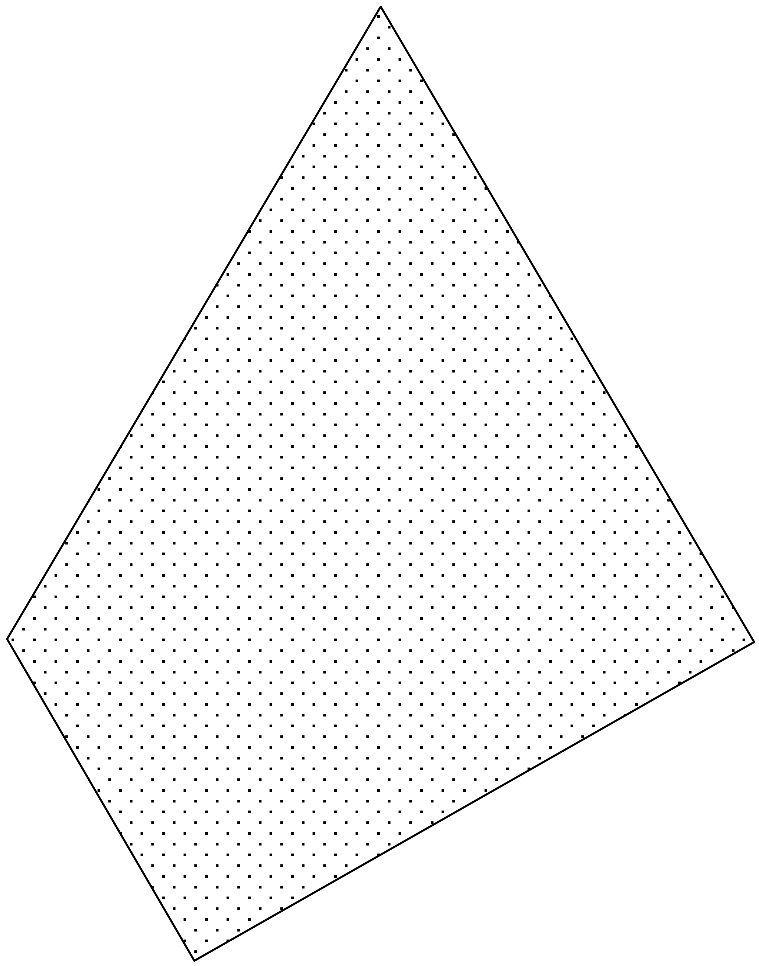
25. Make the shape so that that **three quarters of it is green** and **the rest is purple**. What fraction of it is purple? Try to arrange the pieces in at least two different ways.



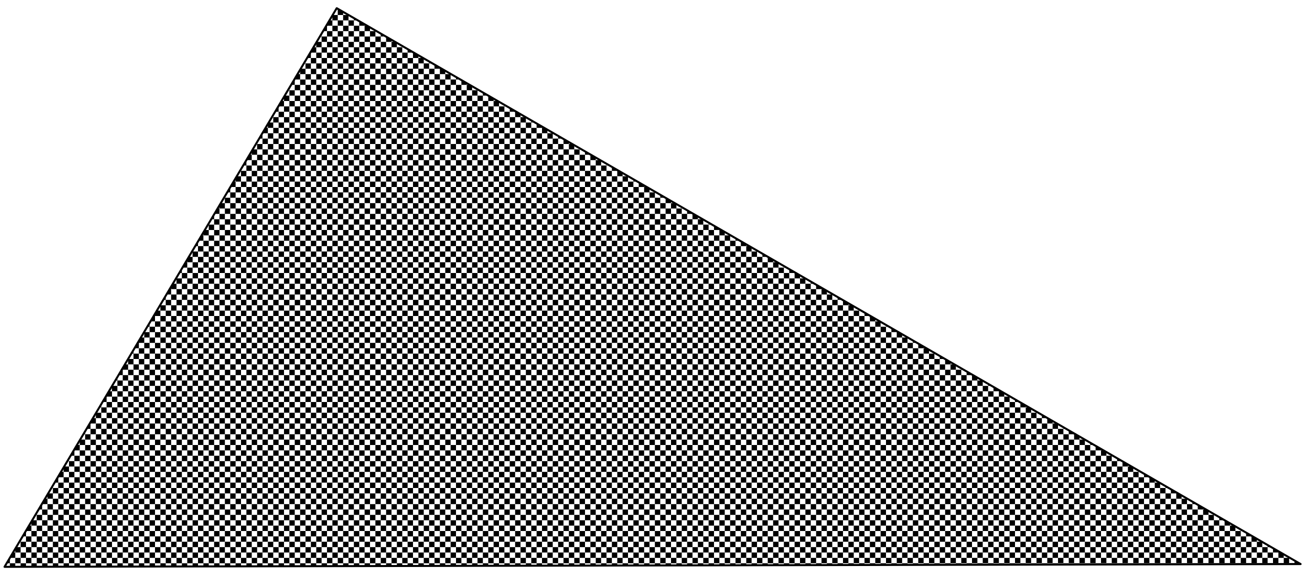
26. Make the shape so that **one third of it is orange and two thirds of it is yellow**.



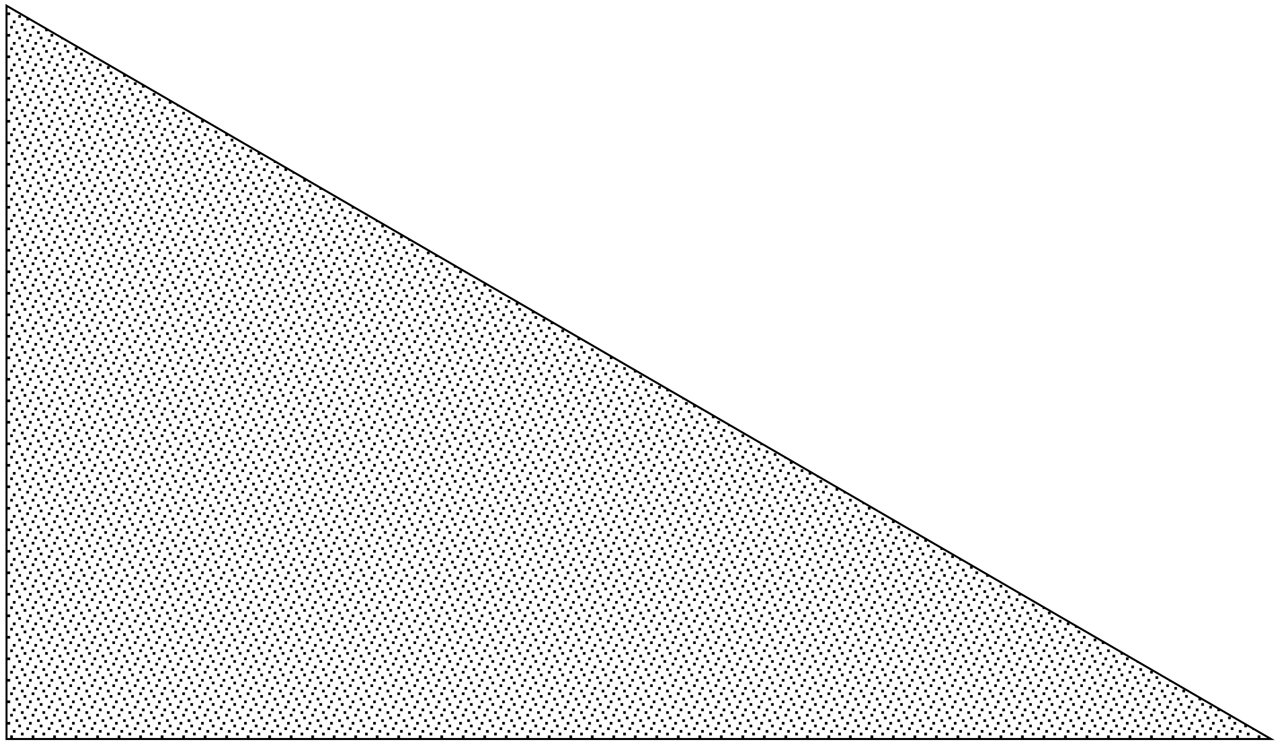
27. Make the shape so that **three thirds of it is purple**.



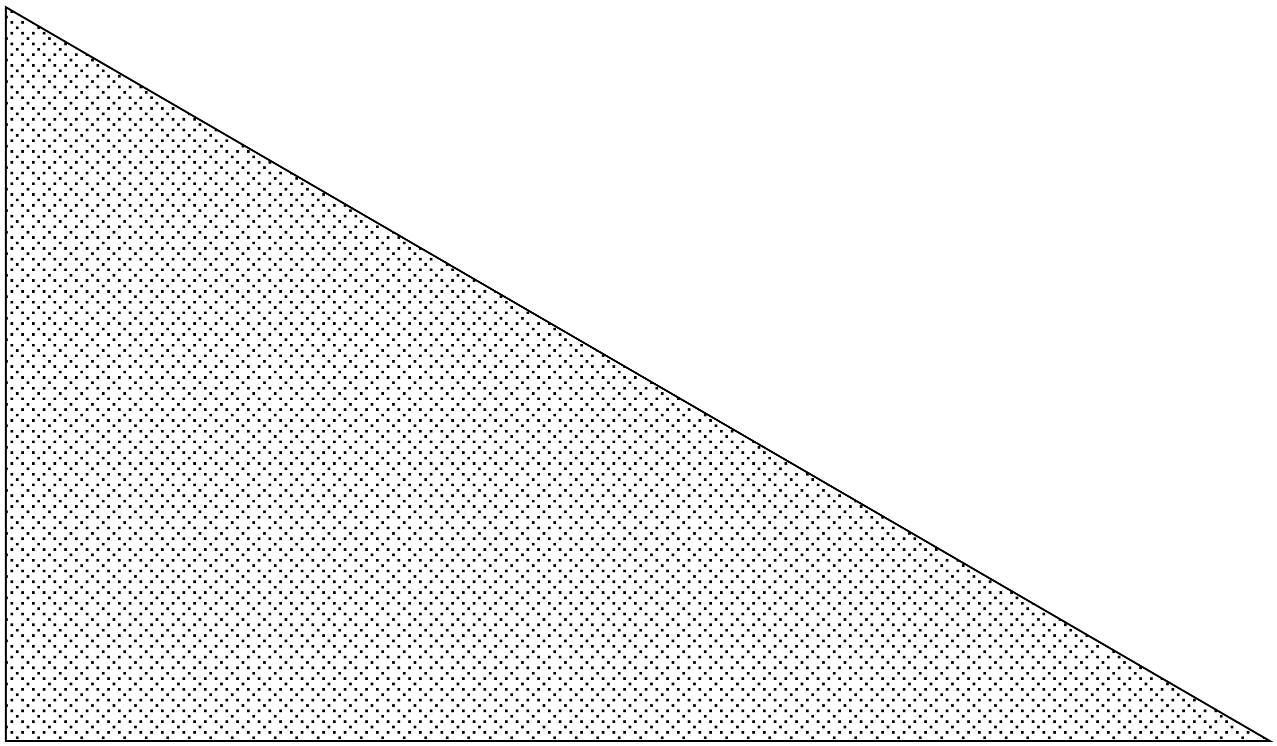
28. Make the shape so that **one third of it is yellow and the rest of it is orange**.



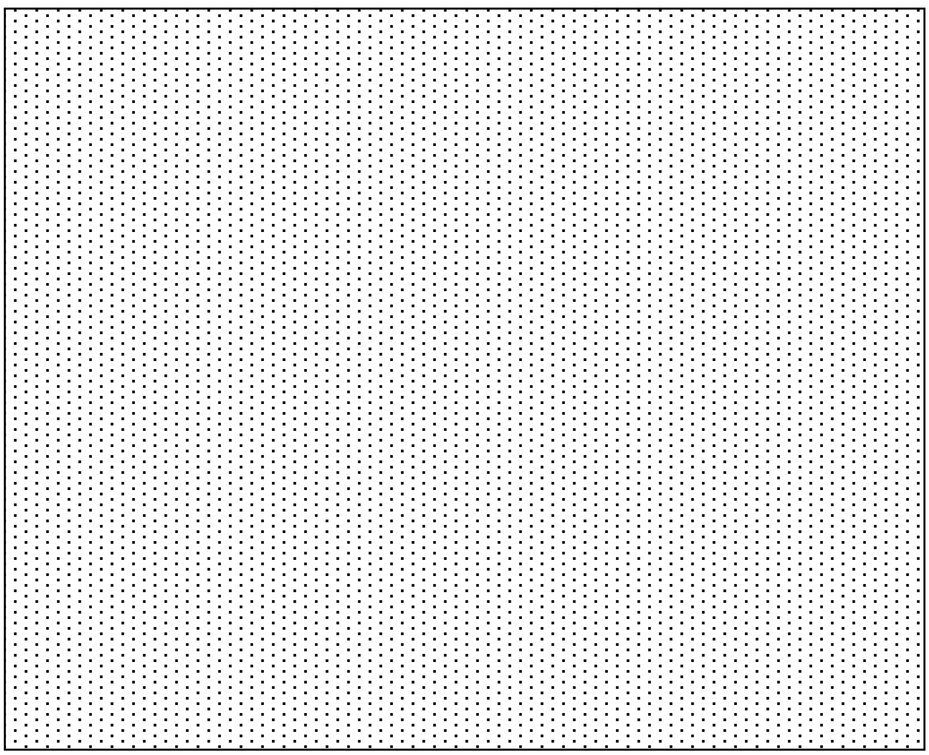
29. Make the shape so that **a quarter of it is yellow and the rest of it is orange**.



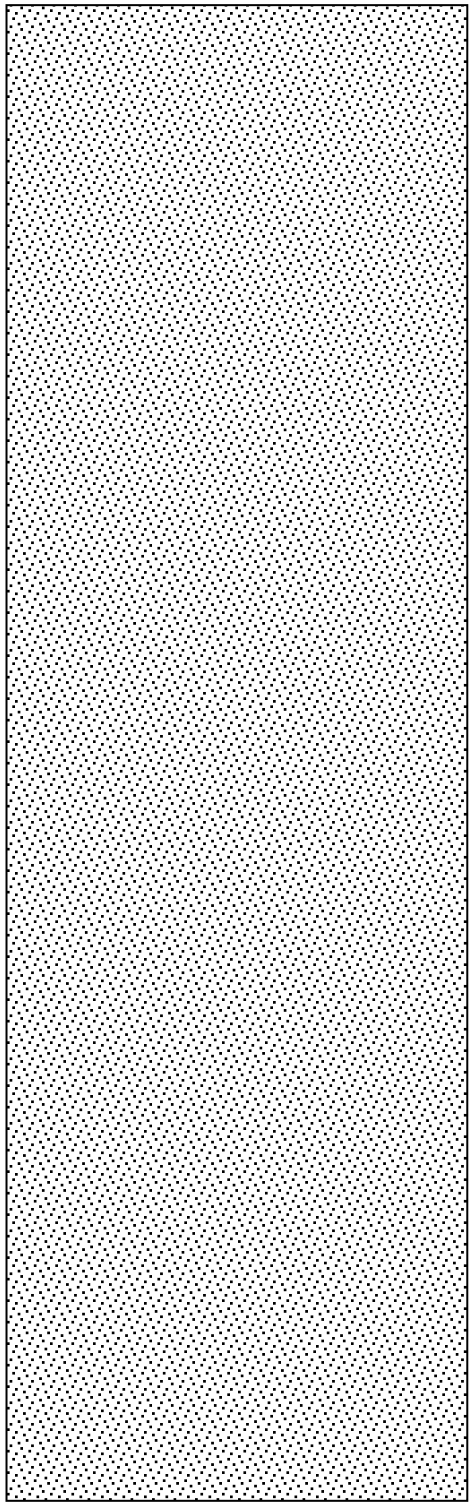
30. Make the shape so that **half of it of it is purple and half of it is green**.



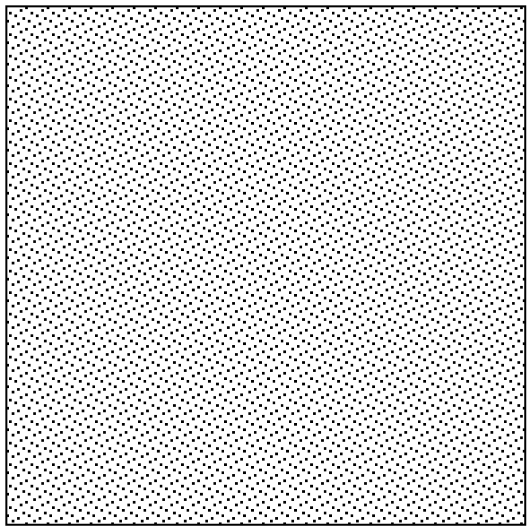
1. Make the shape so that that **half of it is orange** and **half of it is green**.



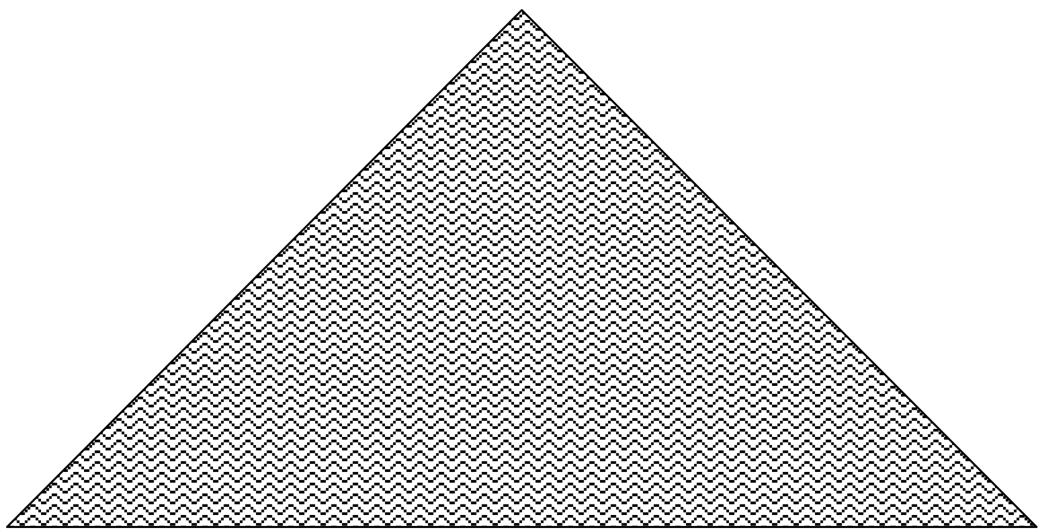
2. Make the shape so that that **half of it is orange** and **half of it is green**.



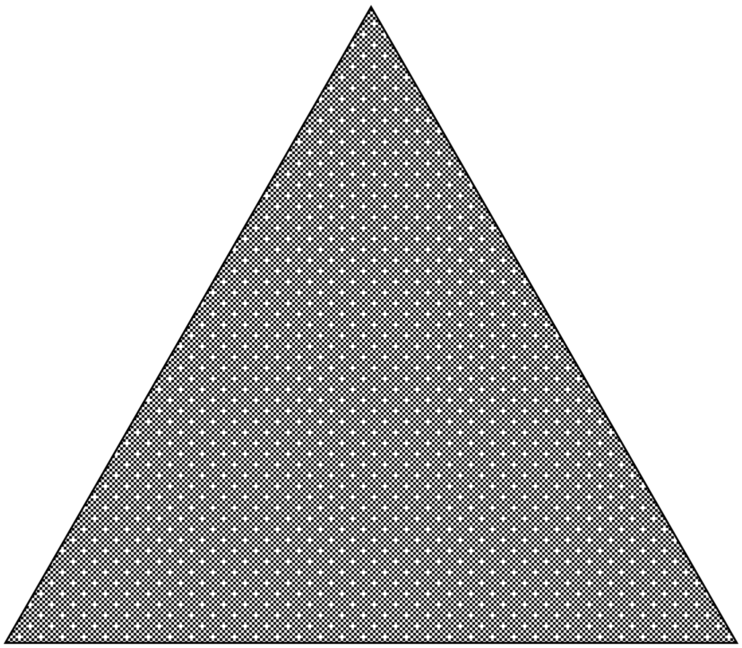
3. Make the shape so that that **half of it is purple** and **half of it is green**.



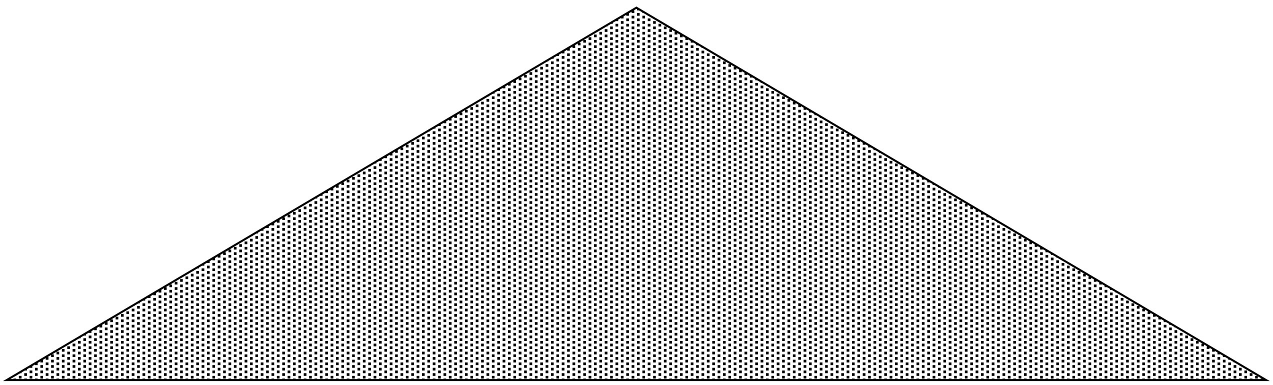
4. Make the shape so that that **half of it is purple** and **half of it is green**.



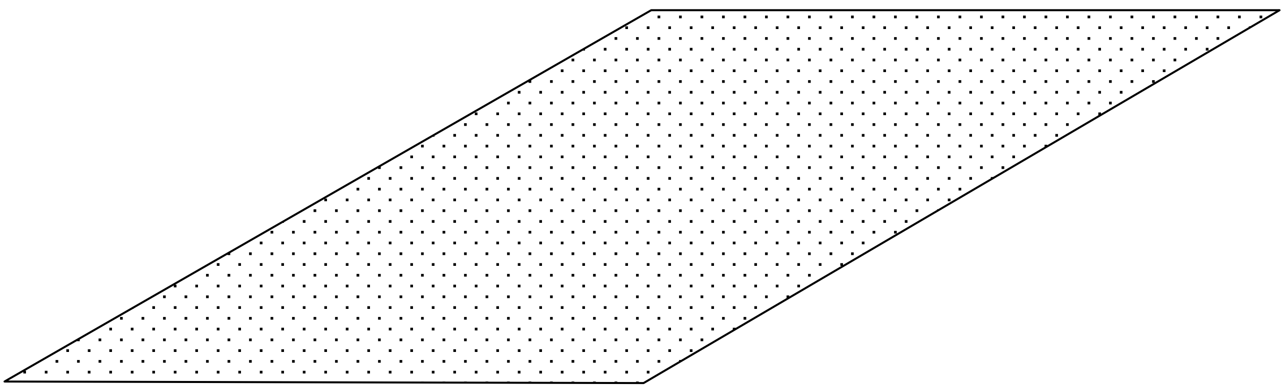
5. Make the shape so that **half of it is orange** and **half of it is yellow**.



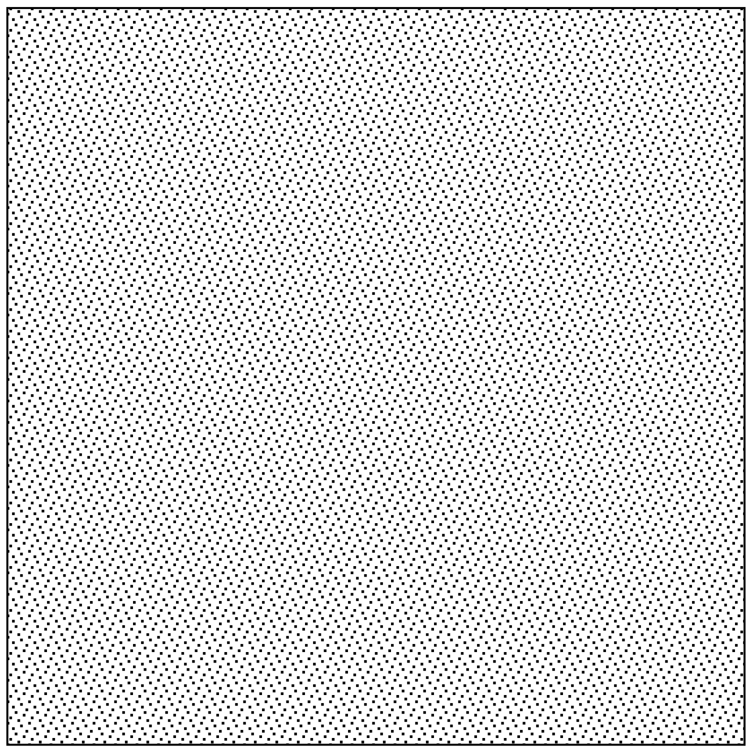
6. Make the shape so that **half of it is orange** and **half of it is yellow**.



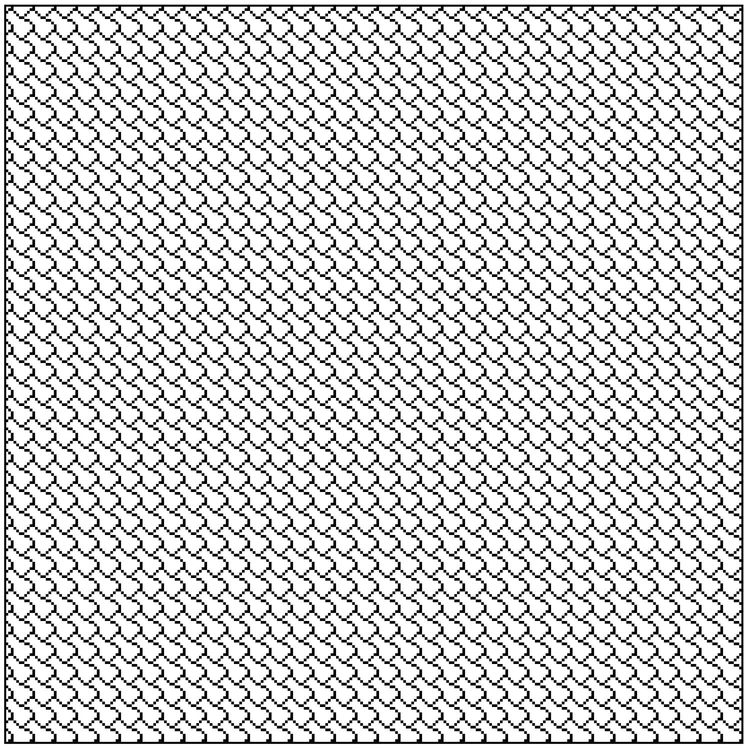
7. Make the shape so that **half of it is orange** and **half of it is yellow**.



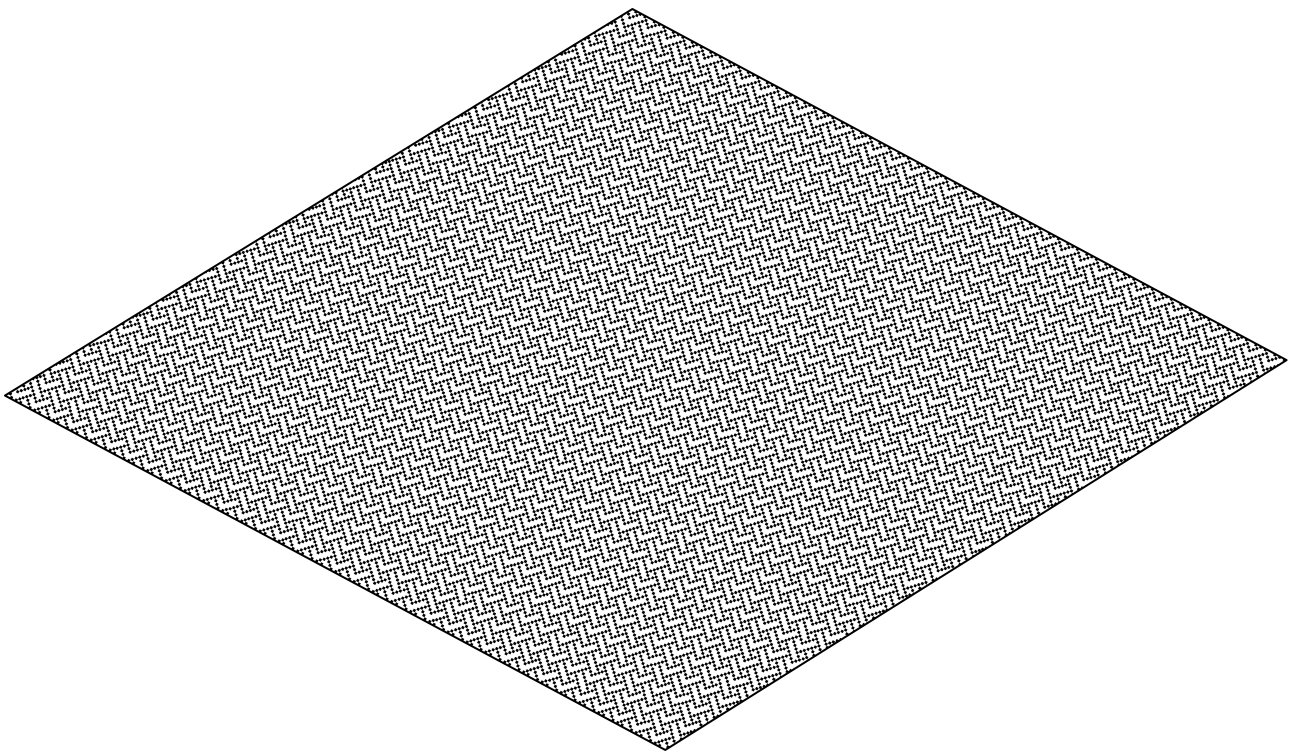
8. Make the shape so that half **of it is purple and half of it is green**. Try to construct an answer in 2 different ways.



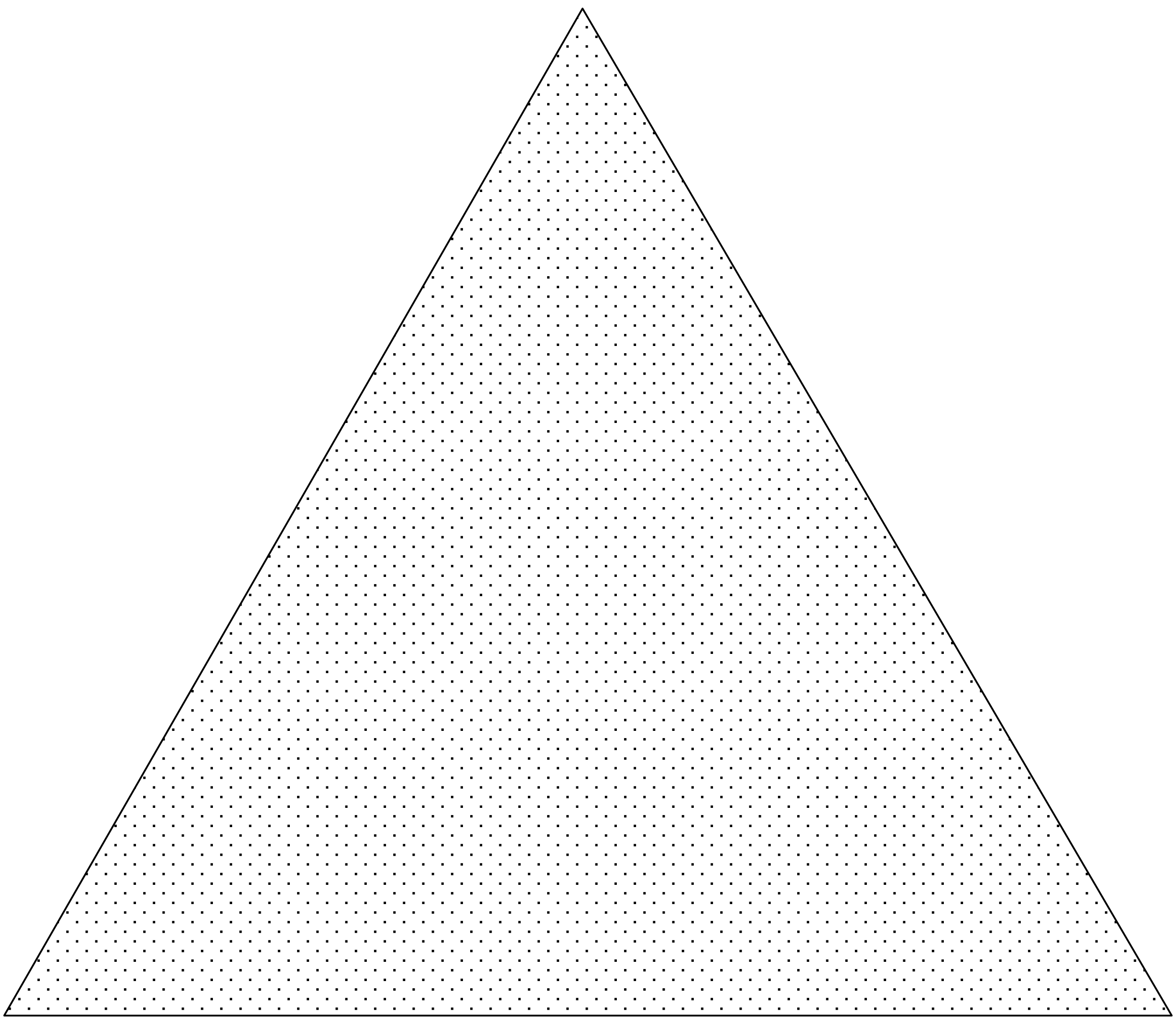
9. Make this shape so that **one fourth of it is green and three fourths of it is purple**.



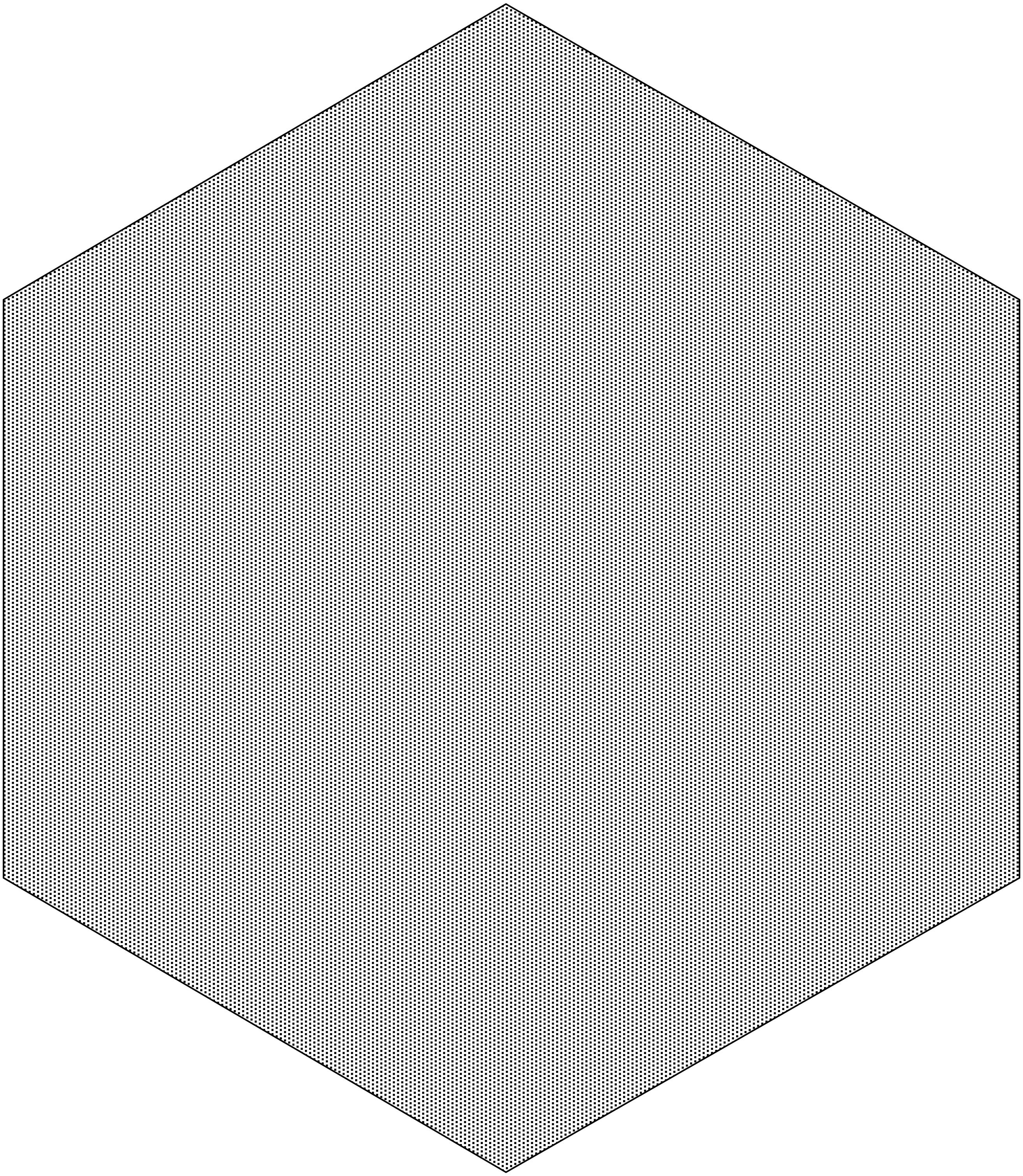
10. Make this shape so that **half of it is purple** and**half of it is yellow**.



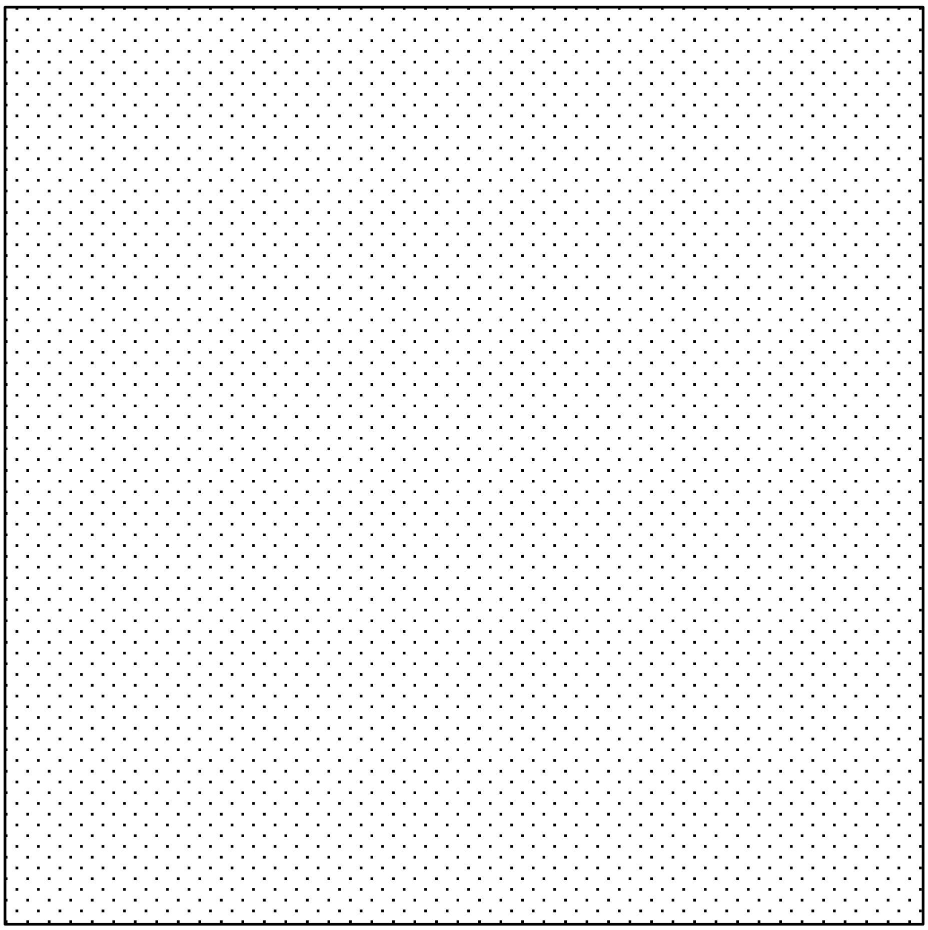
11. Make the shape so that **half of it is purple** and **half of it is green**.



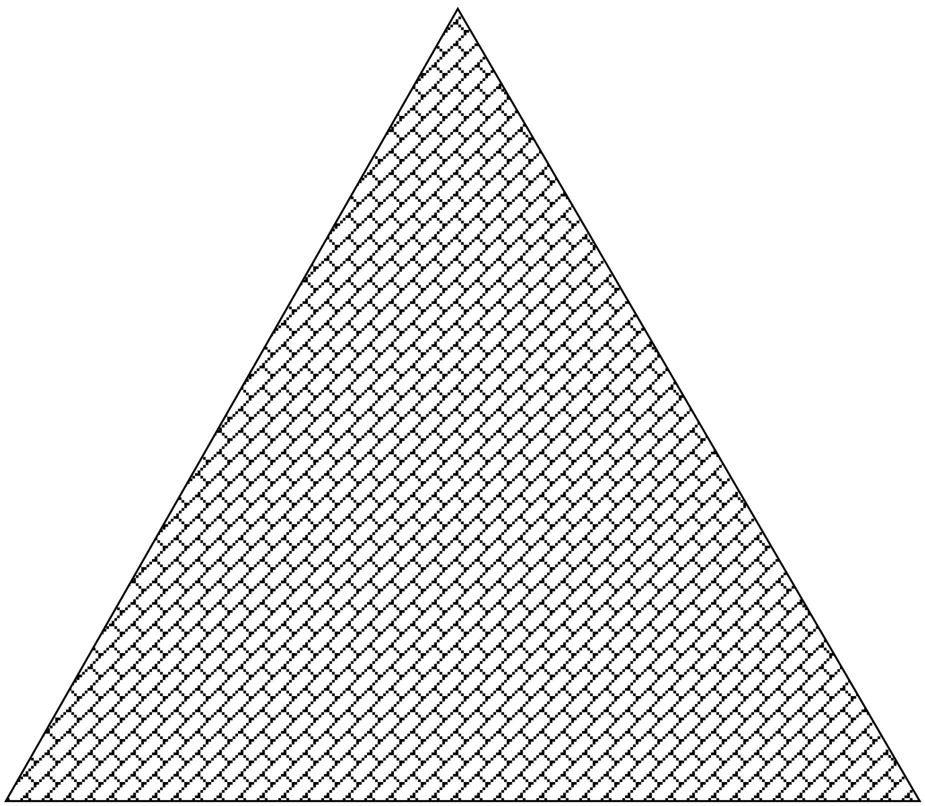
12. Make the shape so that **a quarter of it is purple**, **a quarter of it is green, a quarter of it is yellow**, and **a quarter of it is orange**.



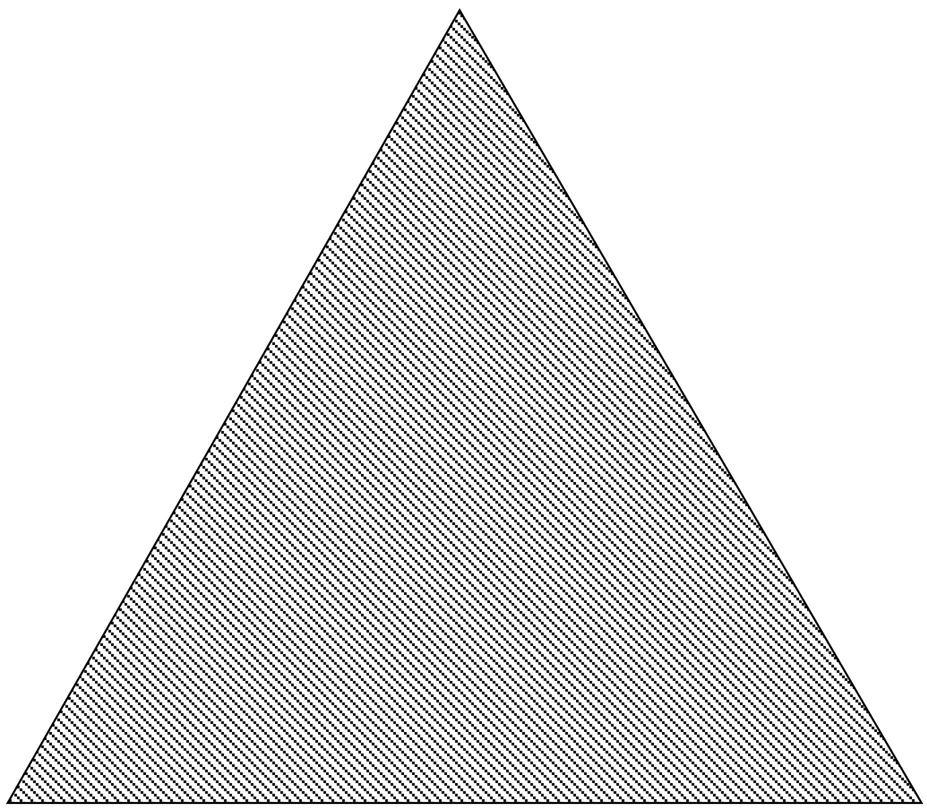
13. Make the shape so **a quarter of it is yellow, and the rest is purple**. What fraction of it is purple?



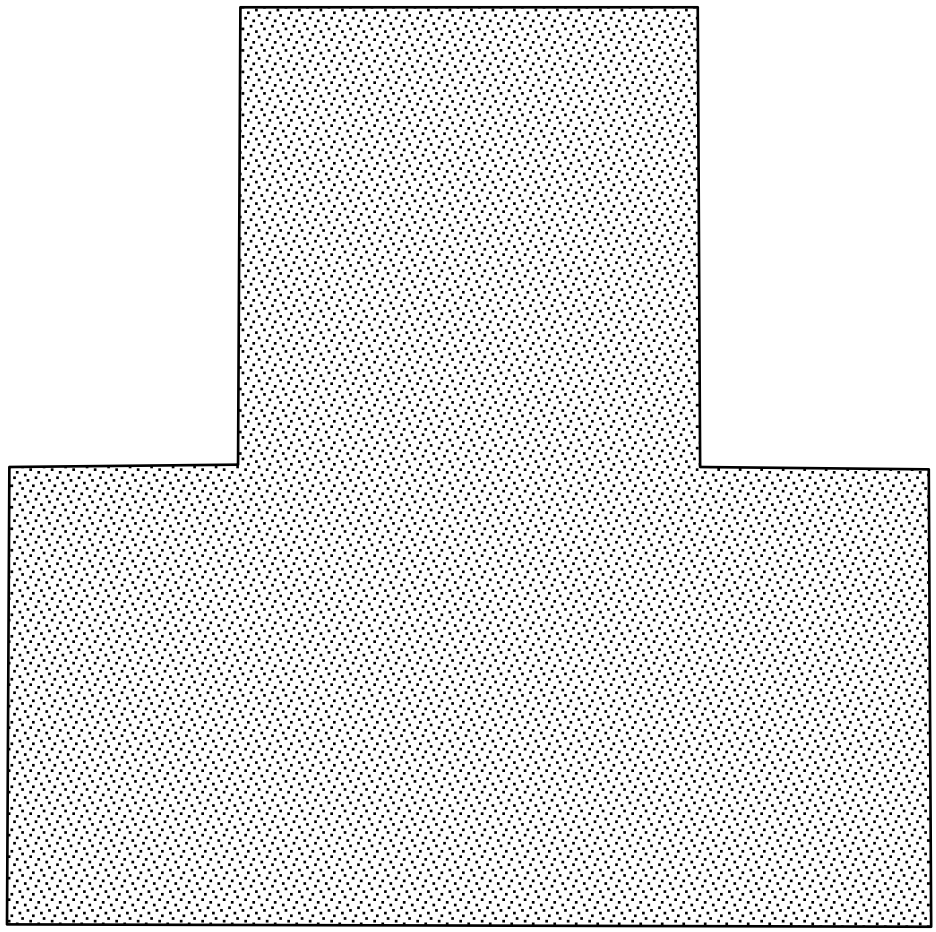
14. Make the shape so that **a quarter of it is orange, a quarter of it is yellow**, **a quarter of it is purple**, and **a quarter of it is green**.



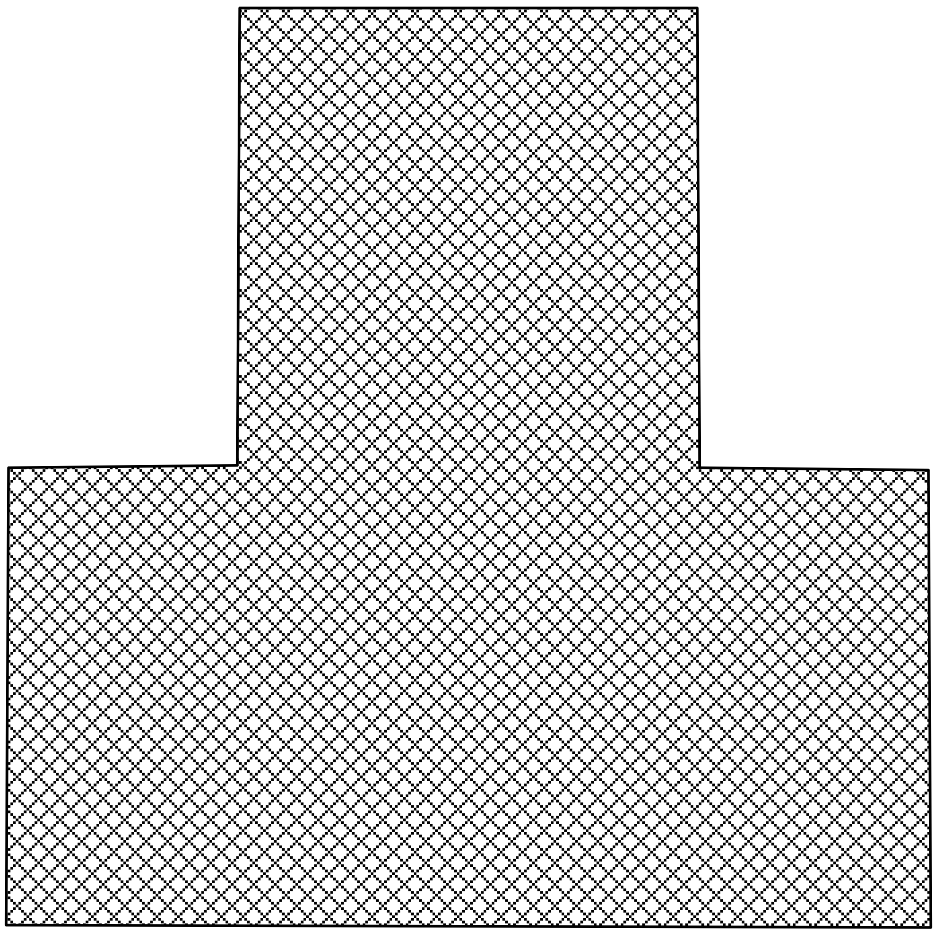
15. Make the shape so that a **half of it is orange, a quarter of it is yellow**, and **a quarter of it is purple**.



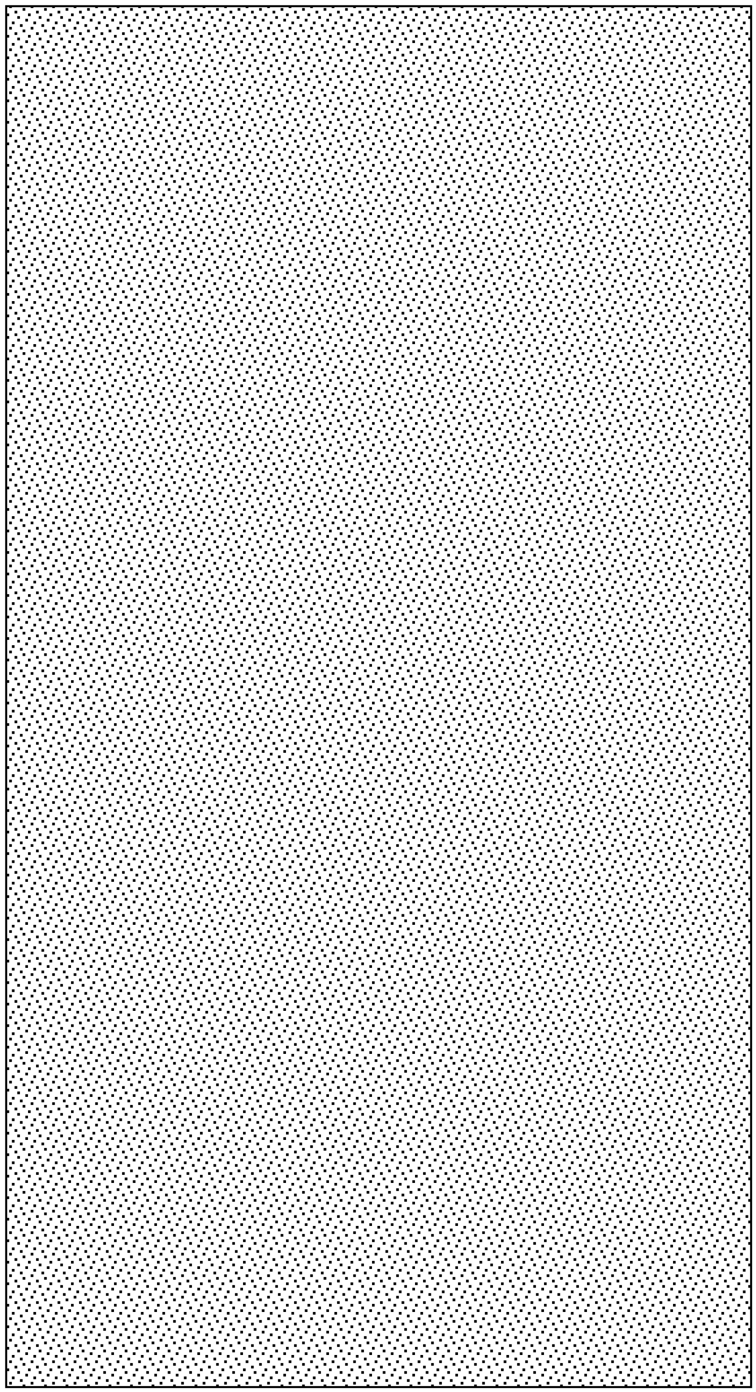
16. Make the shape so that **three thirds of it is yellow**.



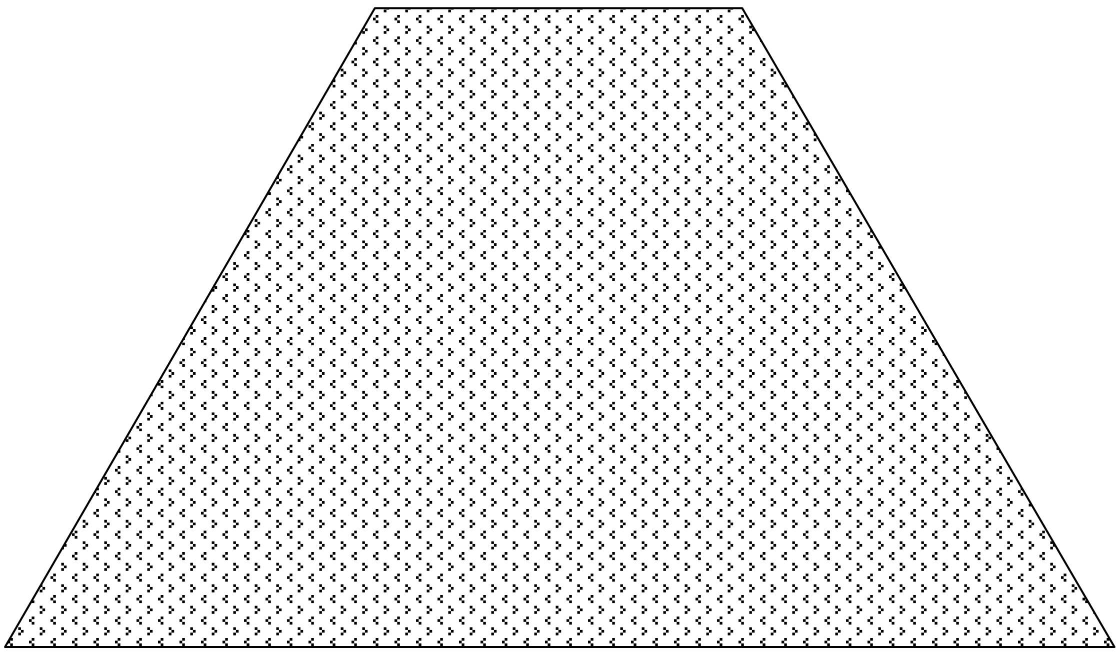
17. Make the shape so that **one third of it is purple, and the rest of it is yellow**. Try to find all the different ways to arrange the tiles.



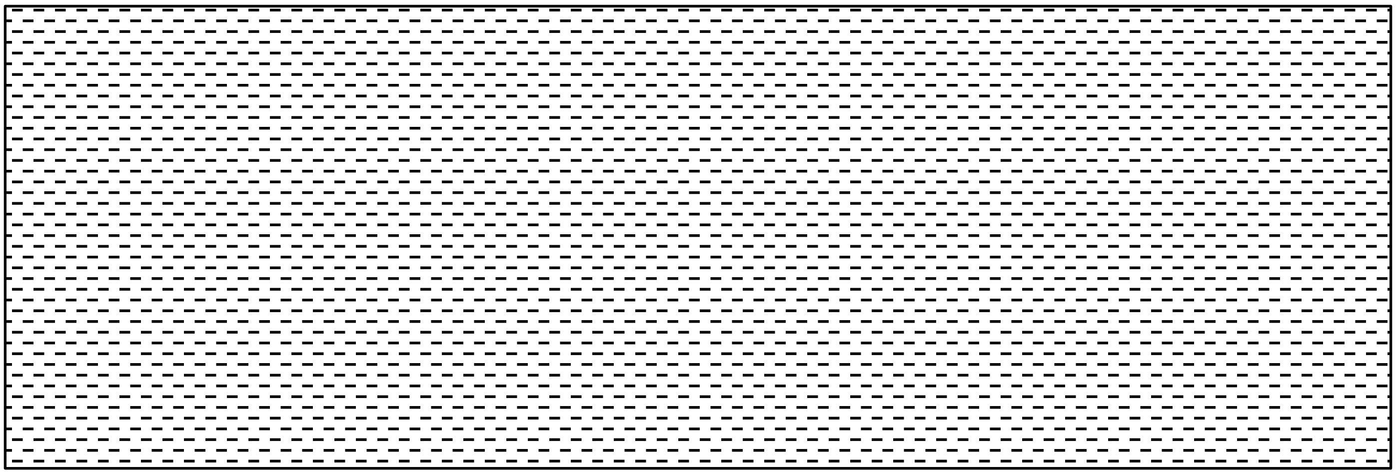
18. Make the shape so that **one third of it is orange, and two thirds of it is green**. Try to find all the different ways to arrange the tiles.



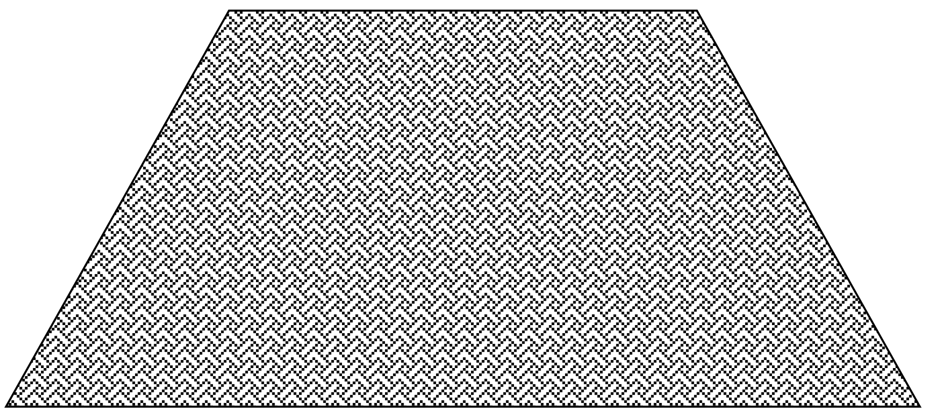
19. Make the shape so that **one quarter of it is green, and the rest of it is purple**. What fraction of it is purple? Try to find all the different ways to arrange the tiles.



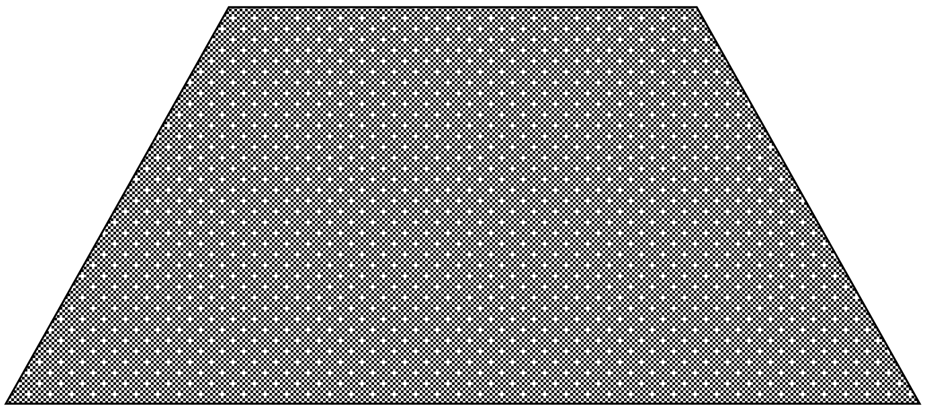
20. Make the shape so that **three thirds of it is purple**.



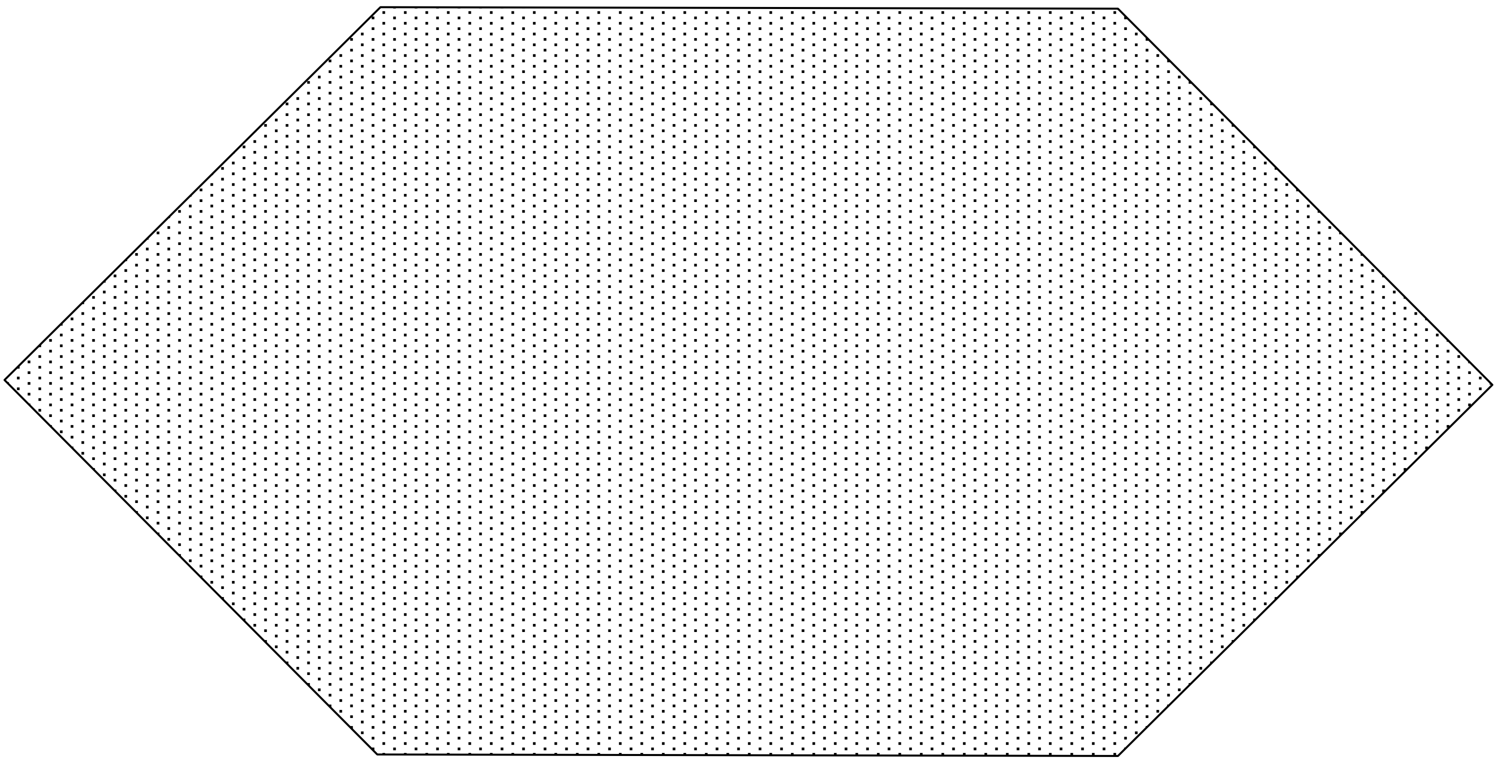
21. Make the shape so that **one third of it is purple and two thirds of it is orange**.



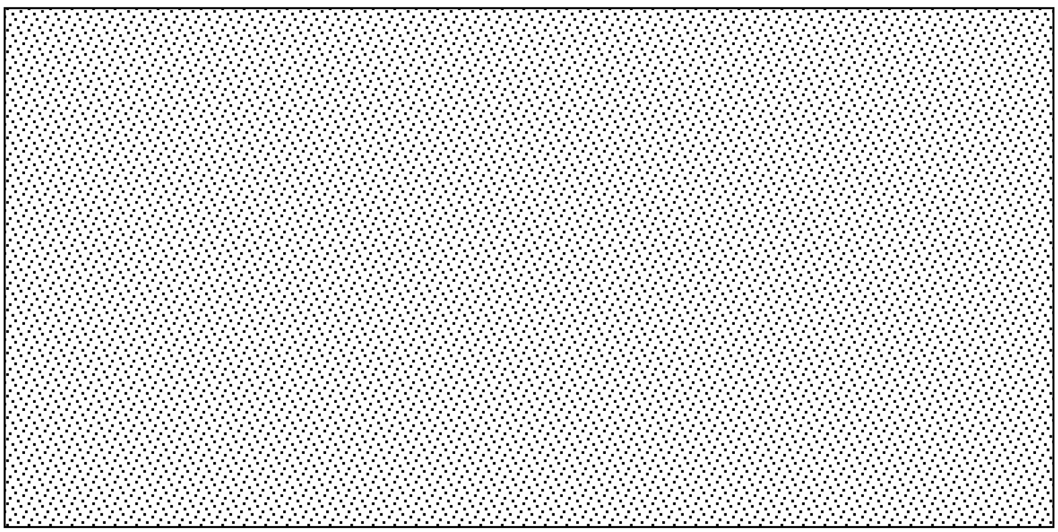
22. Make the shape so that **three thirds of it is yellow**.



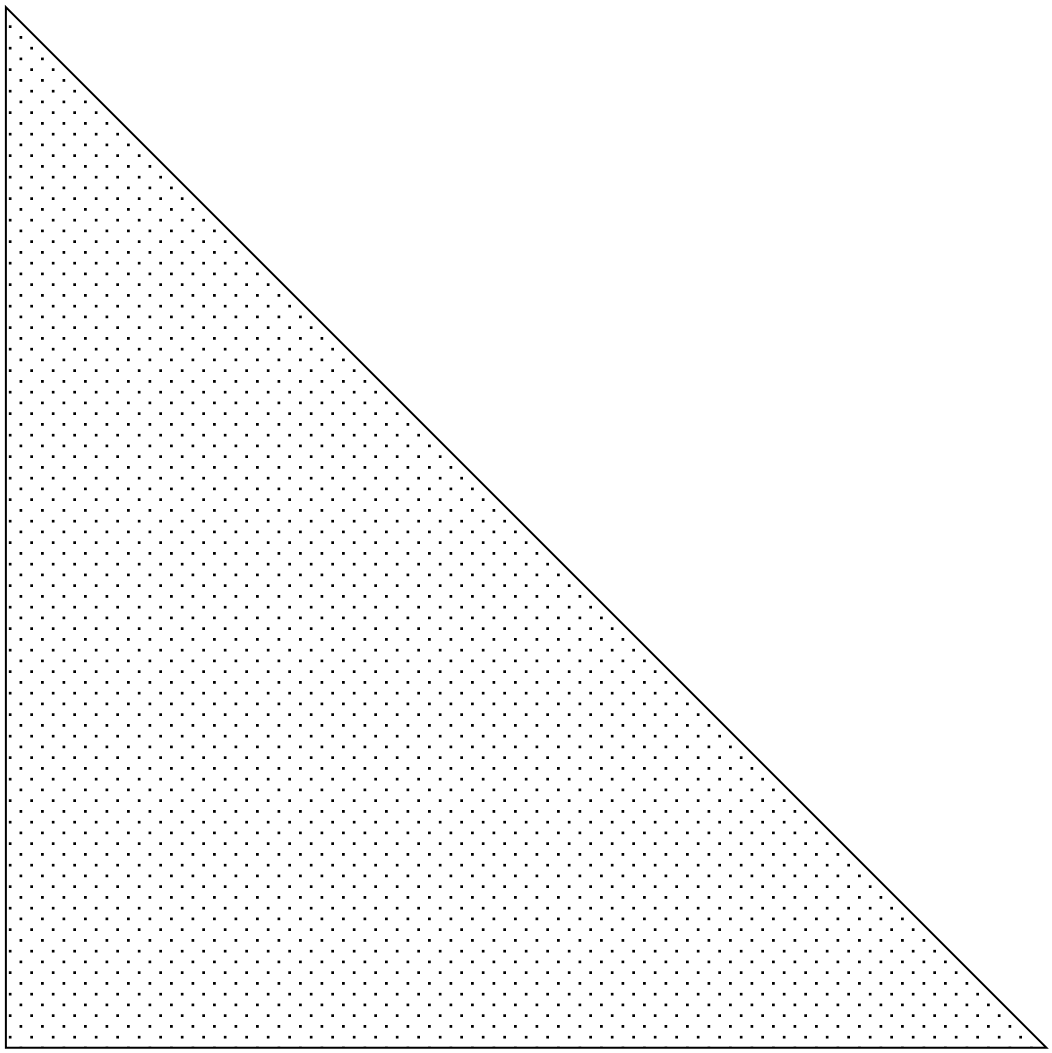
23. Make the shape so that that **half of it is orange** and **half of it is yellow**. Try to arrange the pieces in at least two different ways.



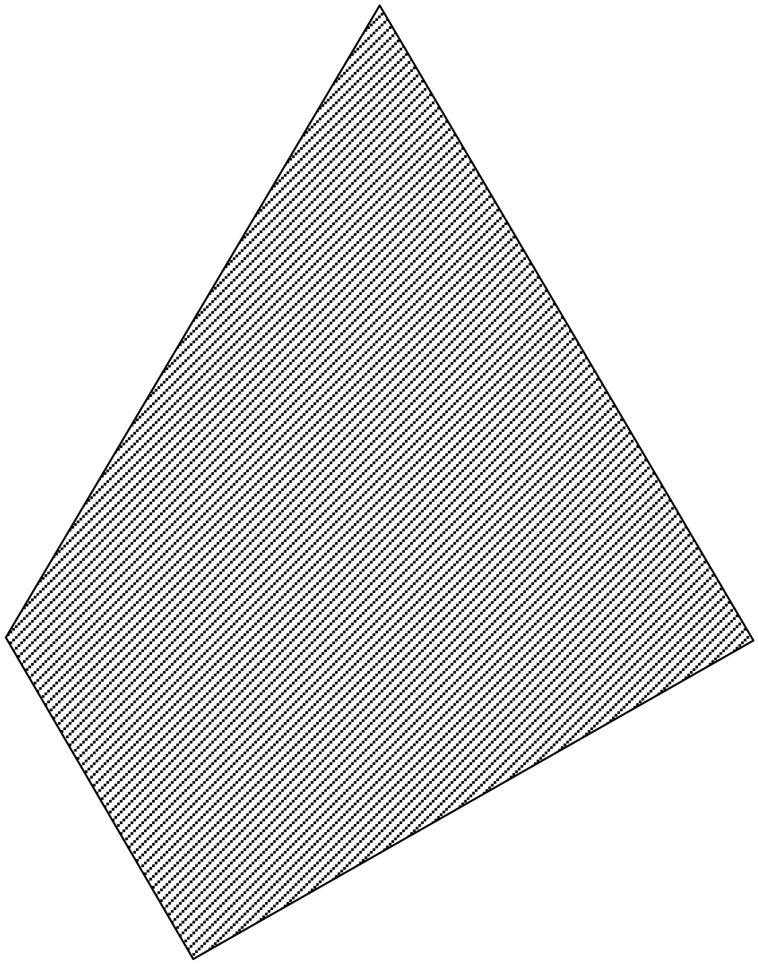
24. Make the shape so that that **half of it orange** and **half of it is yellow**. Try to arrange the pieces in at least two different ways.



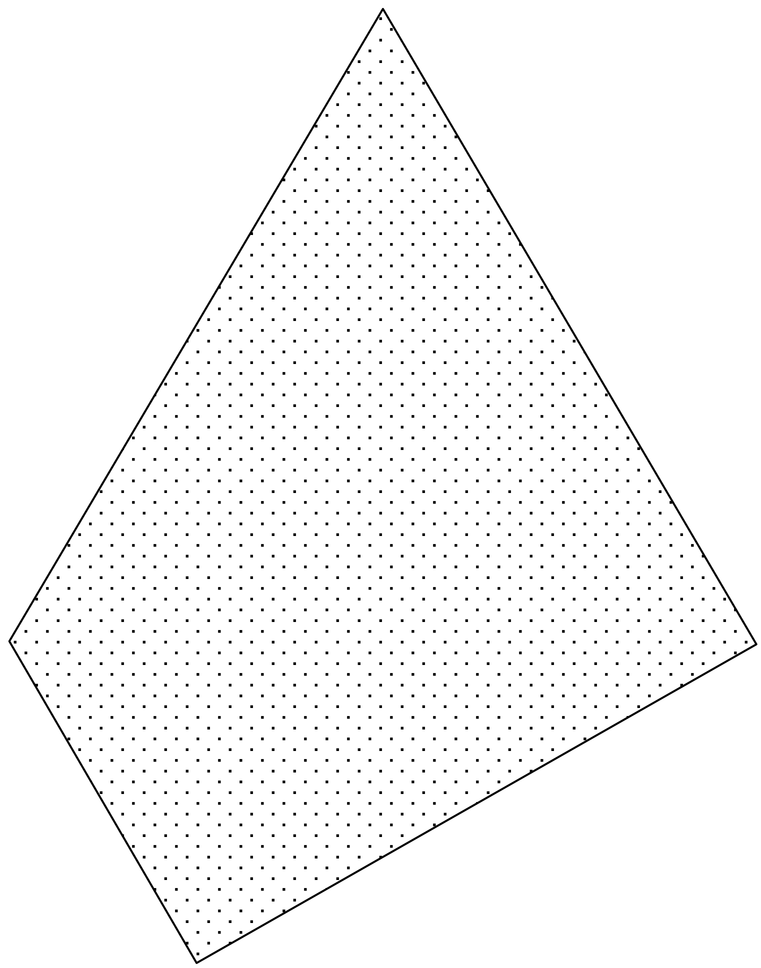
25. Make the shape so that that **three quarters of it is yellow** and **the rest is orange**. What fraction of it is orange? Try to arrange the pieces in at least two different ways.



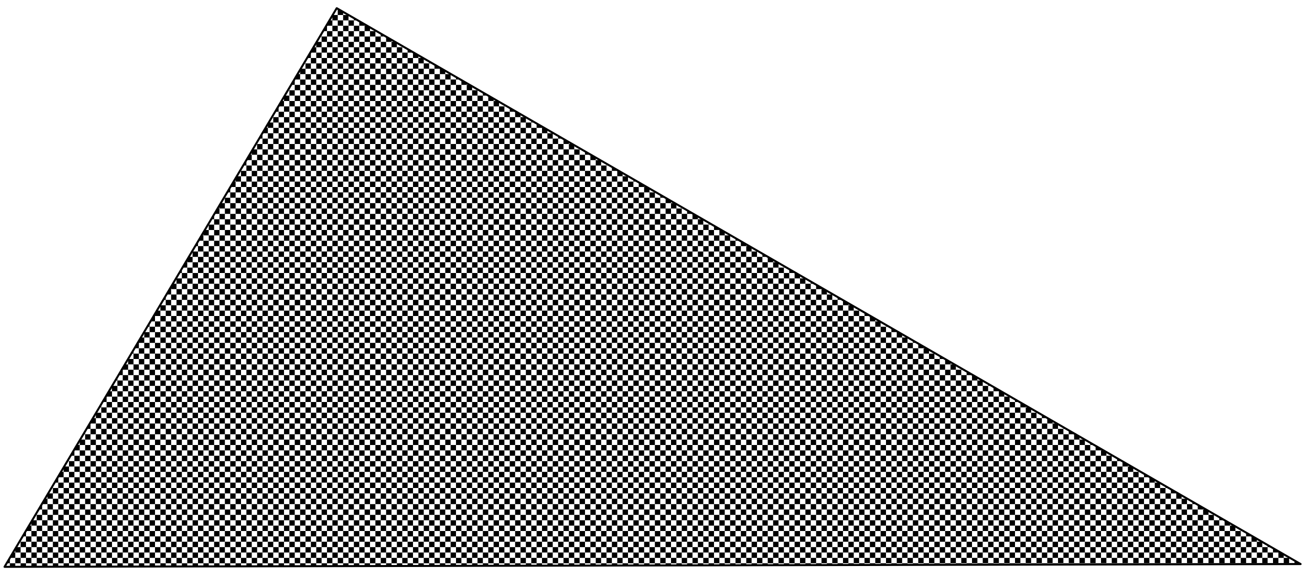
26. Make the shape so that **one third of it is purple and two thirds of it is green**.



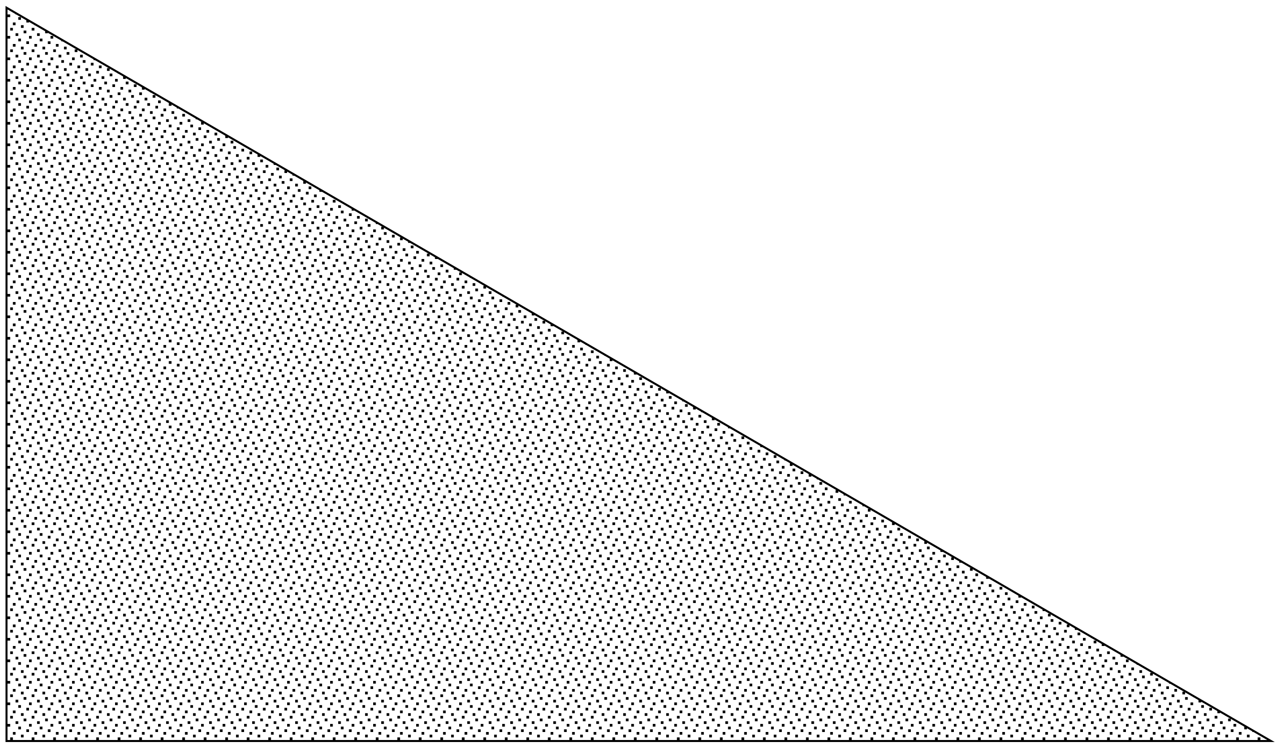
27. Make the shape so that **three thirds of it is green**.



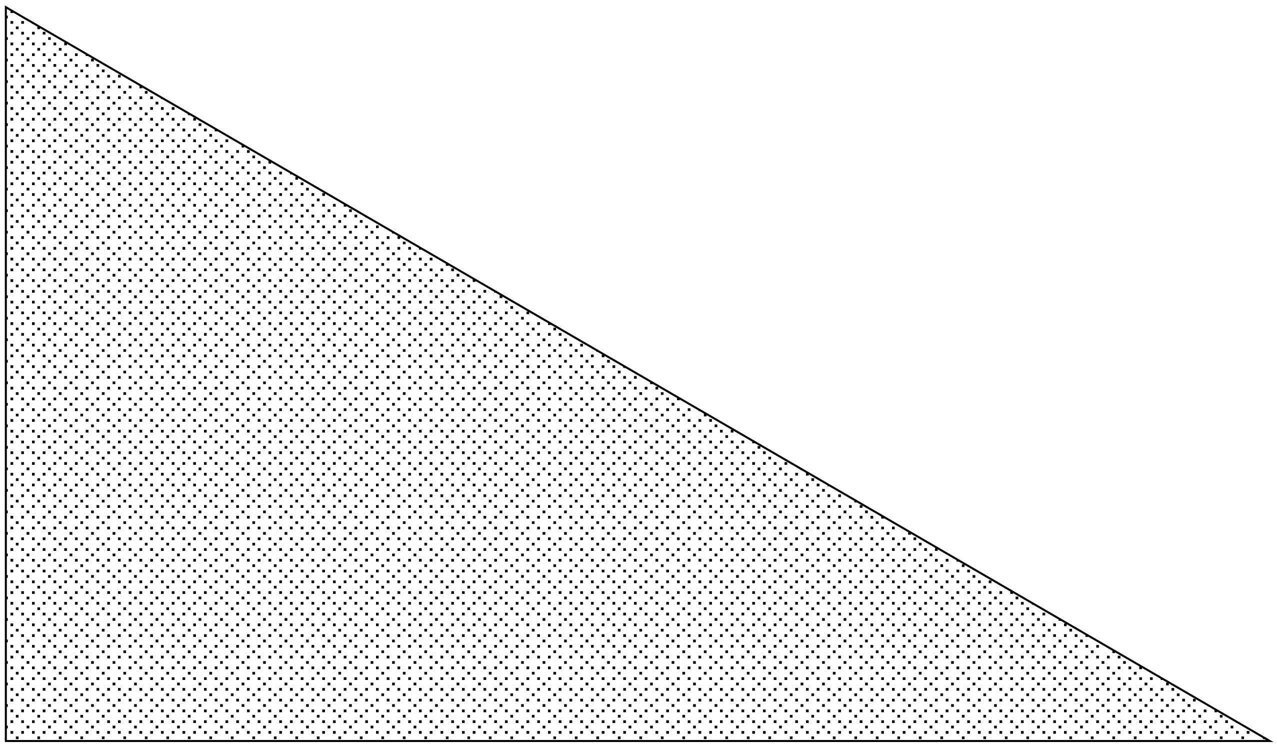
28. Make the shape so that **one third of it is purple and the rest of it is green**.



29. Make the shape so that **a quarter of it is purple and the rest of it is green**.

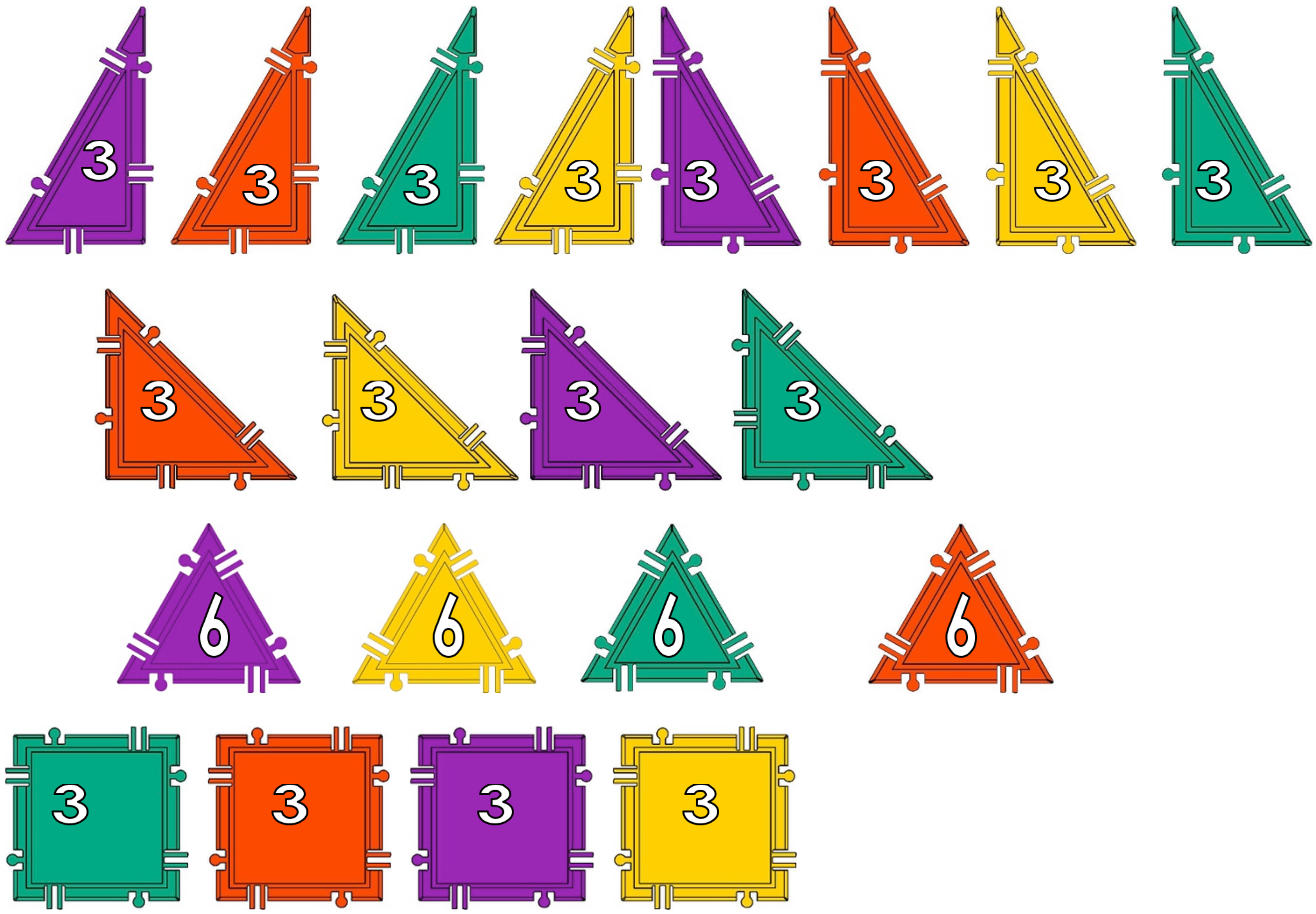


30. Make the shape so that **half of it of it is yellow and half of it is orange**.



Level 2

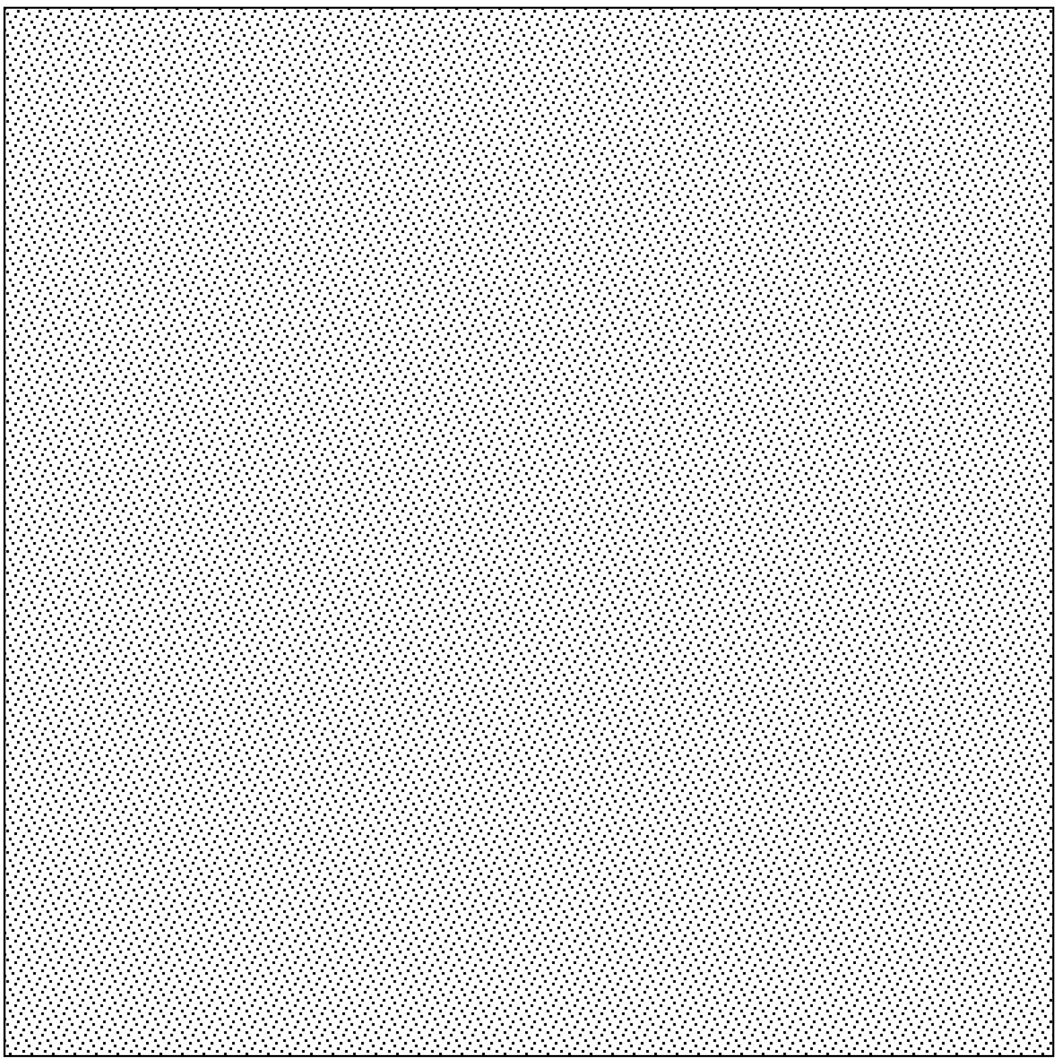
These exercises are designed for students in grade 3 and upwards. They require the following tiles per group:



31.

a. Make the square so that **1/4 of it is yellow, 1/4 of it is green, 1/4 of it is orange, and 1/4 of it is purple.**

b. Now make the same square so that **3/8 of it is green, 3/8 of it is yellow, and a quarter of it is purple**.



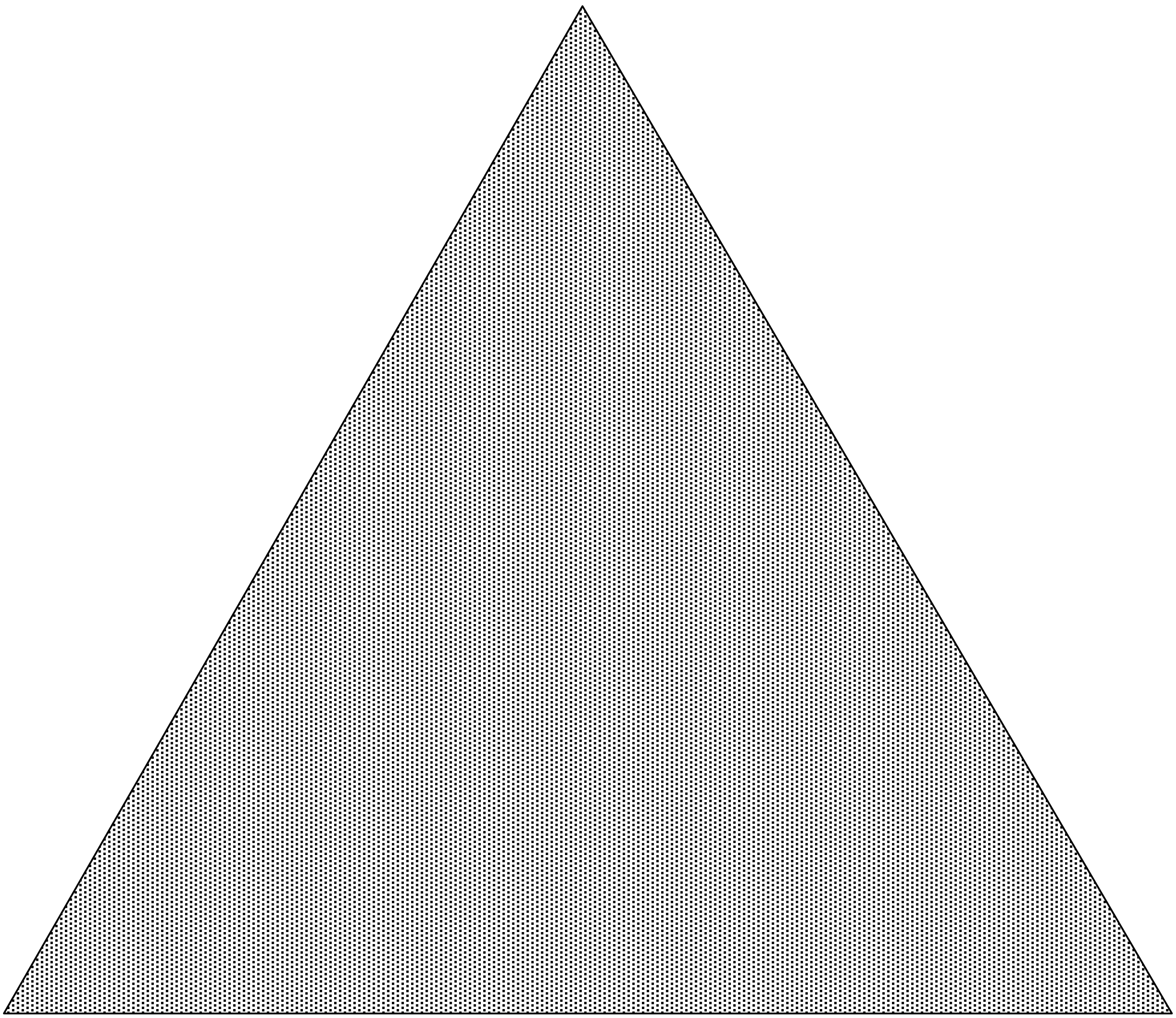
32. Use the square from the previous problem to help you put the following fractions in order from smallest to largest:

$$\frac{1}{4} \quad \frac{1}{8} \quad \frac{3}{4} \quad \frac{1}{2} \quad \frac{3}{8}$$

33.

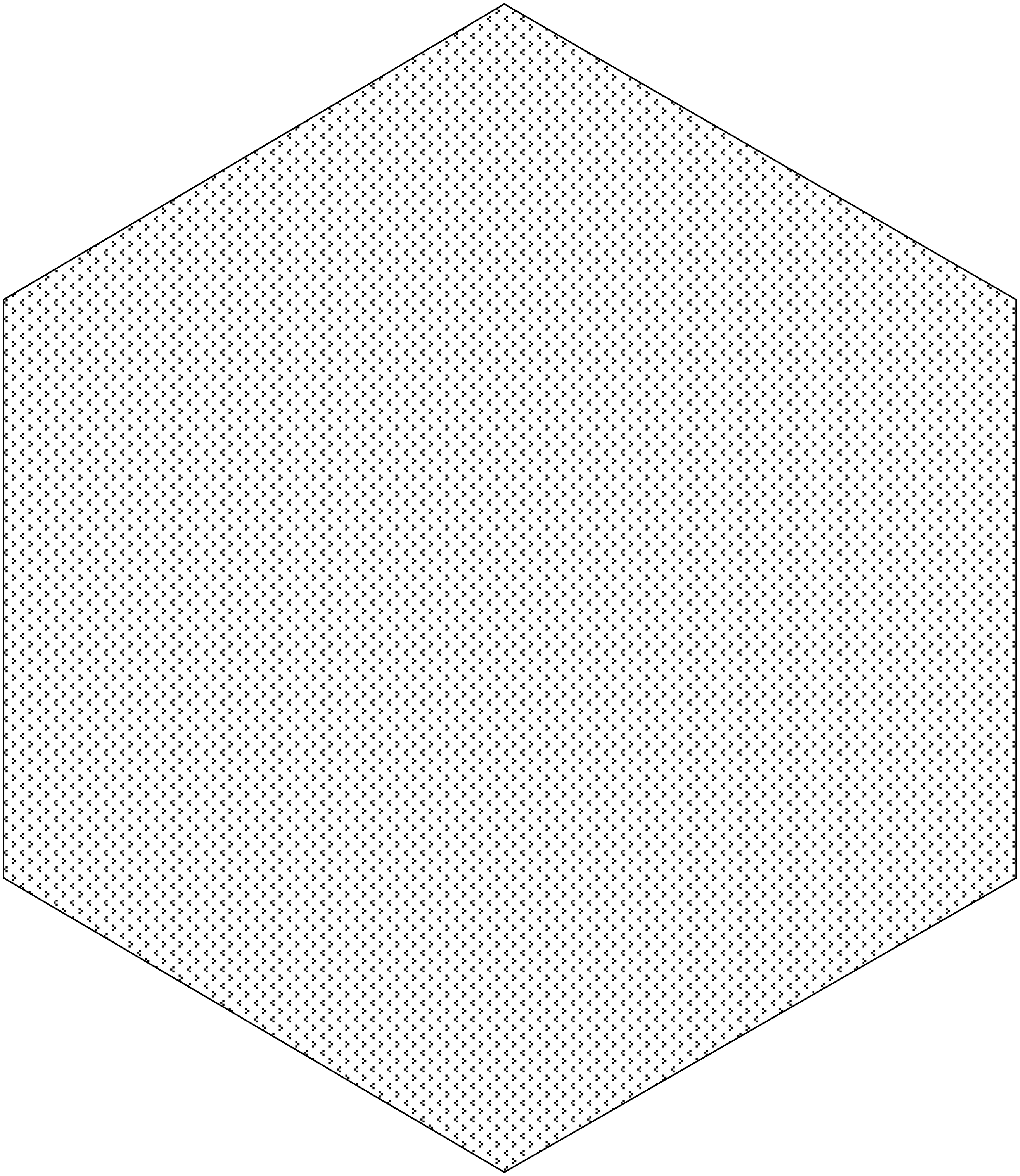
a. Make the triangle so that **1/6 it is yellow and 5/6 of it is green**.

b. Now make the same triangle so that a **third it is yellow and the rest is green**. Try to construct the solution in as many different ways as you can.



34.

a. Make the hexagon so that **1/3 of it is orange, 1/4 of it is yellow**, and **5/12 of it is green**.



b. Now make the hexagon that **1/6 of it is yellow, 1/2 of it is purple,** and **1/3 of it is orange.**

c. Now make the hexagon so that a **third of it is purple, a third of it is orange, a fourth of it is yellow, and the rest is green.** What fraction is green?

35. Use the hexagon from the previous problem to help you put the following fractions in order from smallest to largest:

$$\frac{7}{12} \quad \frac{5}{6} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{3} \quad \frac{11}{12} \quad \frac{1}{12} \quad \frac{1}{6}$$

Shape Challenge Problems

36. Make a square that is a third green, a third orange and a third yellow. Hint: use 9 tiles.

37. Make a square that is 1/8 green, 1/8 orange, 3/8 yellow, and 3/8 purple.

38. Make an equilateral triangle that is 1/9 yellow, 1/3 green, and 5/9 purple.

39. Make an equilateral triangle that is 3/16 purple, 3/8 orange, 3/16 yellow, and 1/4 green.

40. Make a right isosceles triangle that is a third orange, a third purple, and a third yellow. Hint: use 9 tiles.

Food problems

For the next group of problems, show your work by writing down a mathematical equation. Also, model your solution with the tiles. Part of your job to find the shape(s) that best model each problem.

41. Monica ate 1/2 of the pie, and her brother Mike ate 1/3 of the pie. What fraction of the pie was left for their dad?

42. Cathy's family went out for pizza and brought back 2/3 of the pizza as leftovers. Cathy gave ¼ of the leftovers to her dog and ate the rest. What <u>fraction of the original pizza</u> did Cathy eat? What about her dog?

43. Alex bought two mini-pizzas. His friends typically eat 2/3 of a pizza each. How many friends can he serve with his two mini-pizzas?

44. Joe was making cake for his 18 friends, but the recipe he had was only for 6 servings. If the original recipe called for 3/4 cups of flour, how much flour did he need to use to make enough cake?

45. Nina had 2 small pumpkin pies in her fridge. She was having 11 friends over, and figured that 1/6 of a small pie per person was a large enough serving. Did she have enough pie for all her friends? Explain your answer.

Cube problems

It is up to you to decide which size cube to make for each problem.

46. Make a cube that is half orange and half purple.

47. Make a cube that is 1/3 orange, 1/3 purple, and 1/3 yellow

48. Make a cube that is 1/6 yellow, 1/6 orange, 1/6 green, and 1/2 purple.

49. Make a cube that is 1/6 yellow, 1/2 green, and 1/3 purple.

50. Make a cube that is ¼ orange, ¼ purple, ¼ yellow, and ¼ green.

Operations on fractions

In problems 51-60, use the tiles to model the operations.

51.

a. $\frac{1}{2}+\frac{1}{3}=$

b. $\frac{1}{2}-\frac{1}{3}=$

c. $\frac{2}{3}-\frac{1}{2}=$

d. $\frac{3}{2}-\frac{2}{3}=$

52.

a. $\frac{3}{8}+\frac{1}{4}=$

b. $\frac{3}{8}-\frac{1}{4}=$

53.

a. $\frac{1}{3}+\frac{1}{4}=$

b. $\frac{1}{3}-\frac{1}{4}=$

c. $\frac{1}{4}+\frac{1}{6}=$

d. $\frac{1}{4}-\frac{1}{6}=$

e. $\frac{5}{12}+\frac{1}{3}=$

54.

a. $3\times\frac{1}{6}=$

b. $2\times\frac{2}{3}=$

c. $4\times\frac{1}{6}=$

d. $2\times\frac{3}{2}=$

55.

a. $\frac{1}{2}\times\frac{1}{3}=$

b. $\frac{1}{2}\times\frac{2}{3}=$

c. $\frac{1}{2}\times\frac{3}{2}=$

d. $\frac{1}{2}\times\frac{3}{4}=$

56.

a. $\frac{2}{3}\times\frac{3}{4}=$

b. $\frac{3}{4}\times\frac{3}{2}=$

c. $\frac{5}{6}\times\frac{1}{2}=$

d. $\frac{3}{8}\times\frac{2}{3}=$

57.

a. $1\frac{2}{3}\times 2=$

b. $1\frac{1}{4}\times 2=$

c. $1\frac{1}{3}\times 3=$

d. $1\frac{5}{6}\times 2=$

58.

a. $\frac{1}{4} \div 2 =$

b. $\frac{1}{2} \div 3 =$

c. $\frac{2}{3} \div 2 =$

d. $\frac{1}{3} \div 4 =$

59.

a. $\frac{2}{3} \div \frac{1}{3} =$

b. $\frac{1}{2} \div \frac{1}{3} =$

c. $\frac{1}{3} \div \frac{1}{2} =$

d. $\frac{3}{4} \div \frac{1}{2} =$

60.

a. $1\frac{2}{3} \times \frac{1}{2} =$

b. $1\frac{1}{2} \times \frac{2}{3} =$

c. $2\frac{1}{3} \times \frac{1}{4} =$

d. $1\frac{1}{4} \times 1\frac{1}{2} =$

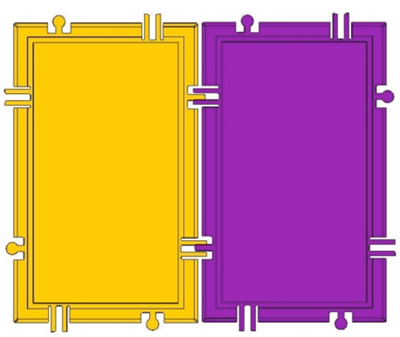
ANSWER KEY

Note that colors of pieces in the answer key may be different from the actual colors you use.

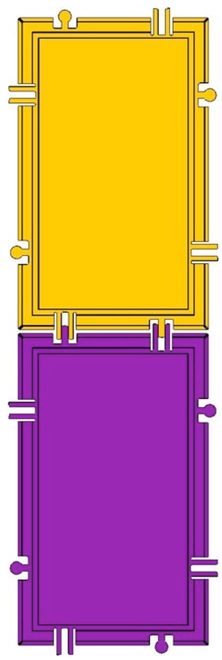
Level 1, Group A

Note: occasionally, the student is asked to solve a tangram in as many different ways as possible. Two solutions are considered "different" if one of them is not a flipped or rotated version of the other. To be "different", two solutions must be constructed in a different way. See, for example, the solution to Problem 8 on the following page.

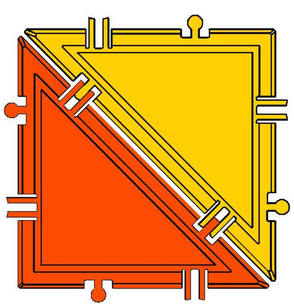
1.



2.



3.



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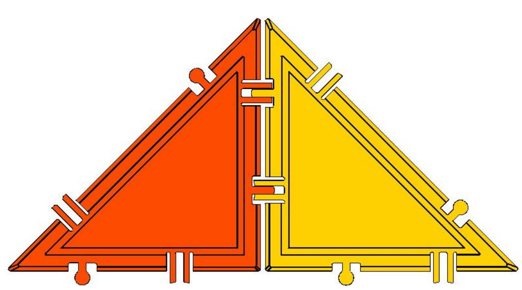
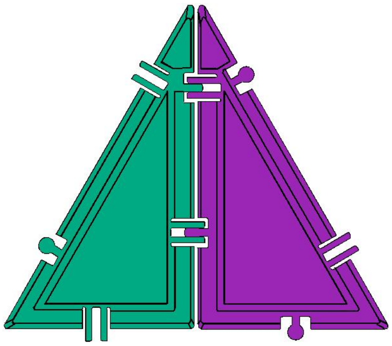
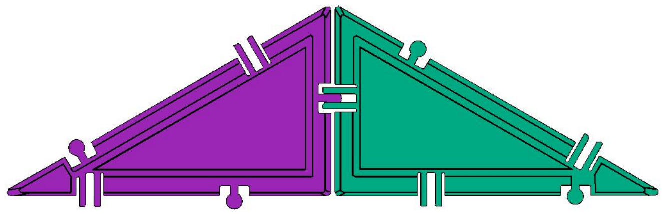


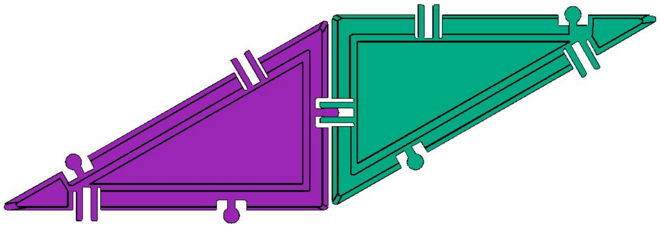
5.



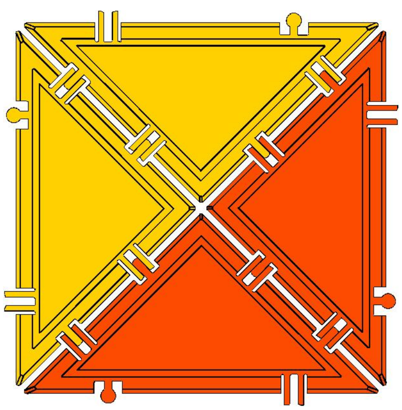
6.

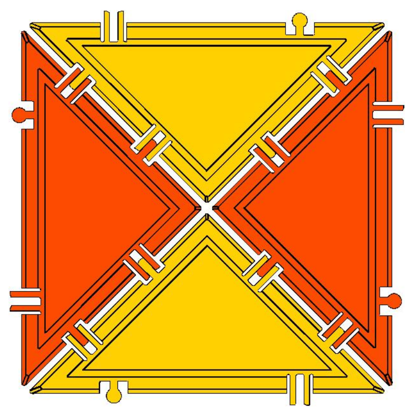


7.

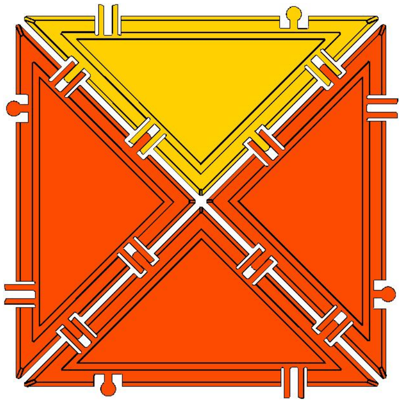


8.

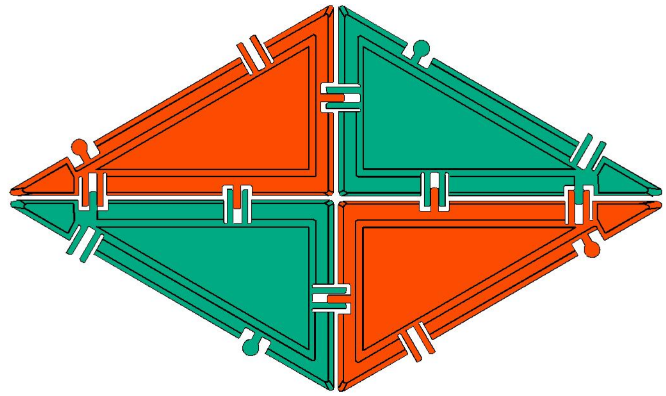




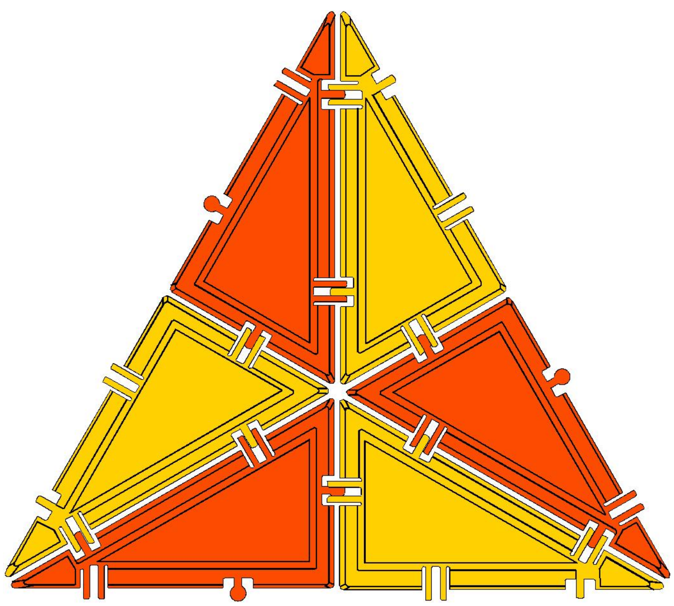
9.



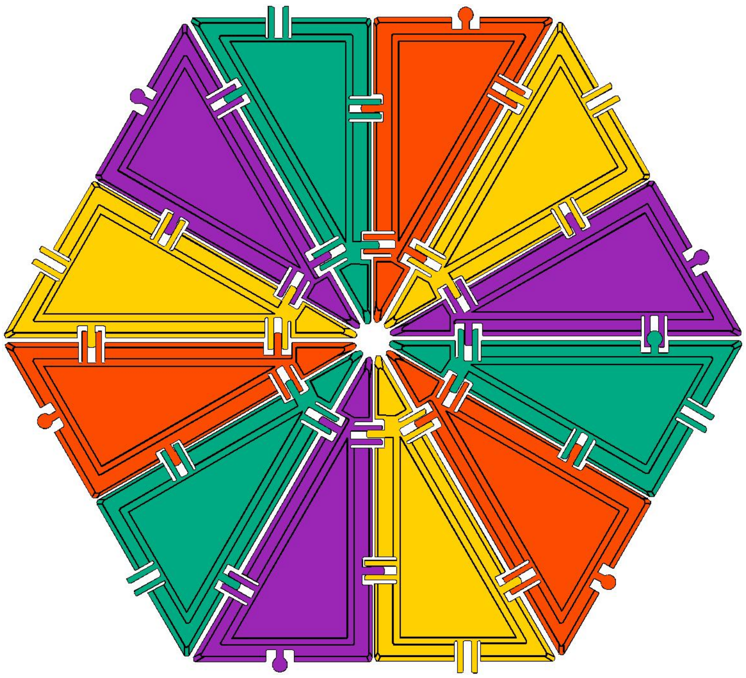
10.

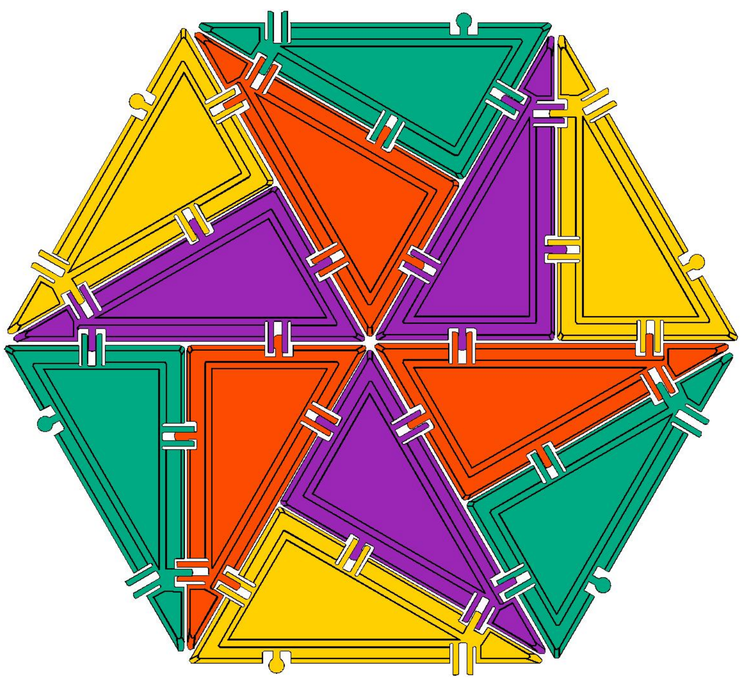


11.

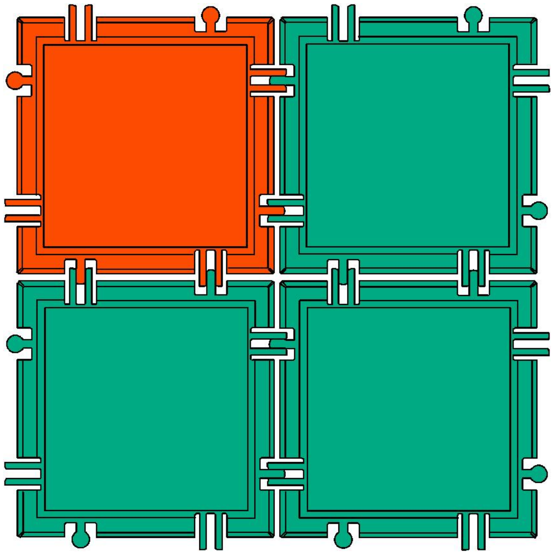


12. There are many different ways to arrange the 12 tiles. Here are some of the prettier ones.

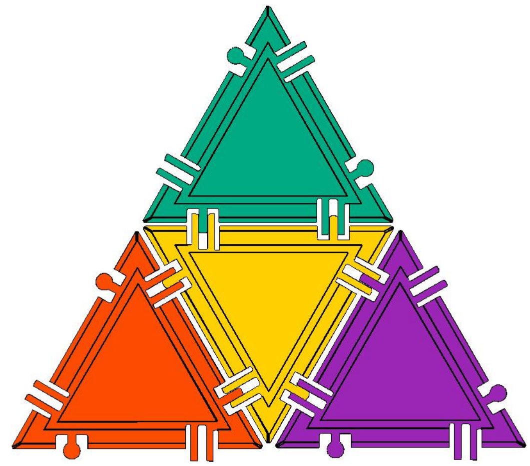




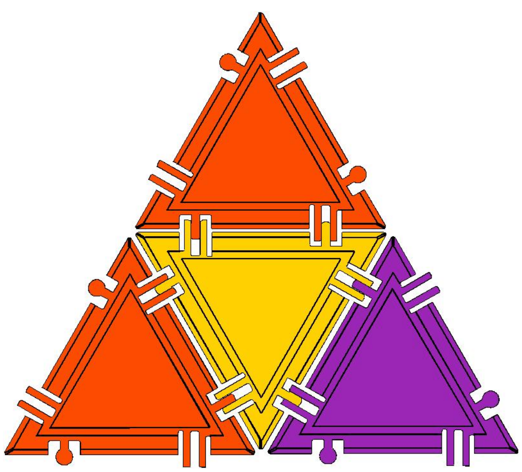
13, ¾ or the square is green



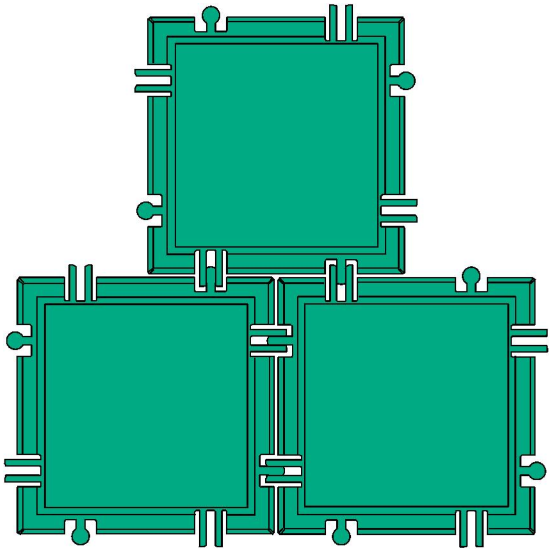
14.



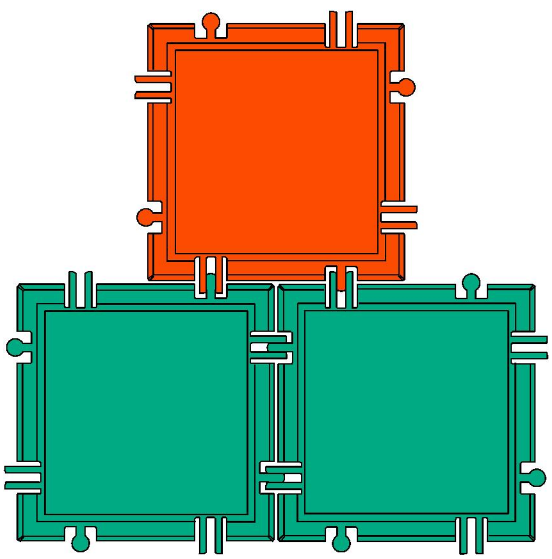
15.

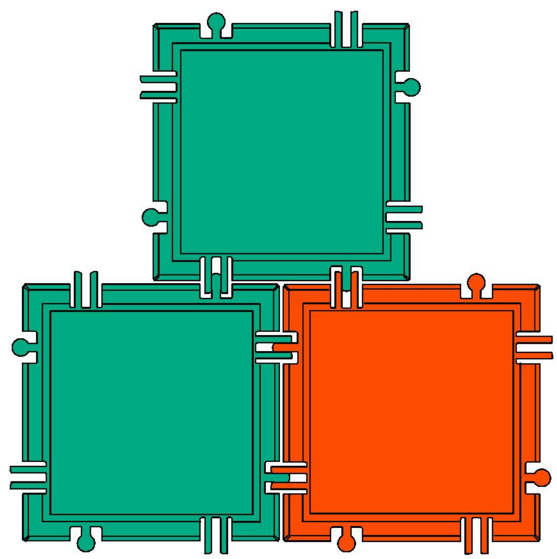


16.

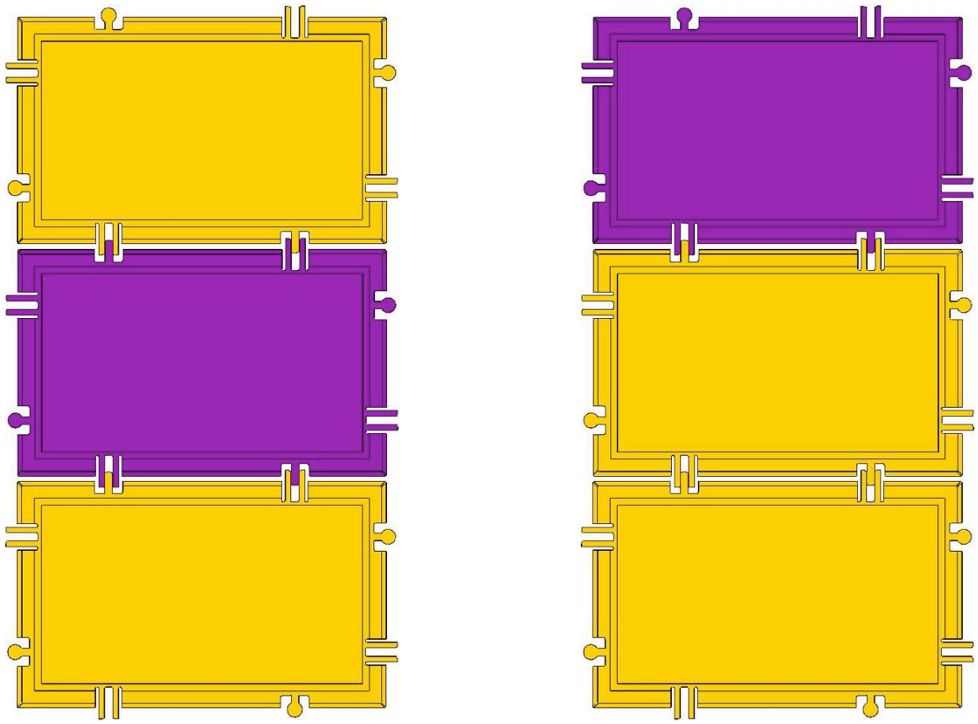


17.

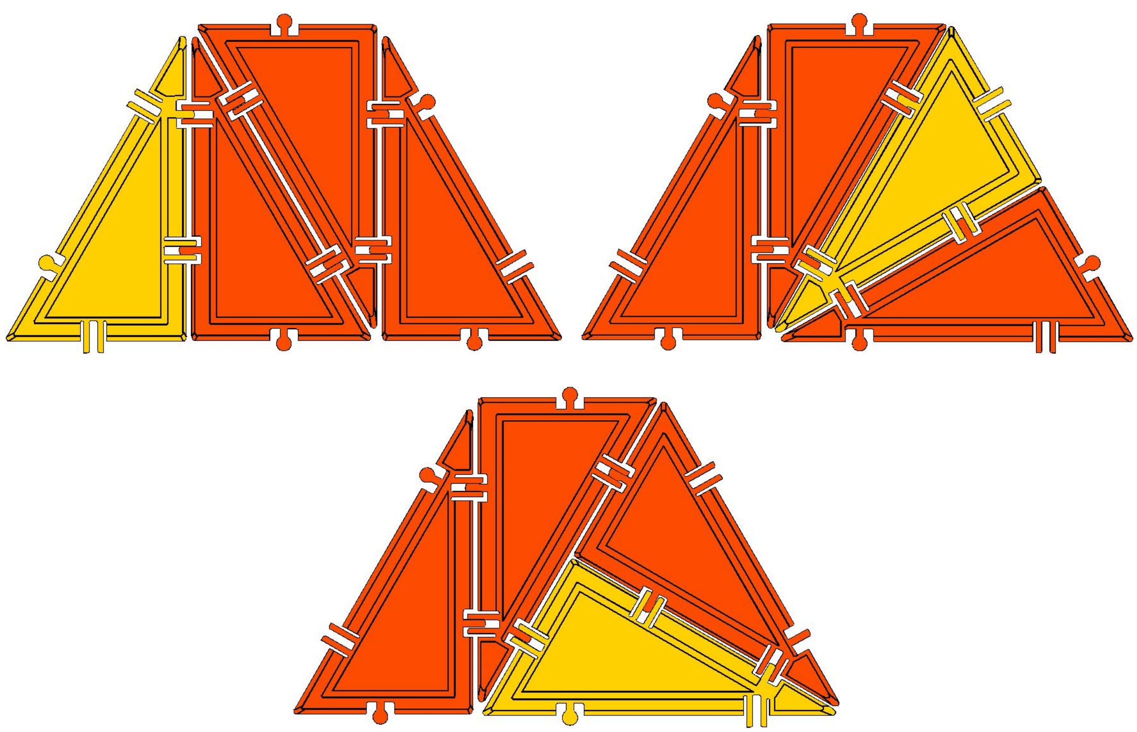




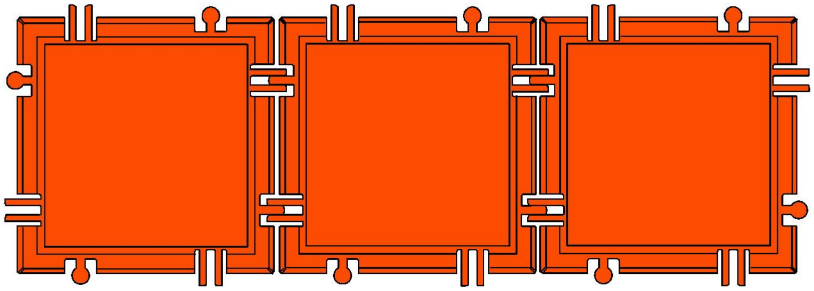
18.



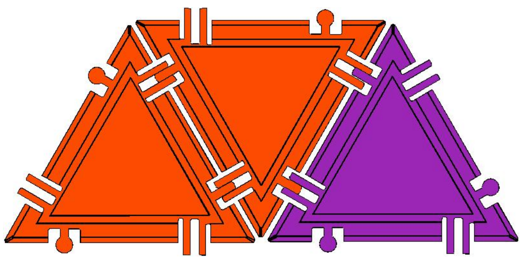
19. ¾ of the trapezoid is orange.



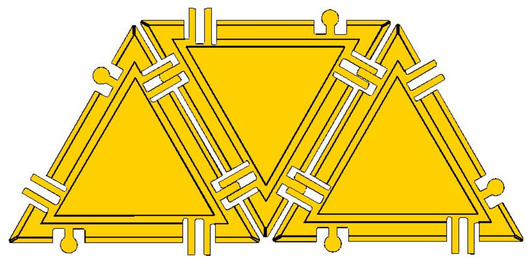
20.



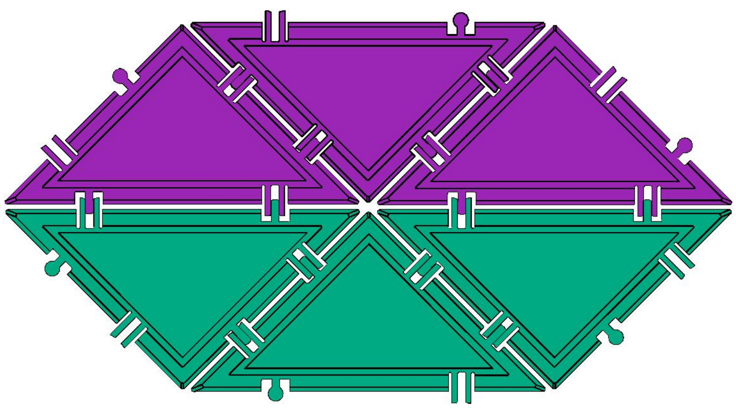
21.

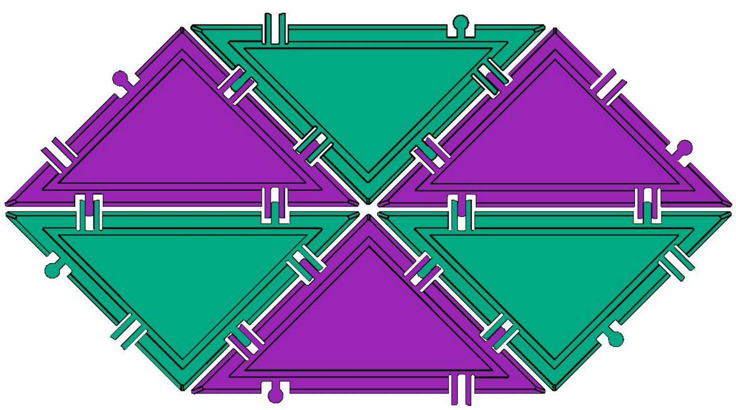


22.



23.





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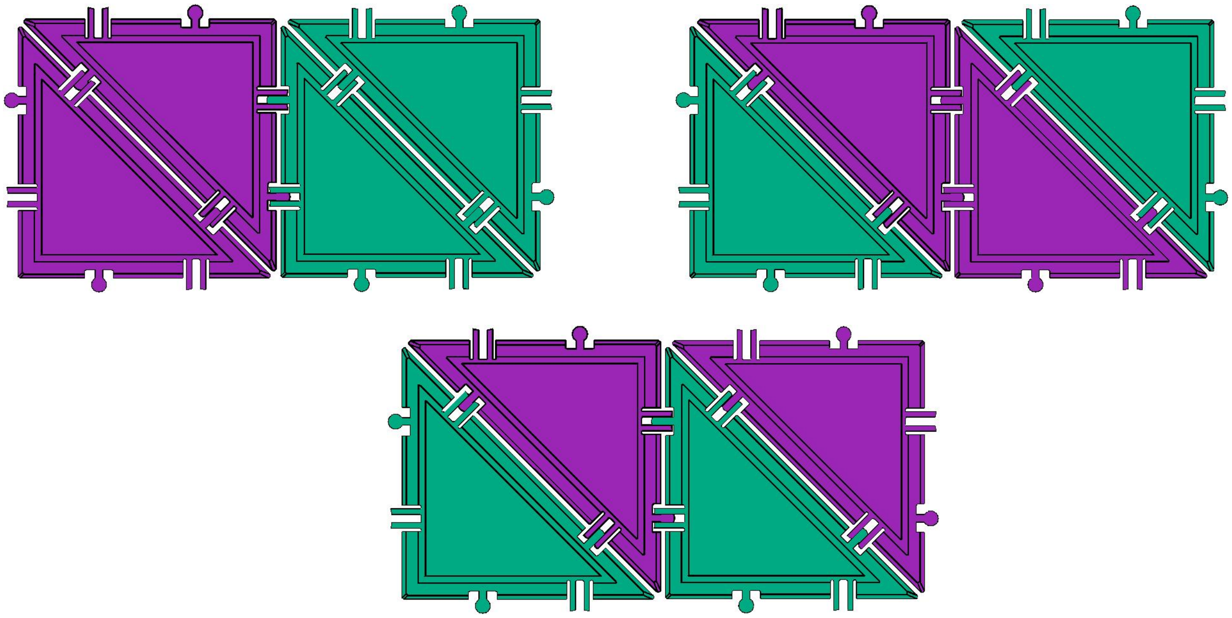
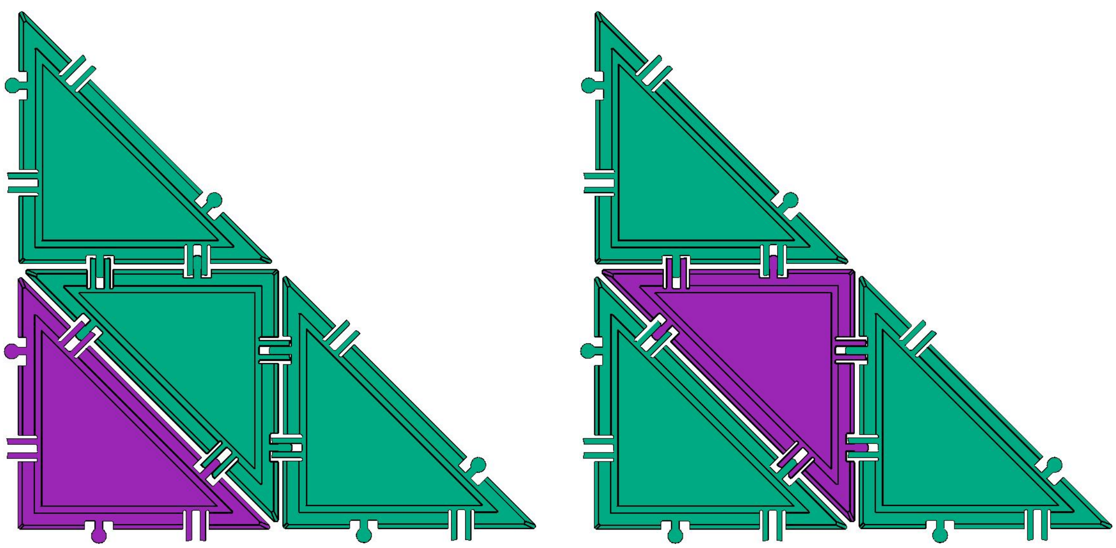
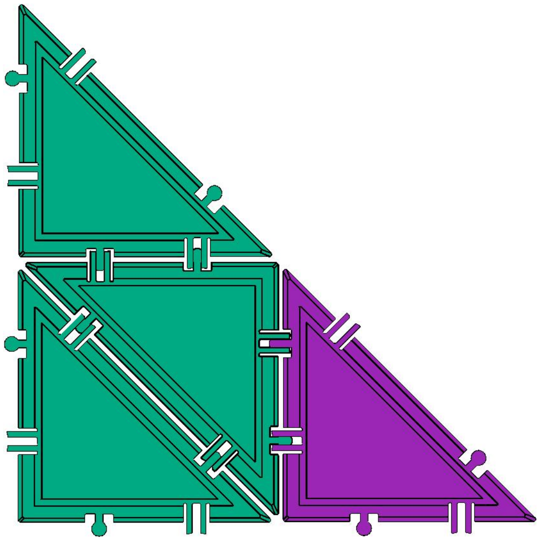
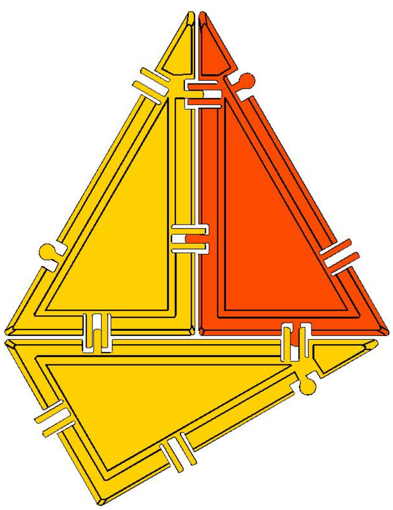


25. ¼ of the triangle is purple.

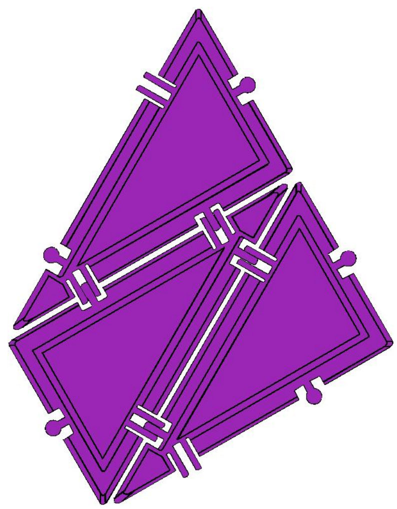




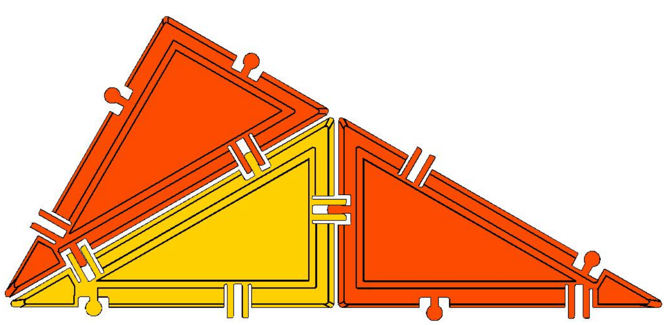
26.



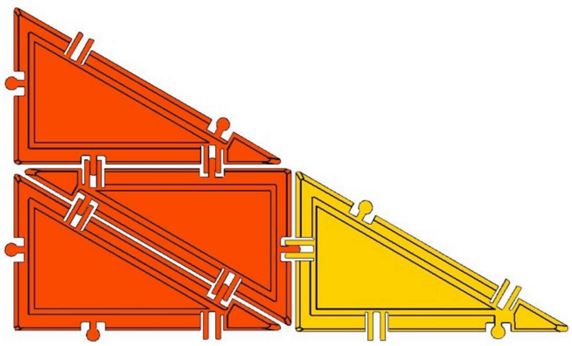
27.



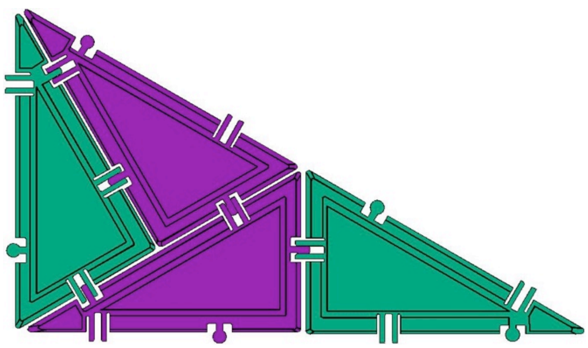
28.

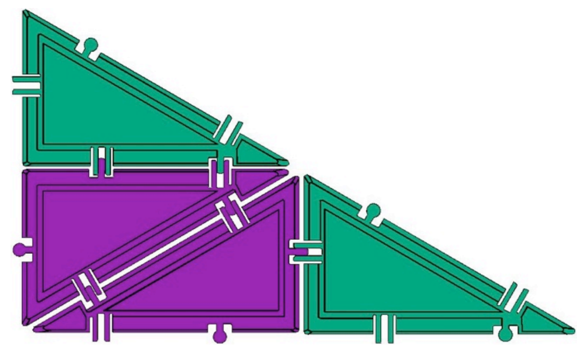


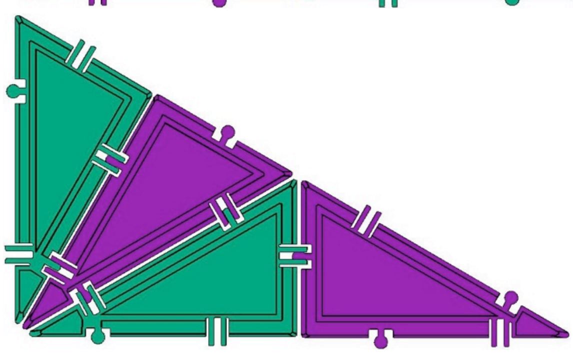
29.

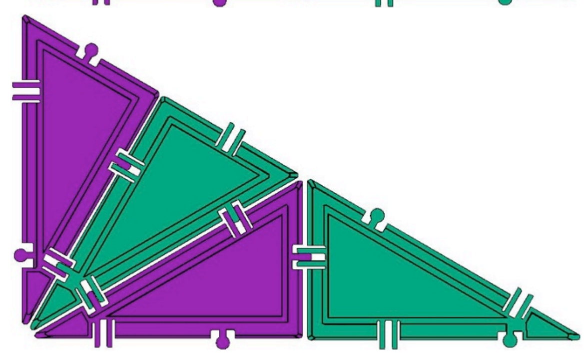


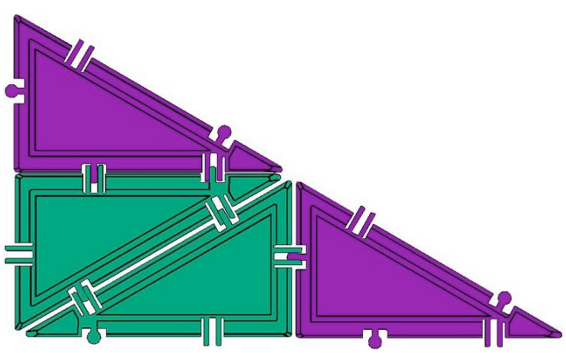
30. There are 6 possible ways to arrange the pieces. Challenge your students to find as many of them as they can.

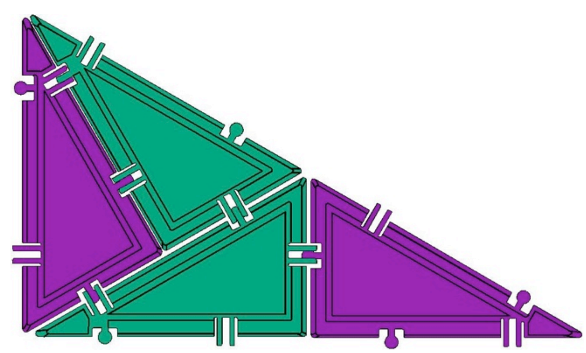








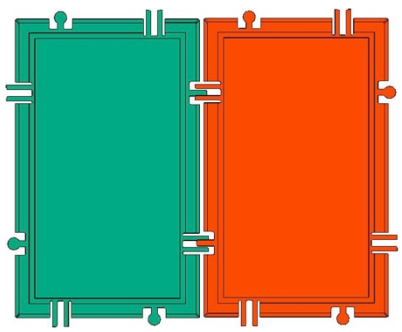




Level 1, Group B

Note: occasionally, the student is asked to solve a tangram in as many different ways as possible. Two solutions are considered "different" if one of them is not a flipped or rotated version of the other. To be "different", two solutions must be constructed in a different way. See, for example, the solution to Problem 8 on the following page.

1.



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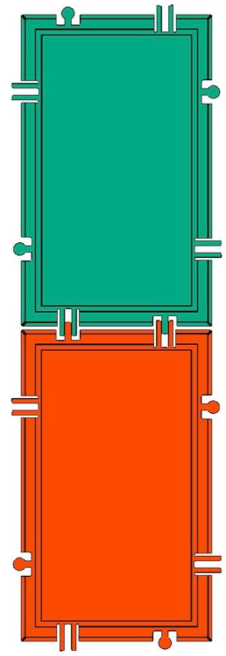

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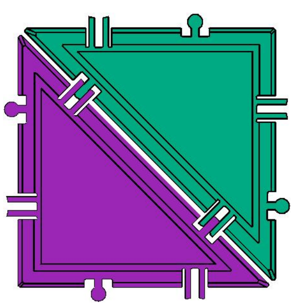

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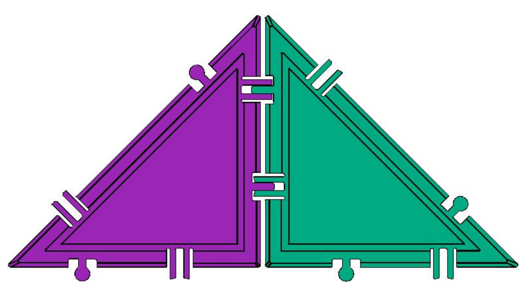

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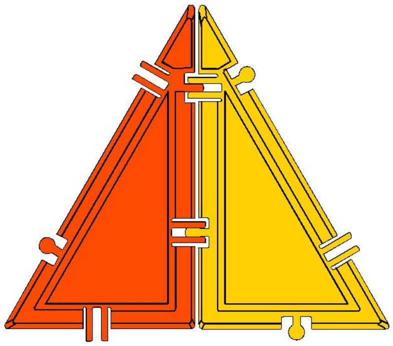

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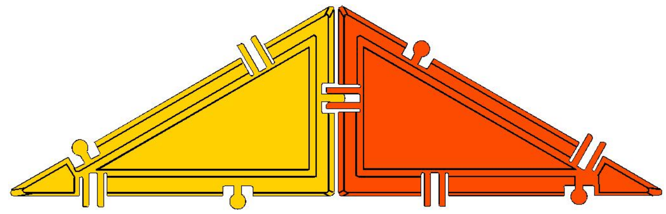

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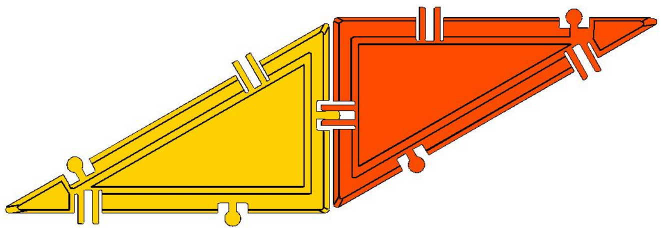

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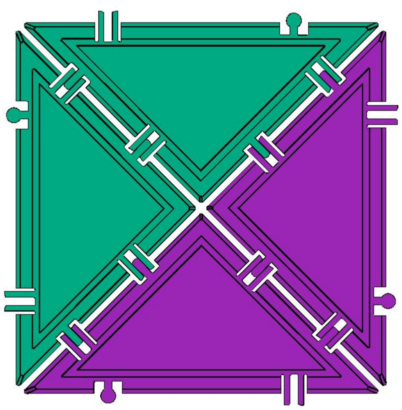

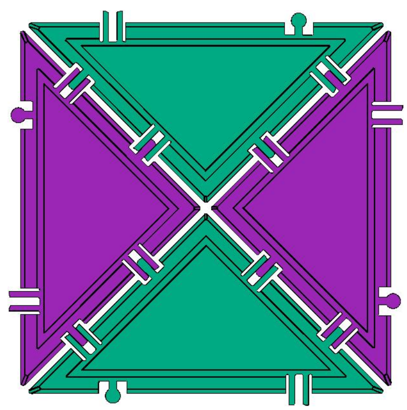

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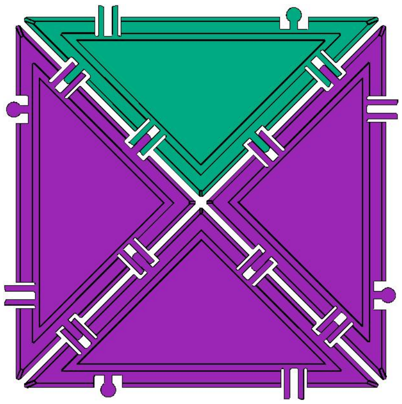

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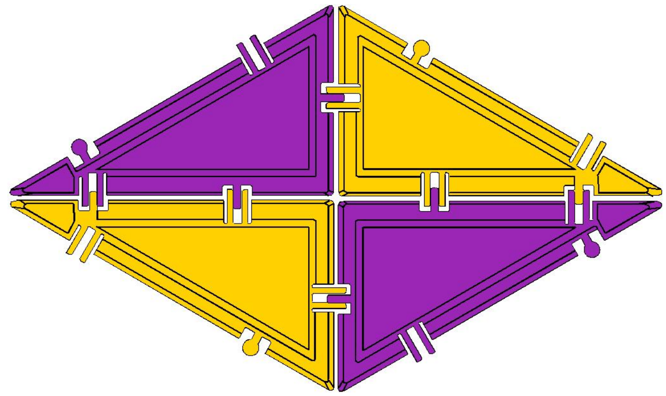

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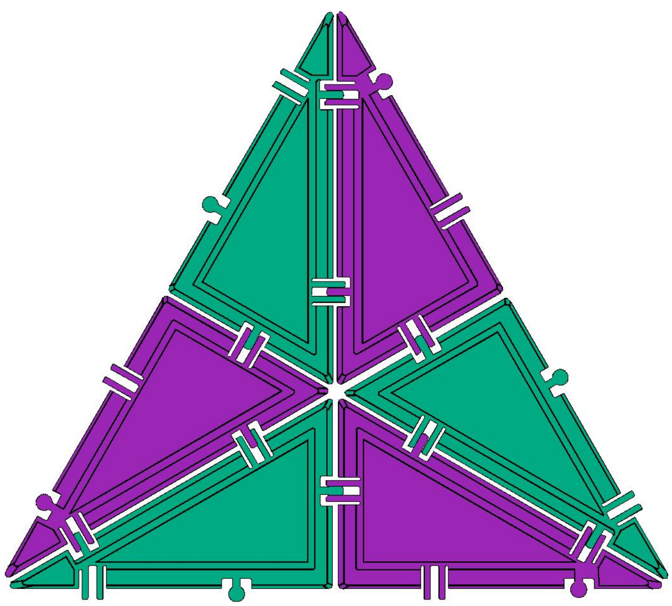
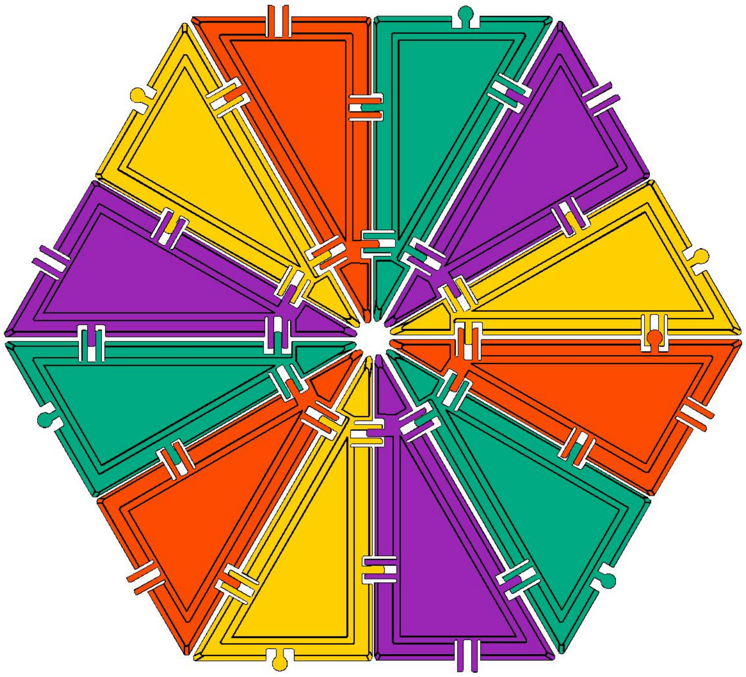
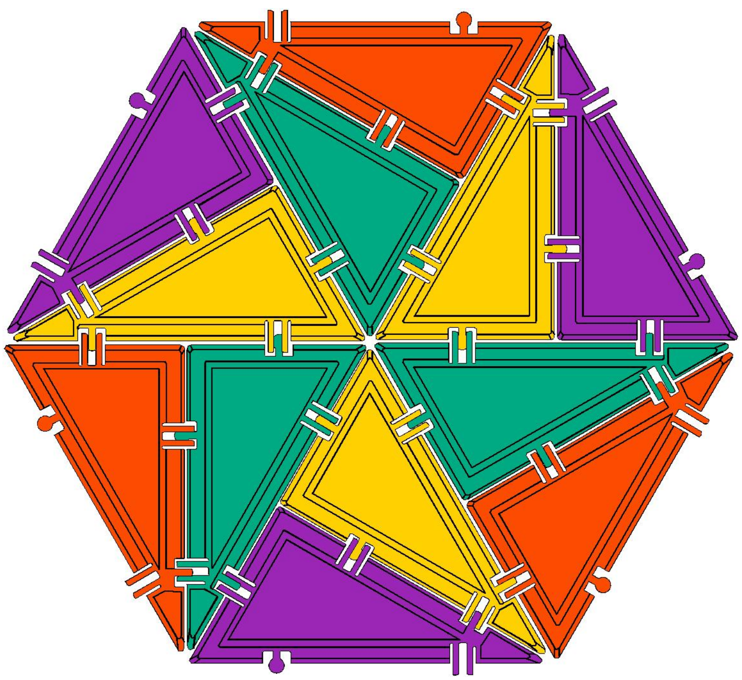
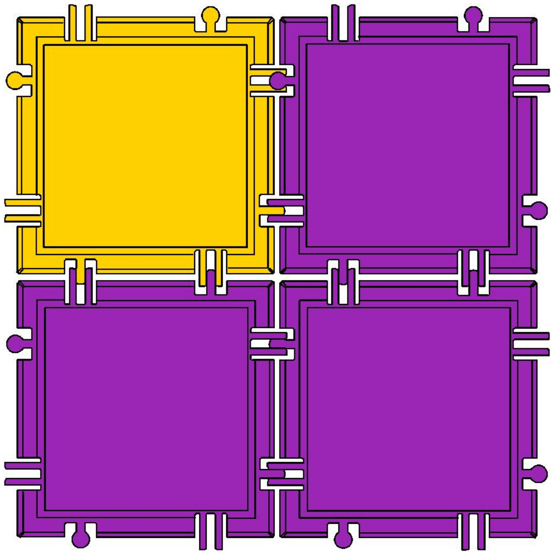


12. There are many different ways to arrange the 12 tiles. Here are some of the prettier ones.





13, ¾ of the square is purple



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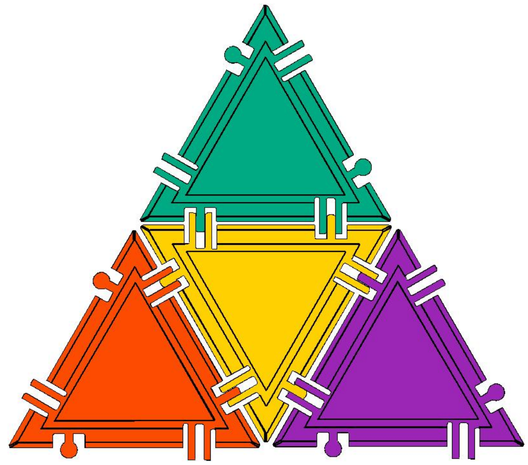

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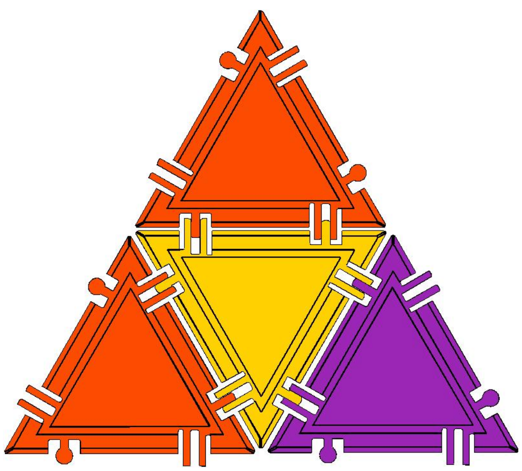

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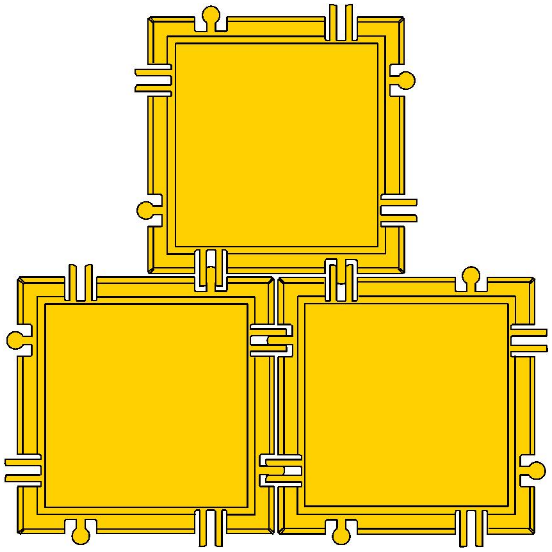
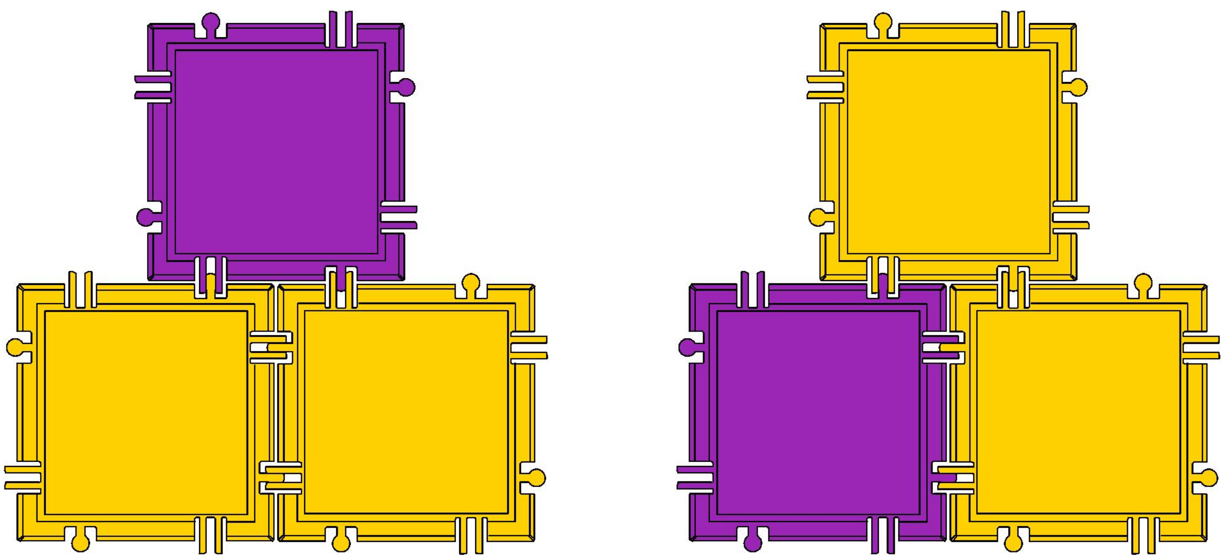
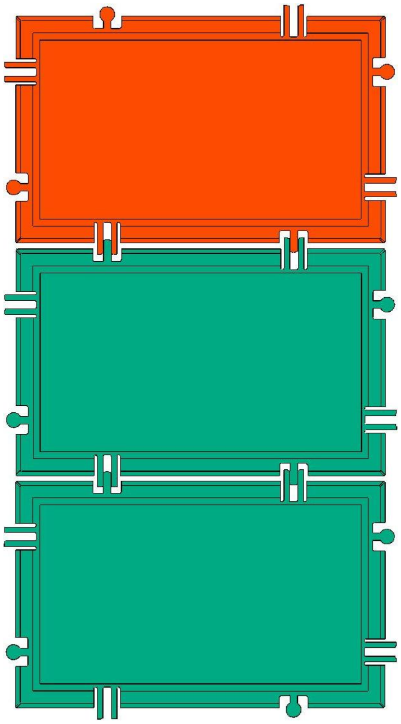
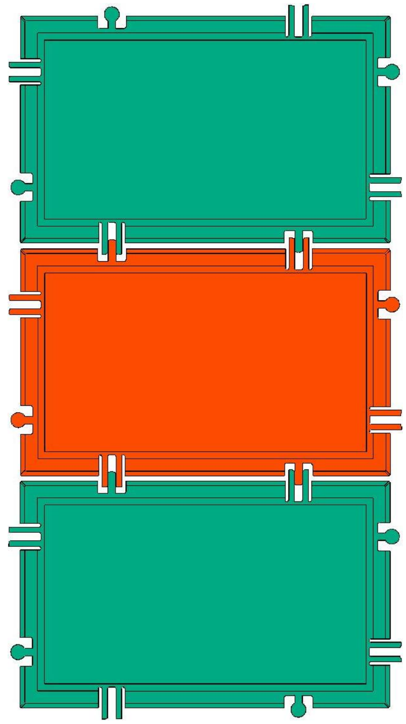


17.

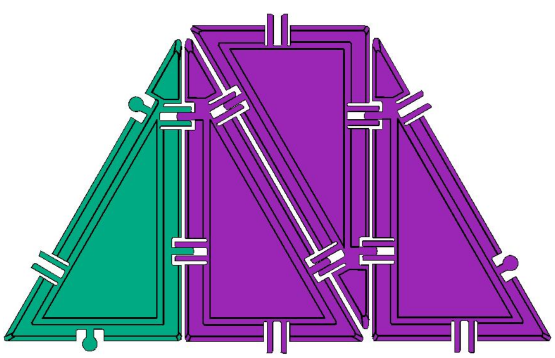


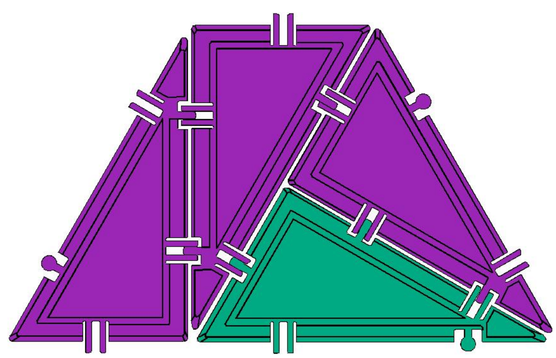
18.

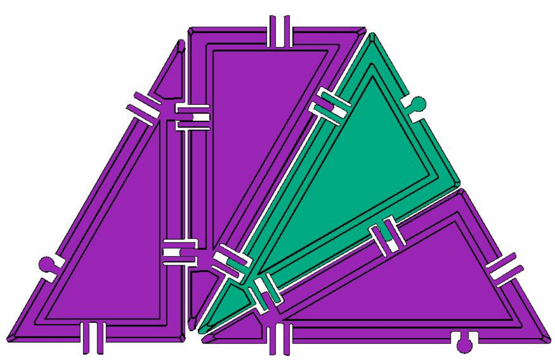




19. ¾ of the trapezoid is purple.







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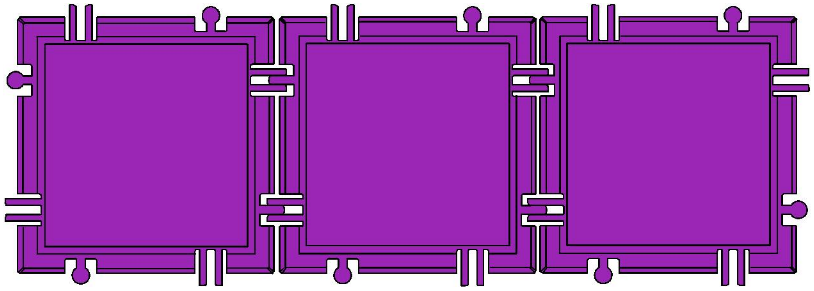

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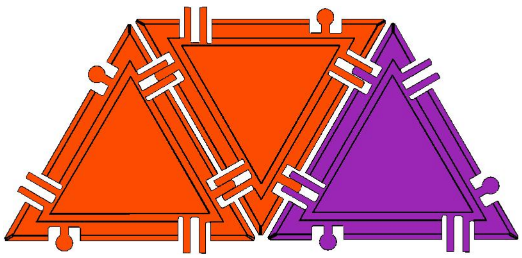

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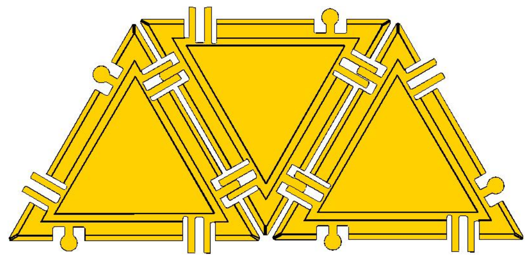

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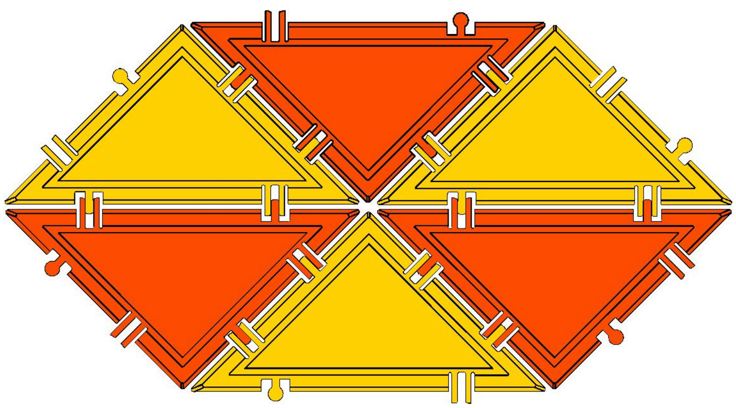

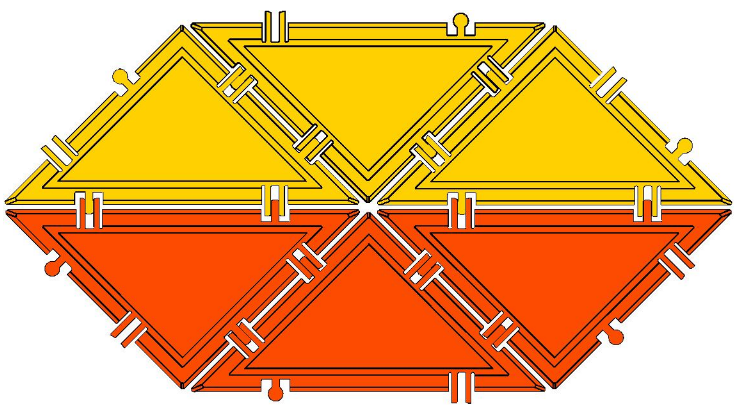

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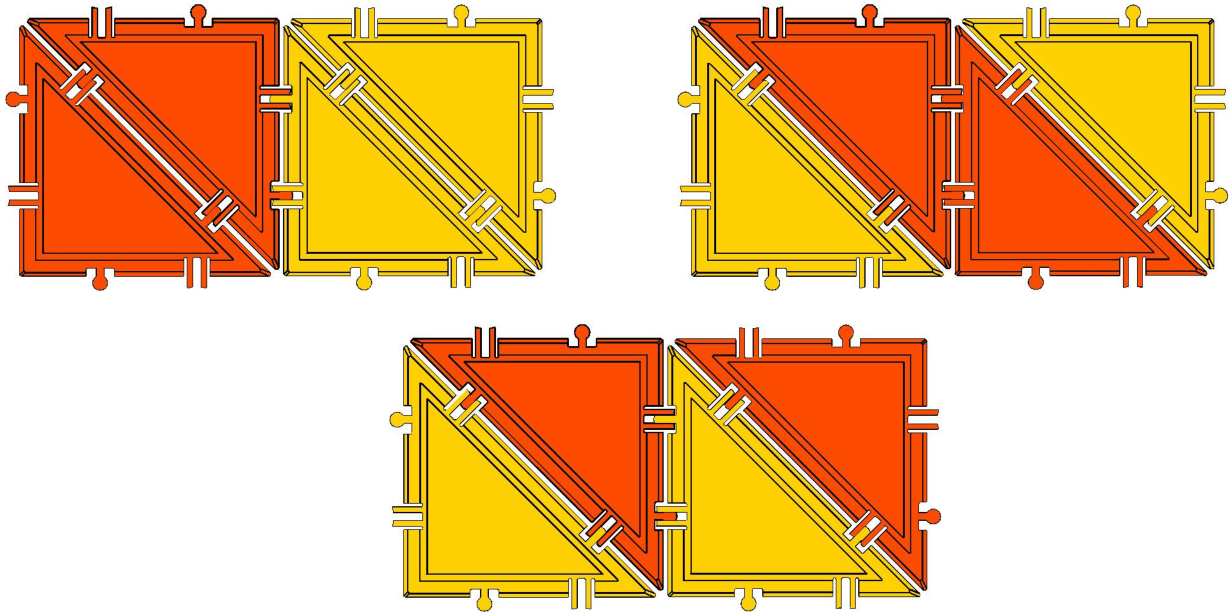
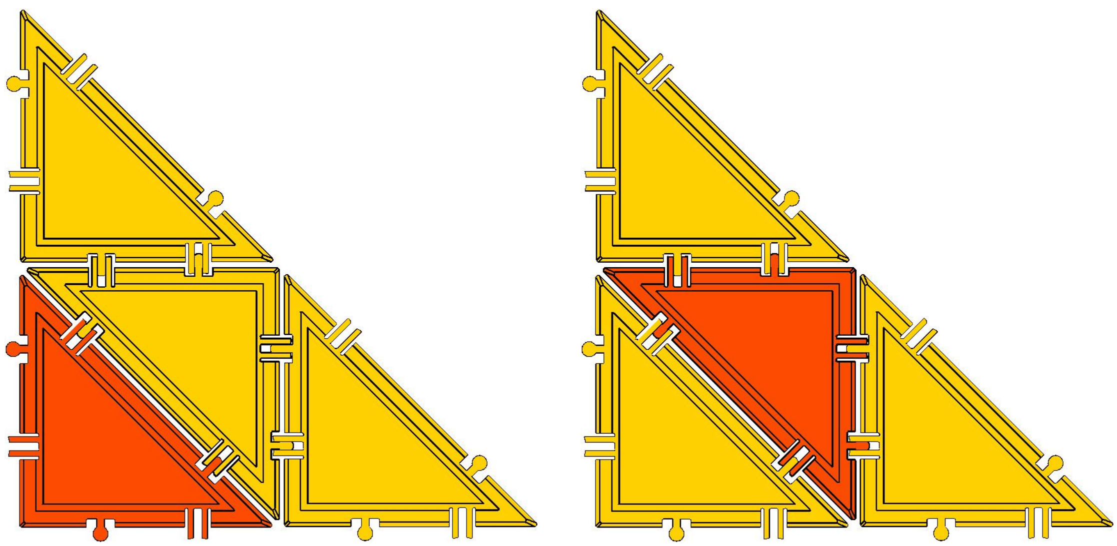
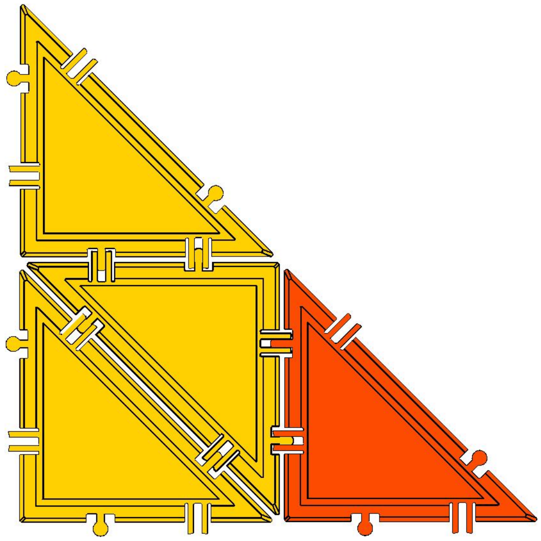


25. ¼ of the triangle is orange.





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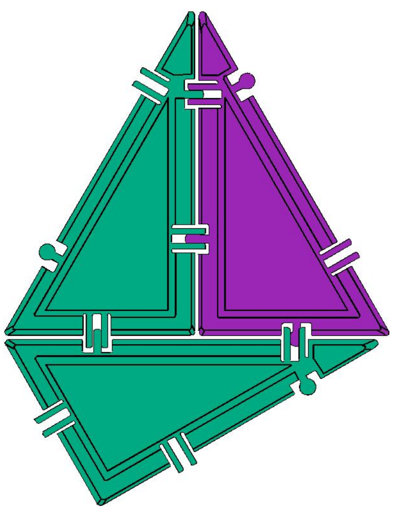
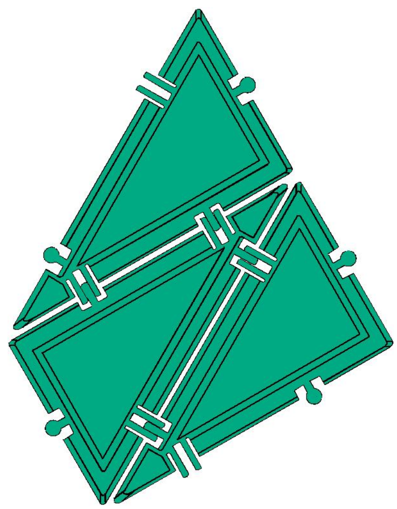
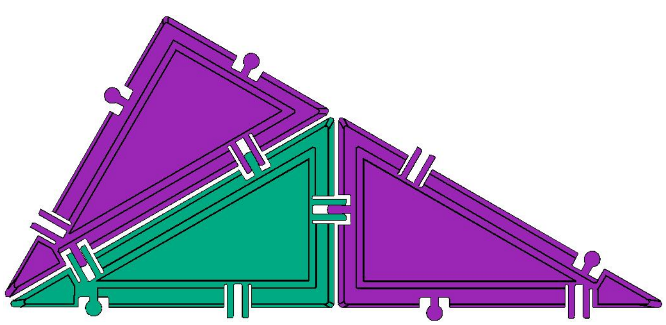


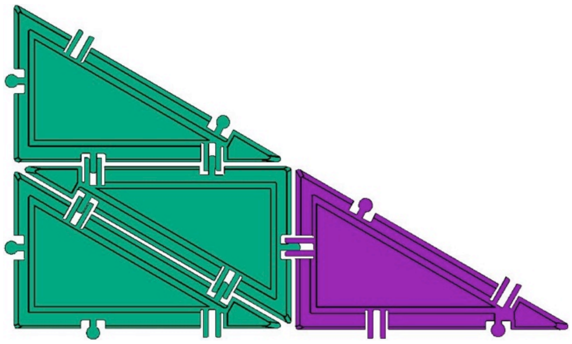
27.



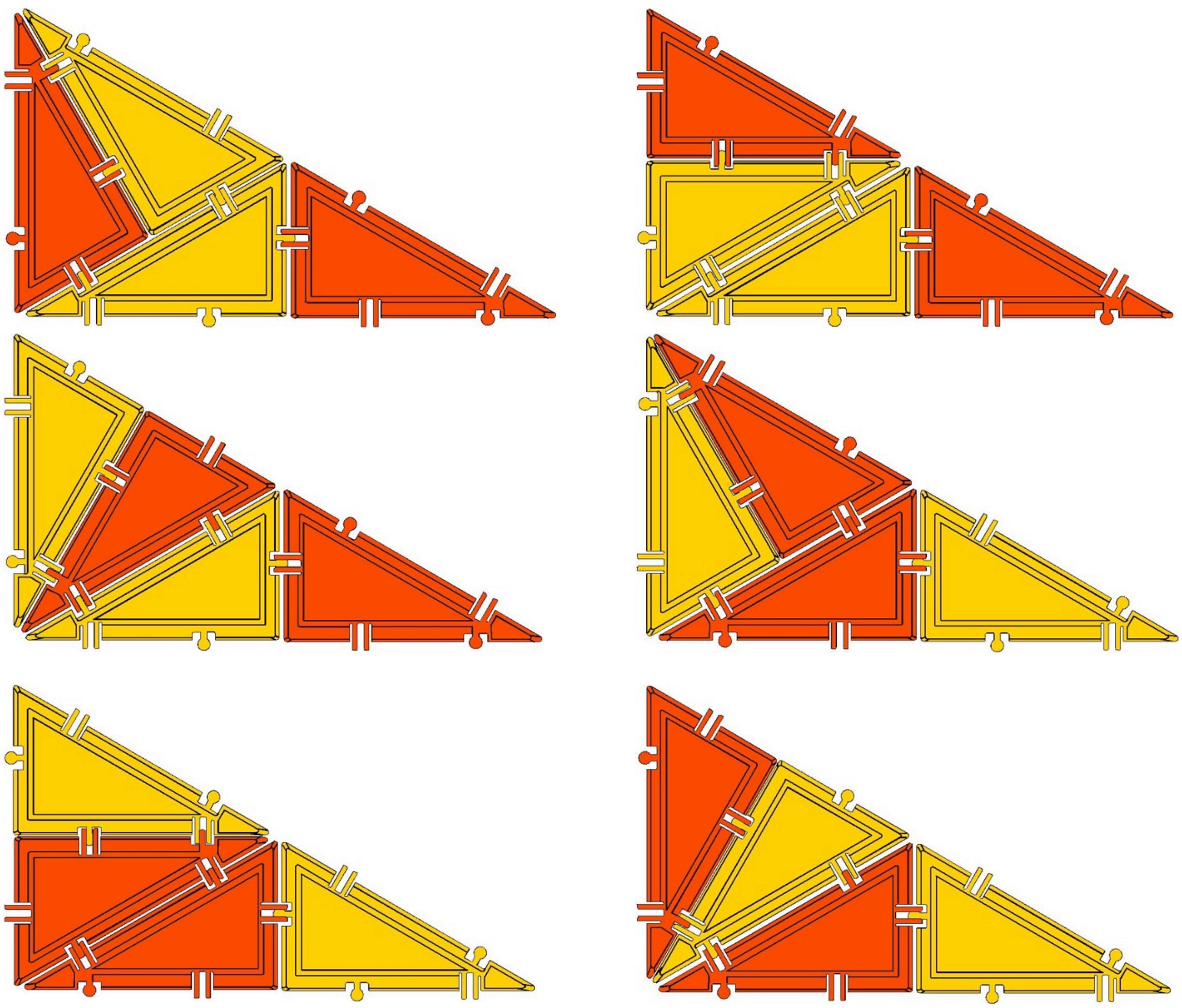
28.



29.

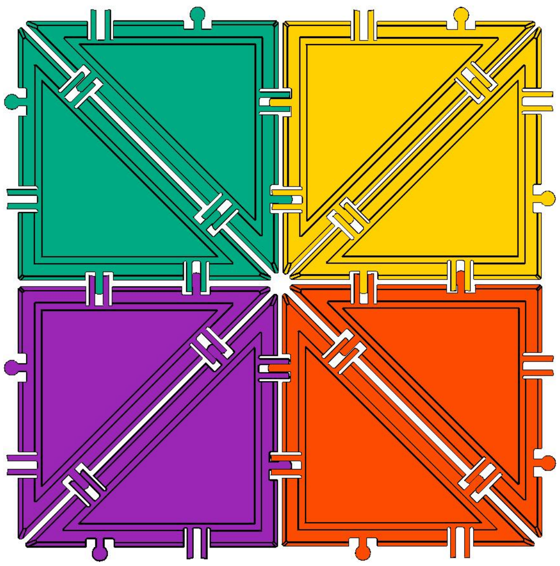


30. There are 6 possible ways to arrange the pieces. Challenge your students to find as many of them as they can.

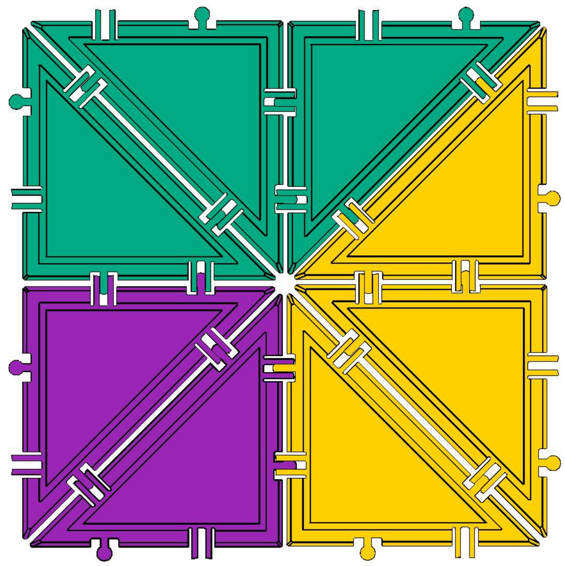


Level 2

31. a.



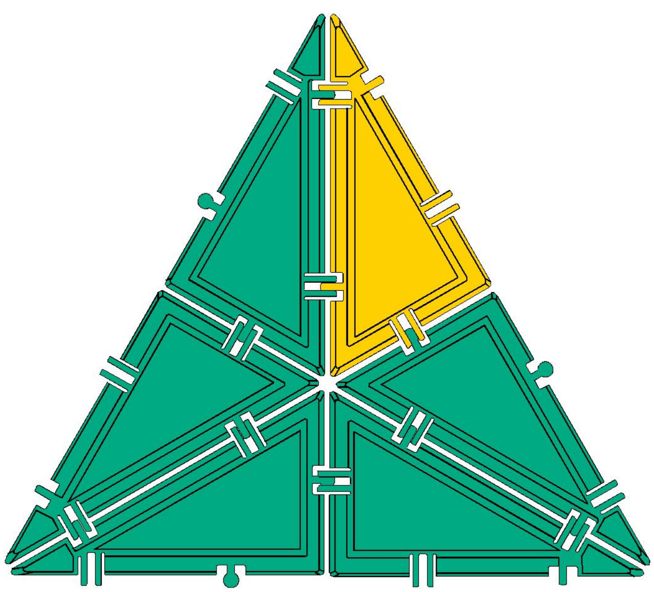
31. b.



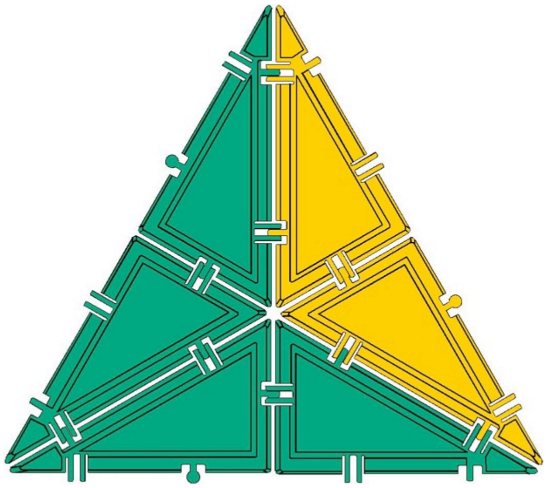
32. After doing the previous problem, students should get a sense that 1/8 of the square represents a single isosceles triangle, 1/4 of the square represents two isosceles triangles, etc.

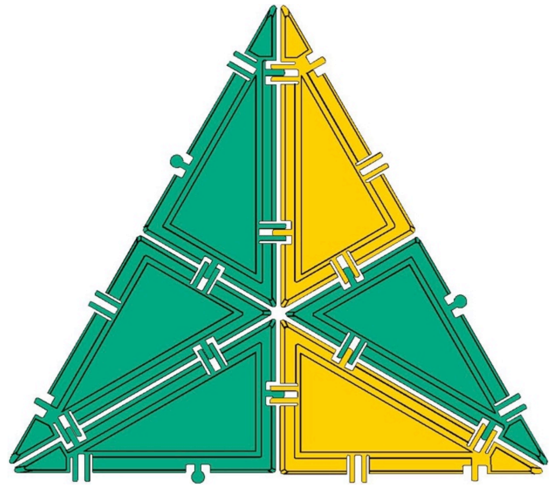
$$\frac{1}{8} \quad \frac{1}{4} \quad \frac{3}{8} \quad \frac{1}{2} \quad \frac{3}{4}$$

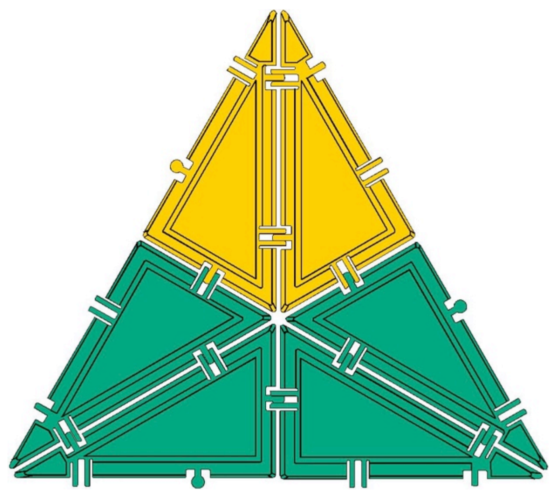
33. a.



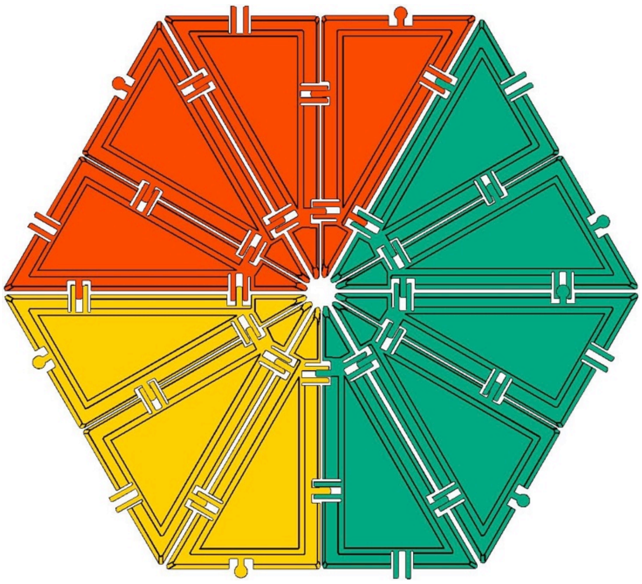
33. b.



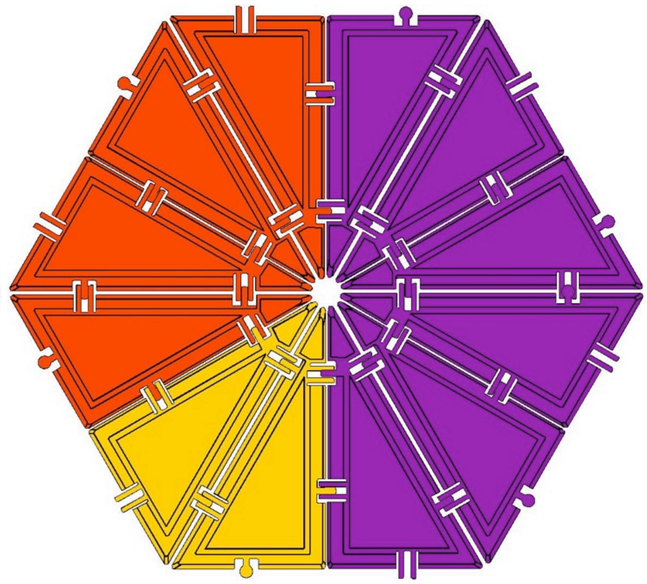




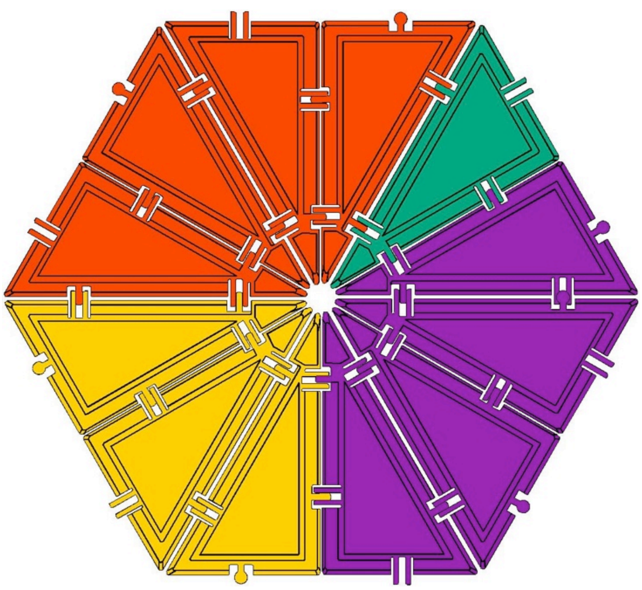
34. a.



34. b.



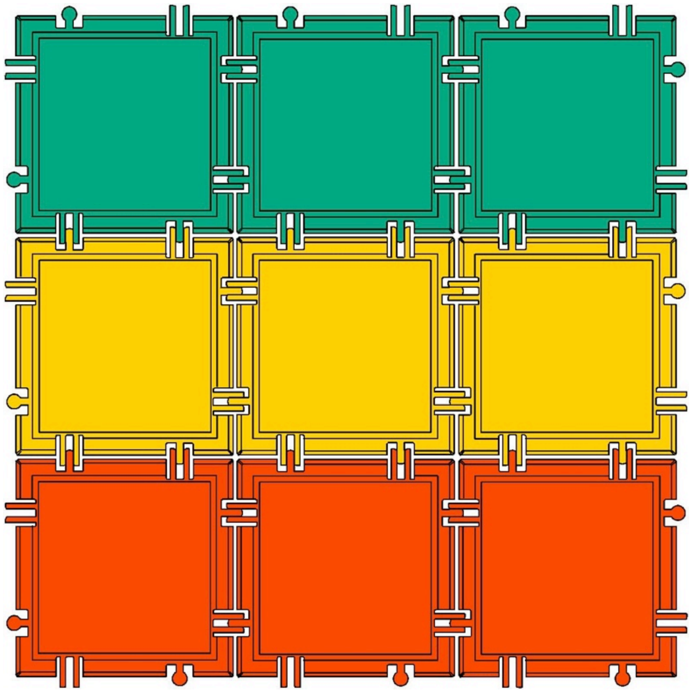
34. c. $\frac{1}{12}$ of the hexagon is green.



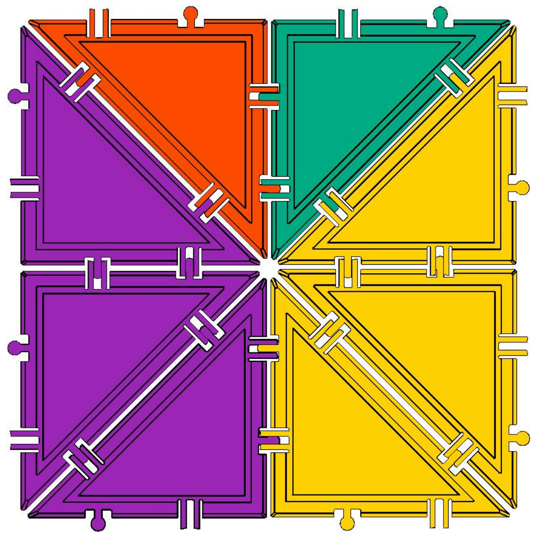
35. After doing the previous problem, students should geta sense that 1/12 of the hexagon is represented by a single scalene triangle, 1/6 of the square-- by two scalene triangles, etc.

$\frac{1}{12}$ $\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{11}{12}$

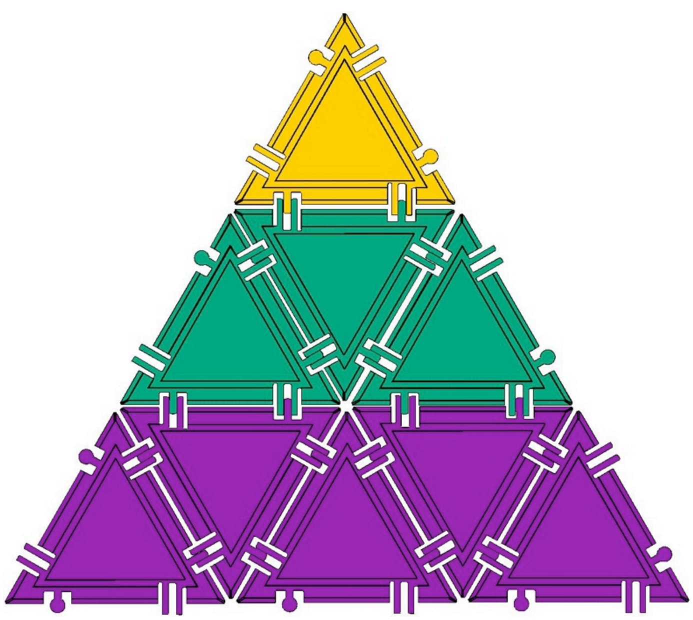
36.



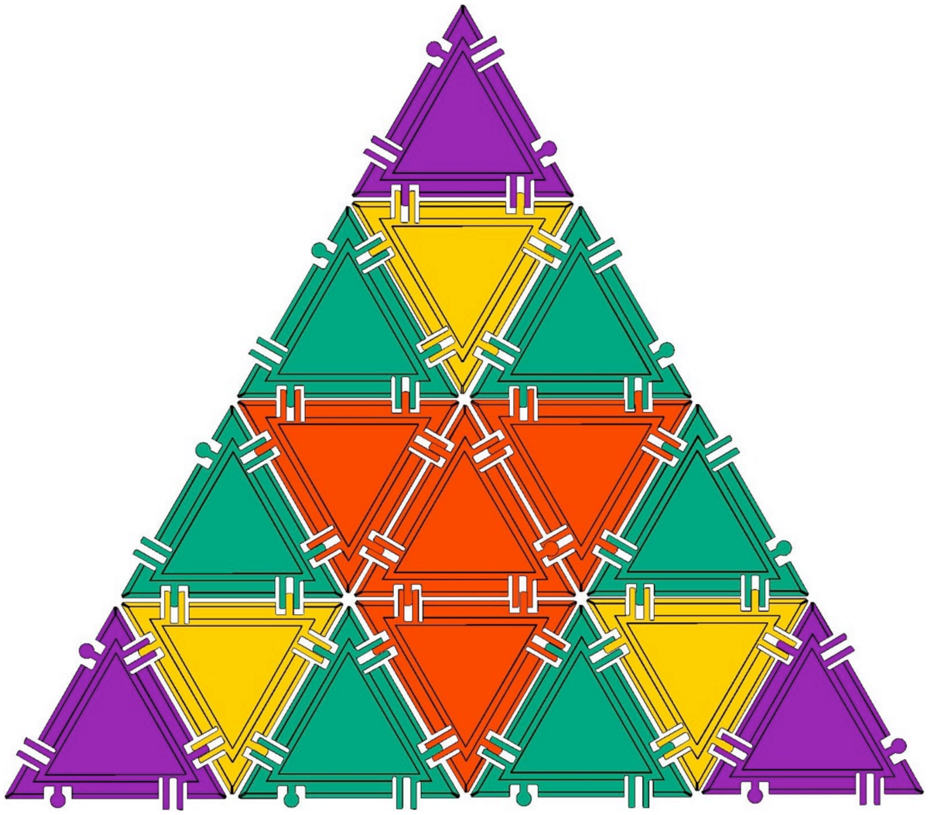
37. Part of this problem is for students to realize on their own that the square needs to be made of 8 tiles.



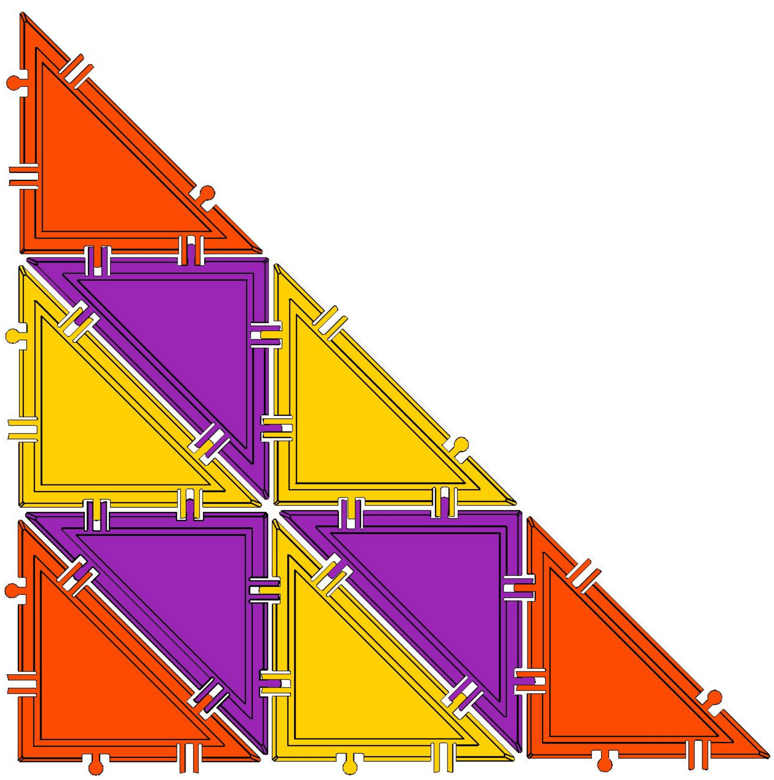
38.



39. The 16 in the denominator gives the clue that 16 small triangles are needed.



40.

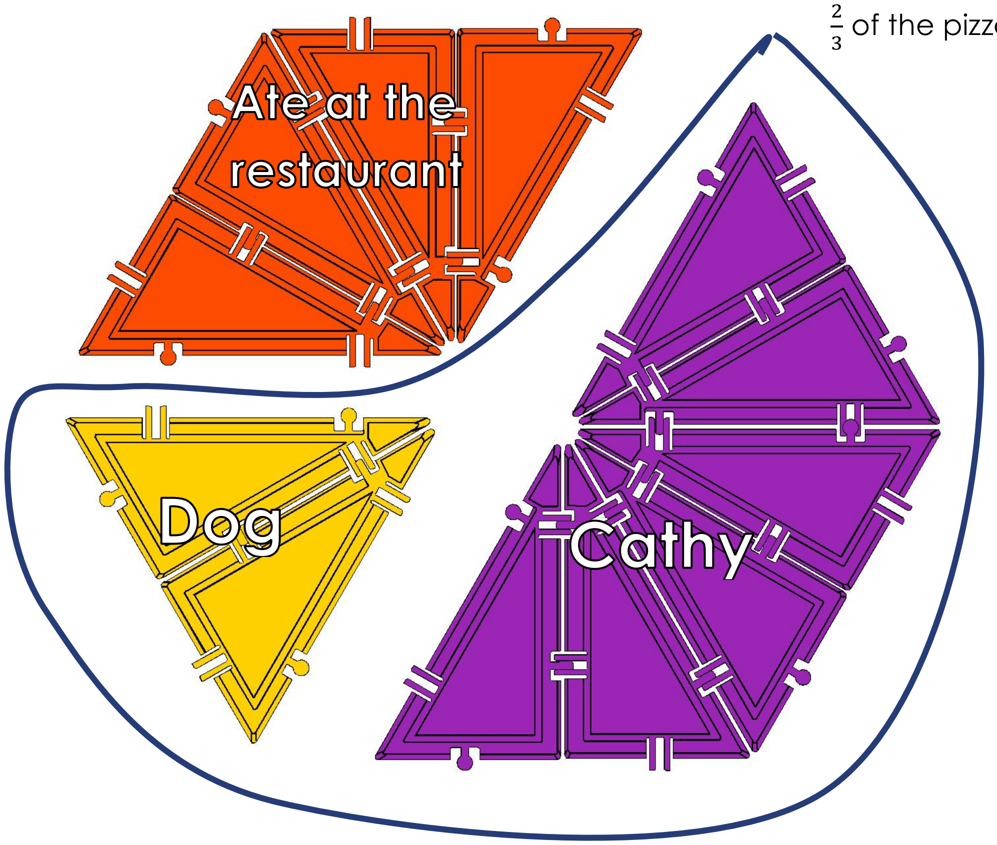


41. The best model for this pie is a hexagon made of 6 triangles. We can't get away with less than 6, since 6 is the smallest number that is divisible by both 2 and 3.

The dad ate $1 - \frac{1}{3} - \frac{1}{2} = \frac{1}{6}$ of the pie.

42. Here we need a hexagon made of 12 equally sized pieces, since the number of pieces will have to be divisible by both 3 and 4.

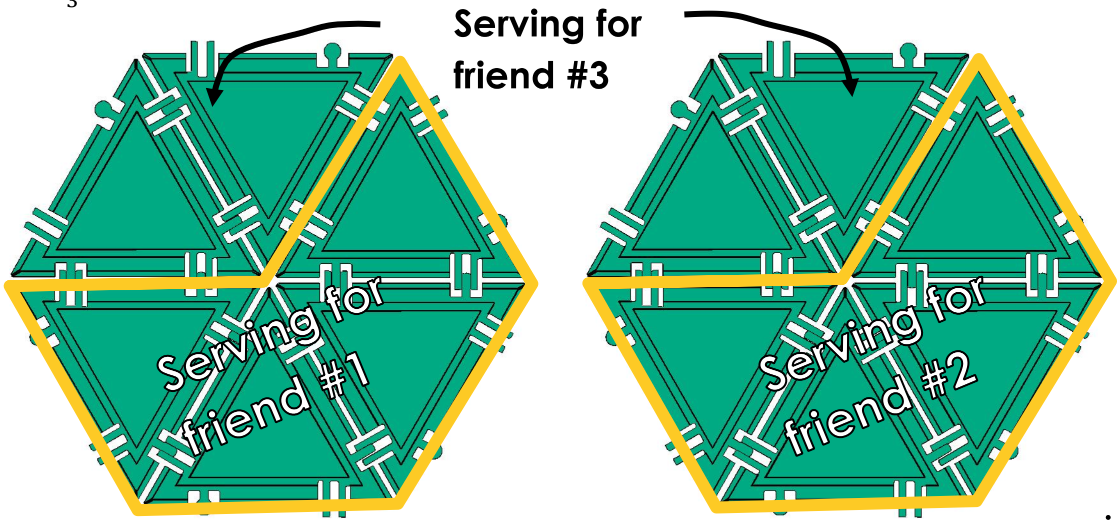
$\frac{2}{3}$ of the pizza (leftovers)



The dog ate ¼ of $\frac{2}{3}$ of the original pizza, which is $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$ of the original pizza.

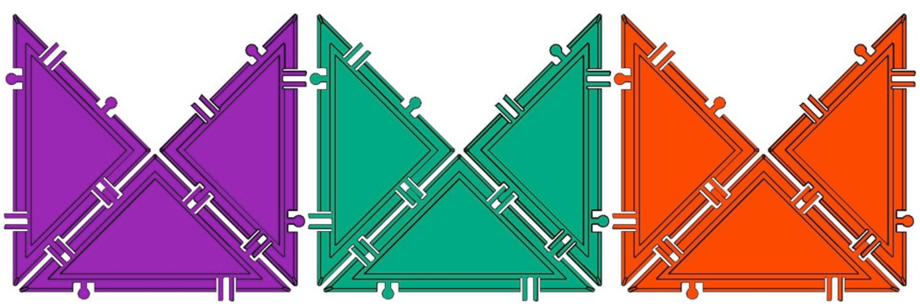
43. This situation is represented by the following mathematical statement:

$2 \div \frac{2}{3} = 3$. In other words, 2/3 goes into 2 three times:

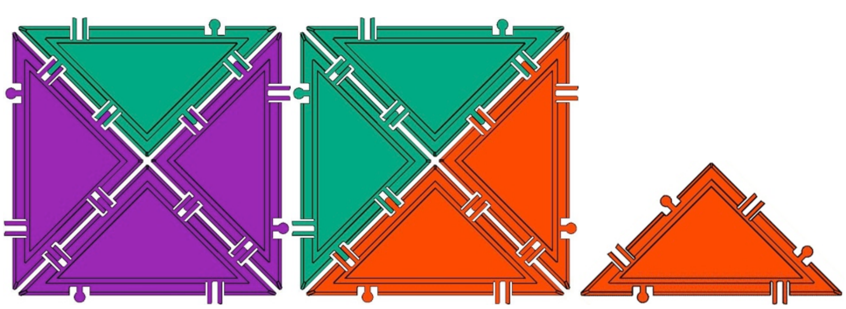


Alex can serve two friends with the parts of the pizzas outlined in yellow, and another friend with the non-outlined parts.

44. Here the best choice to represent a cup would be a square made of 4 equal pieces. Since ¾ cup servers just 6 people, we will need ¾ X 3=2¼ cups for 24 people:

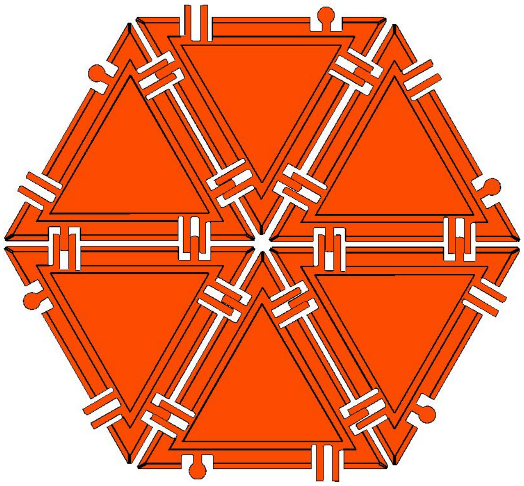


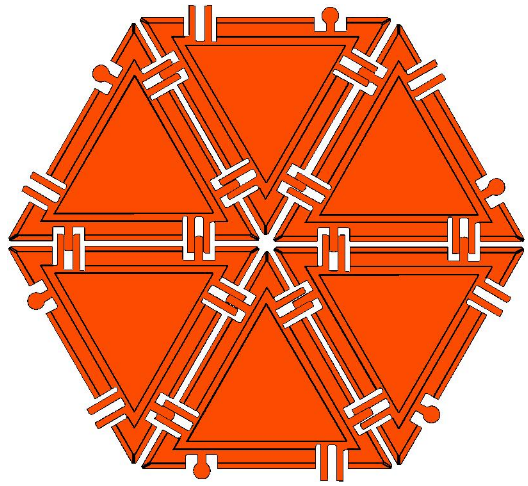
After rearranging the pieces, we get

,

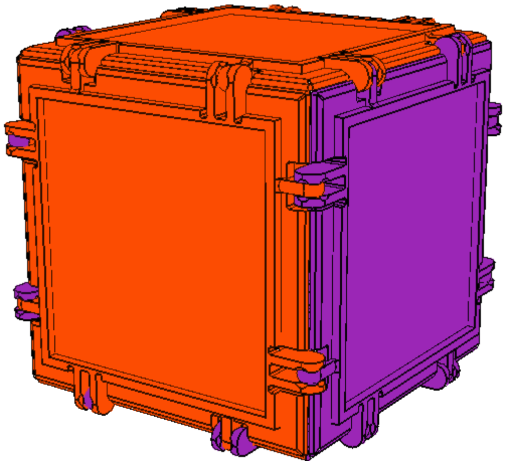
which is a total of 2 ¼ cups.

45. The best way to represent Nina's pumpkin pies is with two hexagons, each consisting of 6 identical triangles. Each triangle represents a serving. All together, we get $2 \div \frac{1}{6} = 12$ triangles, which is enough for all of Nina's 11 friends.

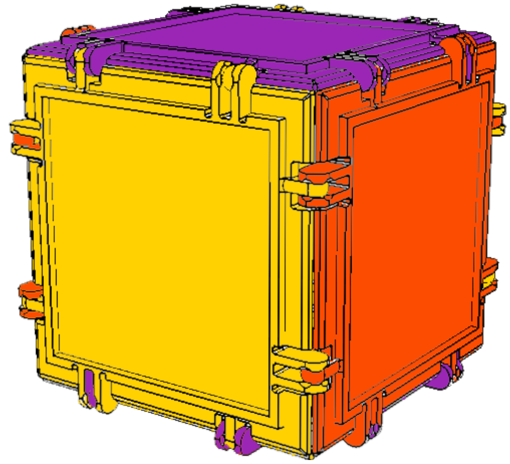




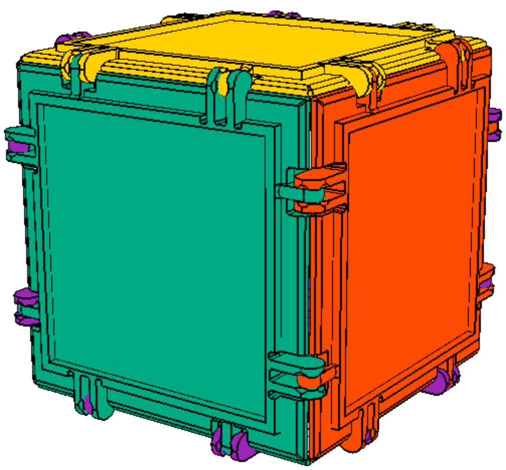
46. 3 orange squares and 3 purple squares.



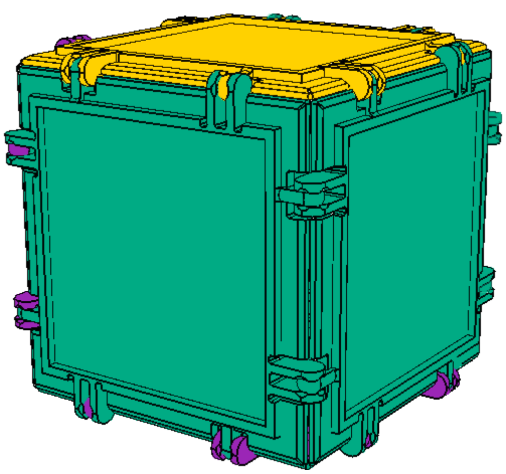
47. 2 each of orange, yellow, and purple squares.



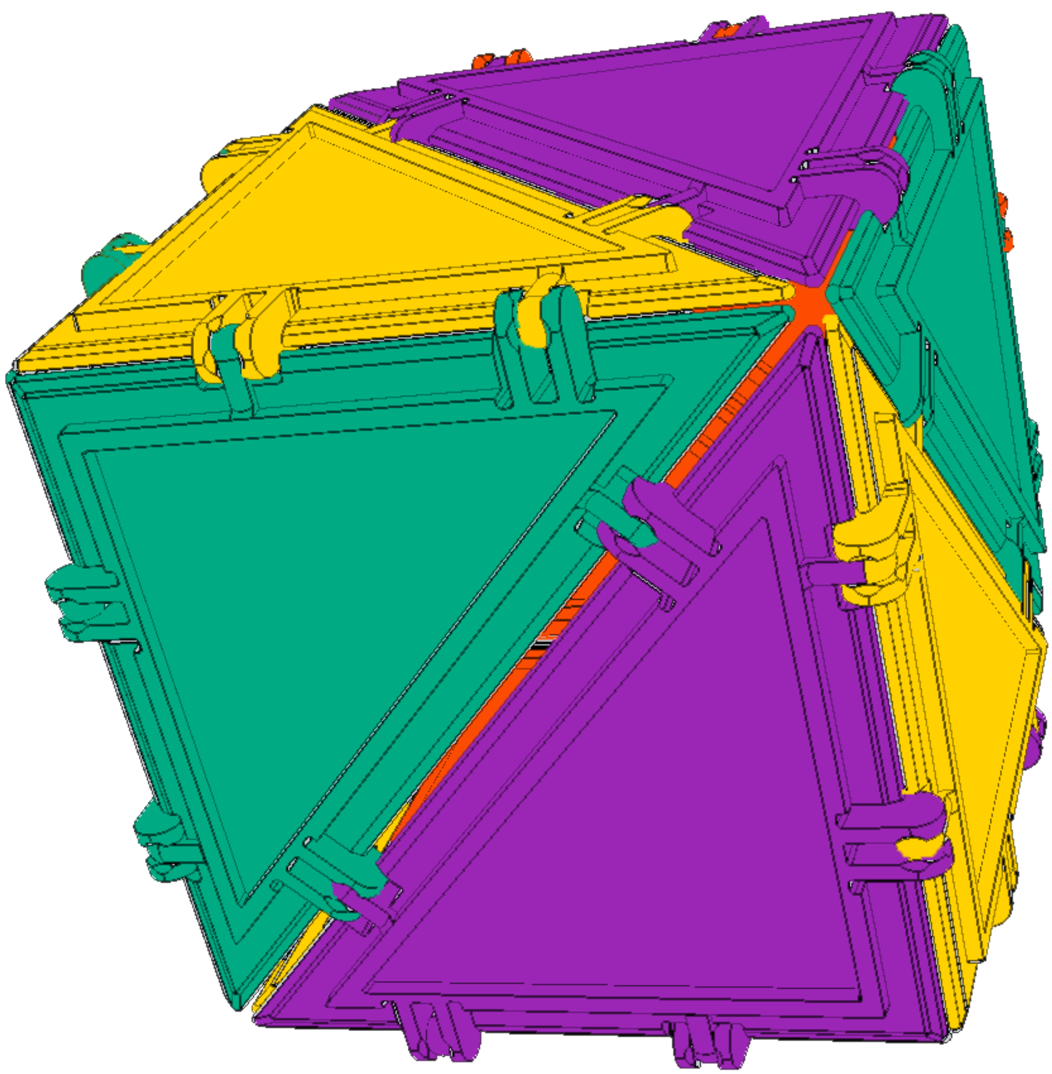
48. 1 each of yellow, orange, and green squares; 3 purple squares (in the back of the cube).



49. 1 yellow square, 3 green squares, and 2 purple squares (in the back of the cube).



50. The challenge here is to realize that we need to make a cube with a number of tiles that is divisible by 4. Our first thought would be to make a cube with 6 square tiles, but 6 is not divisible by 4. So we need to make a cube with a number of tiles that is divisible both by 6 (since we need to build 6 congruent square faces) and by 4 (to satisfy the fraction requirement of the problem). The smallest such number is 12. If we use isosceles triangles, we can build a cube with 12 faces. The cube is shown below. It consists of 3 faces in each color: yellow, purple, orange, and green.

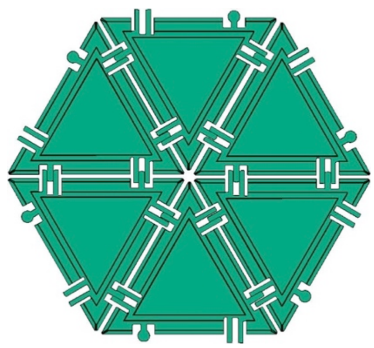


Note: colors do not matter in these problems. Colors shown in solutions below may not reflect actual colors in your set.

An Example for Problem 51.

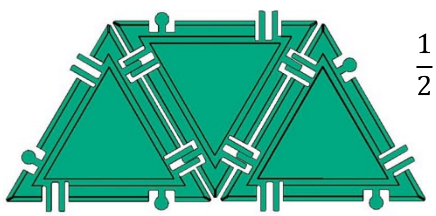
Let us start with $\frac{1}{2} + \frac{1}{3}$ as an example. We need to represent 1 with a figure that is made of number of tiles that is divisible by both 2 and 3. This is necessary because we want to make sure that 1/2 of the tiles and 1/3 of the tiles are whole numbers. The smallest number divisible by both 2 and 3 is 6. So let's take a figure that is made up of 6 identical tiles to represent 1. The most obvious choice is the hexagon .

For problem 51,



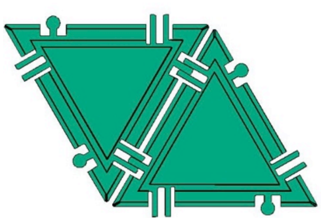
$= 1$

51. a.



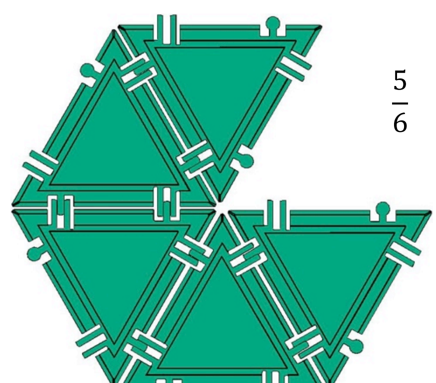
$\frac{1}{2}$

+



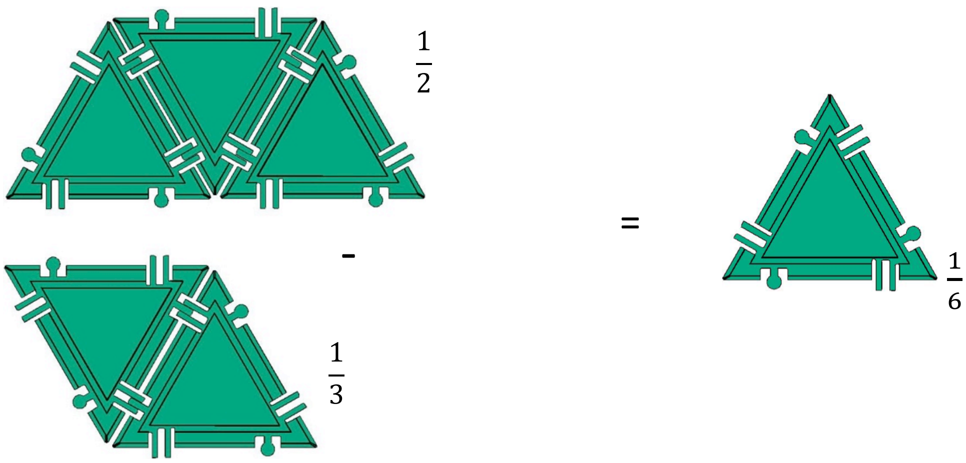
$\frac{1}{3}$

=

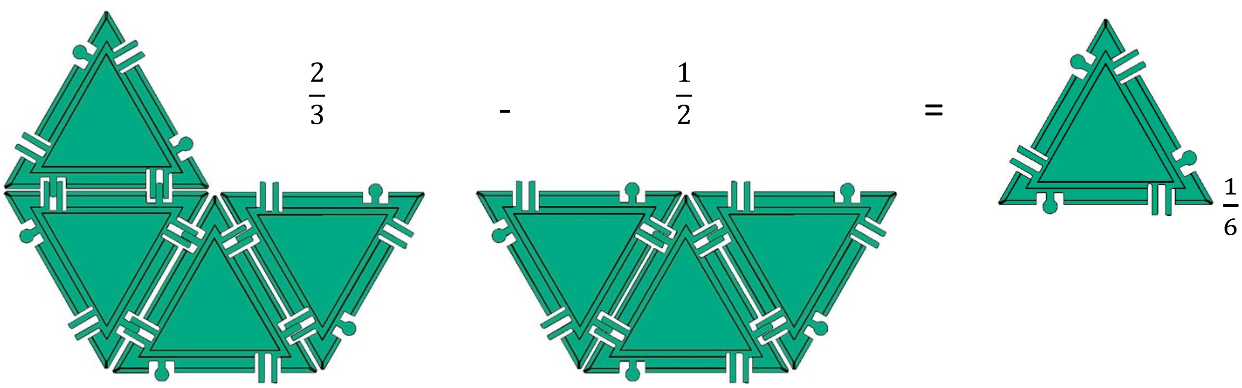


$\frac{5}{6}$

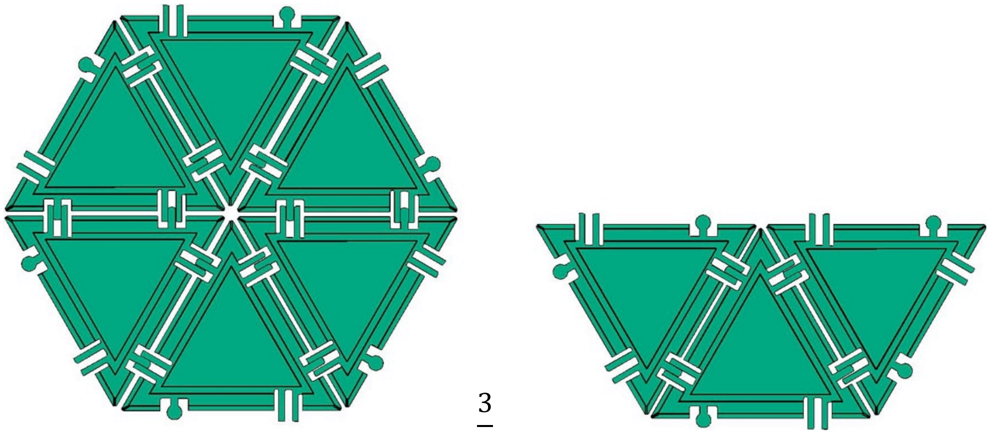
51.b.



51. c.

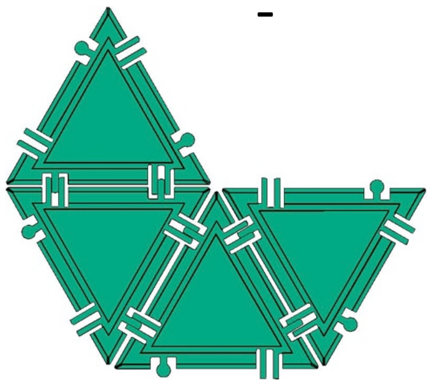


51.d.



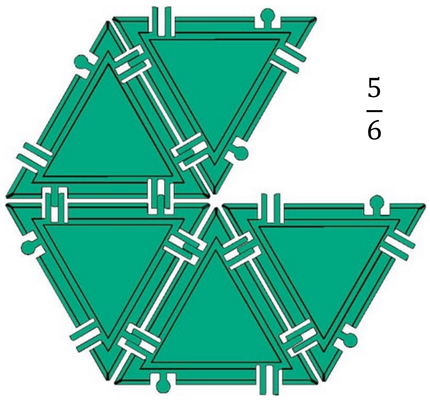
$\frac{3}{2}$

-



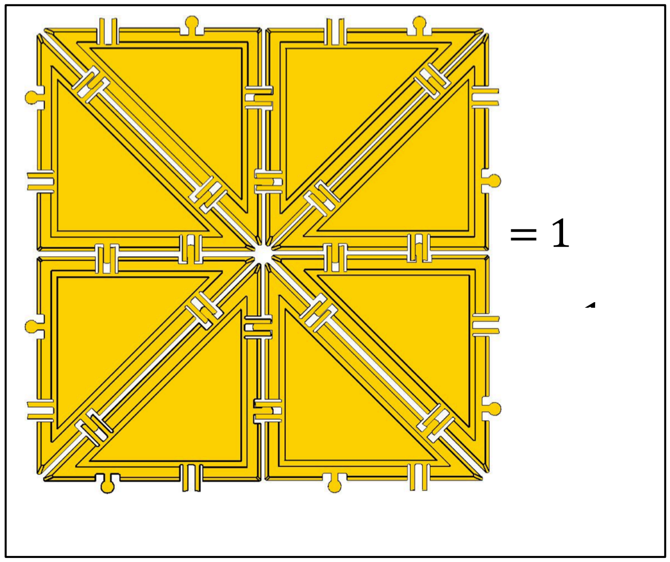
$\frac{2}{3}$

=

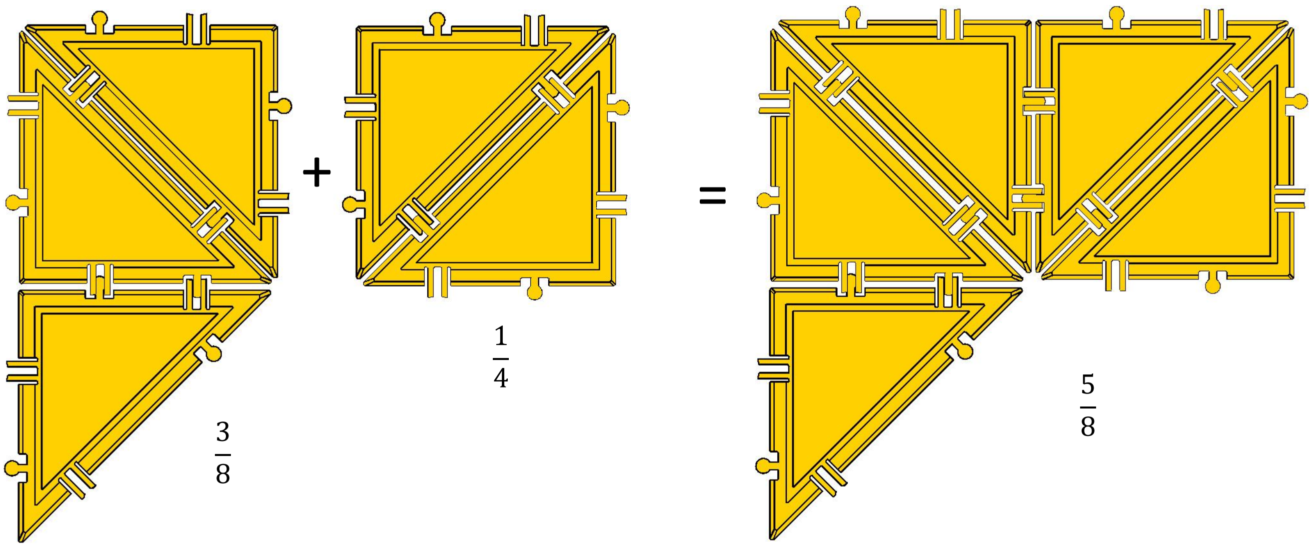


$\frac{5}{6}$

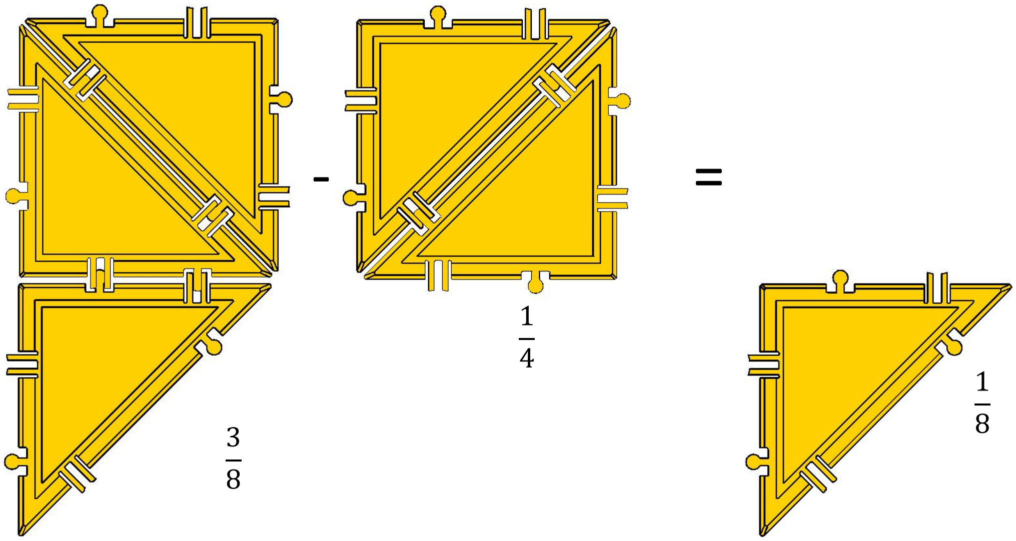
In problem 52,



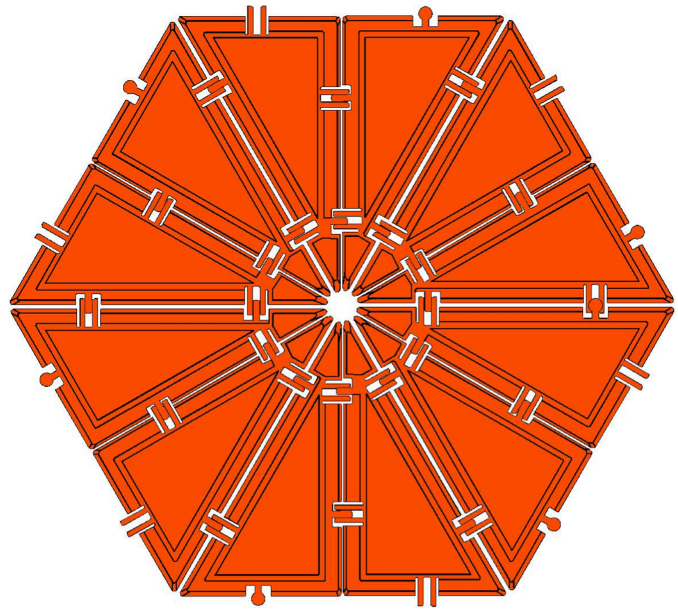
52. a.



52. b.

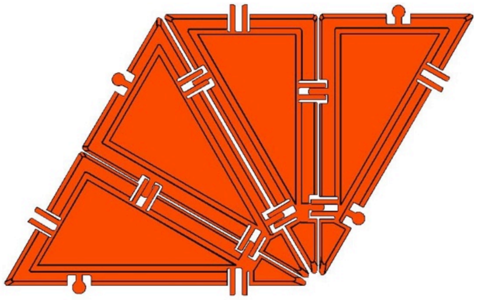


In problem 53,



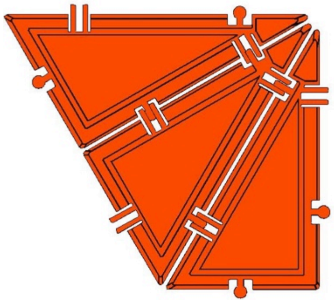
$= 1$

53. a.



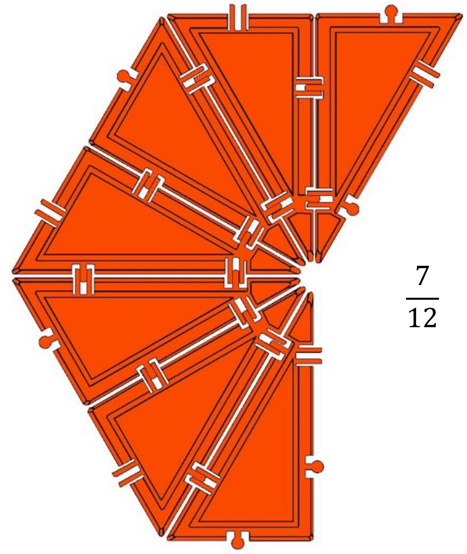
$\frac{1}{3}$

+



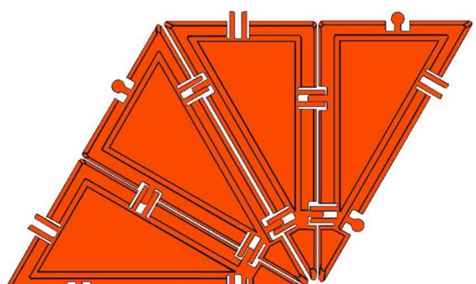
$\frac{1}{4}$

=



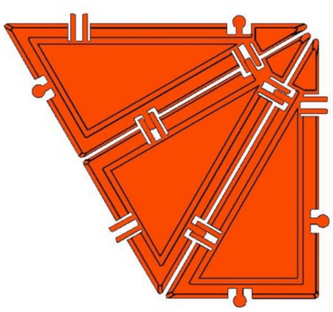
$\frac{7}{12}$

53. b.



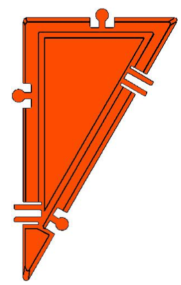
$\frac{1}{3}$

-



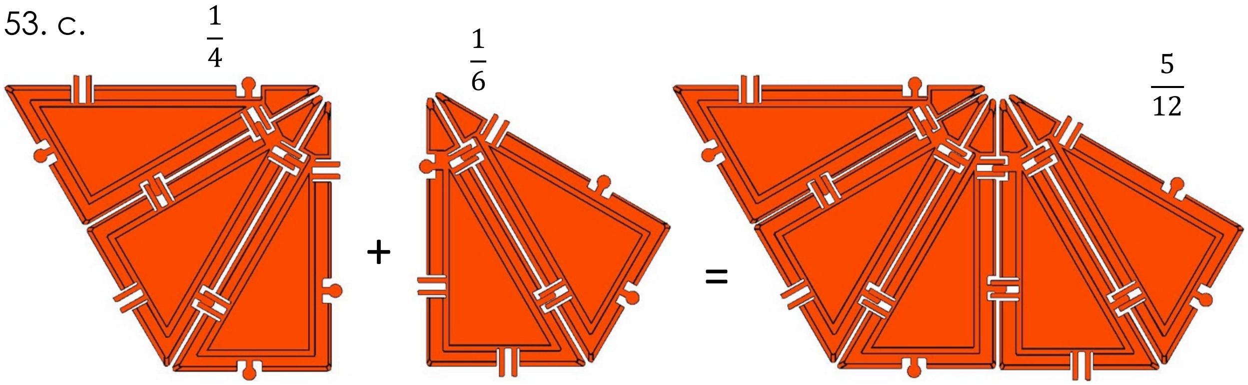
$\frac{1}{4}$

=



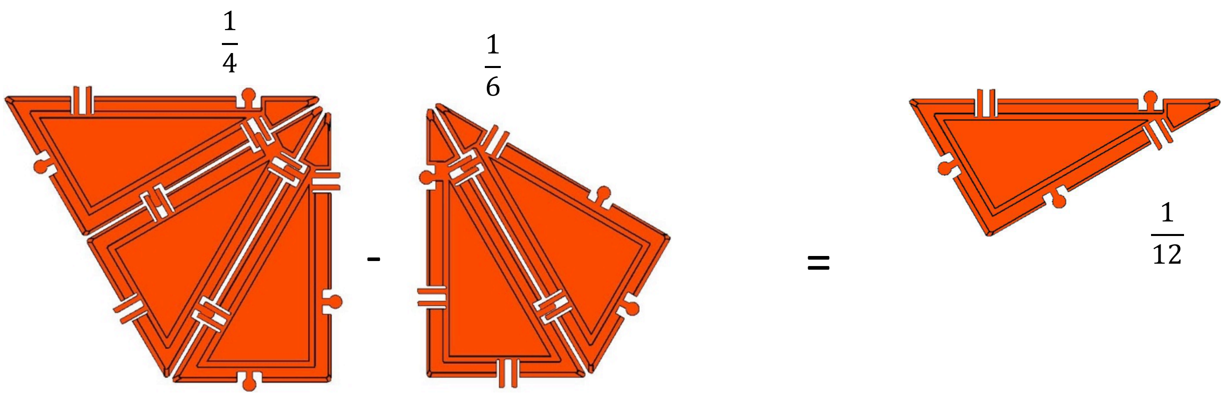
$\frac{1}{12}$

53. c.



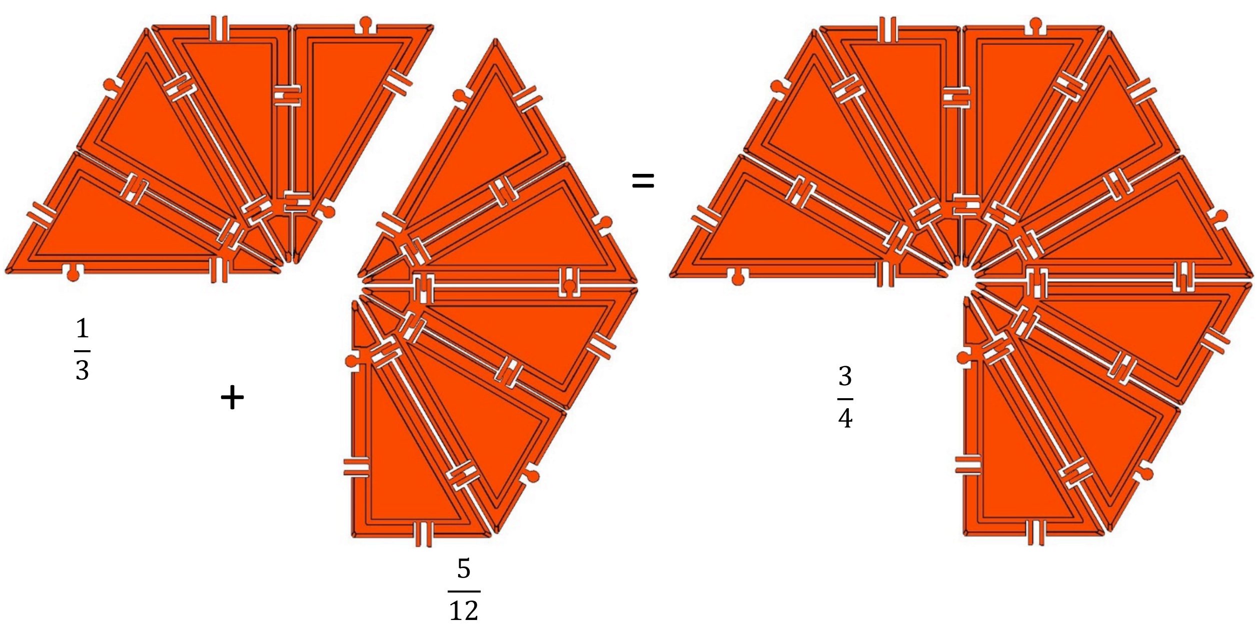
$\frac{1}{4}$
$\frac{1}{6}$
+
=
$\frac{5}{12}$

53. d.



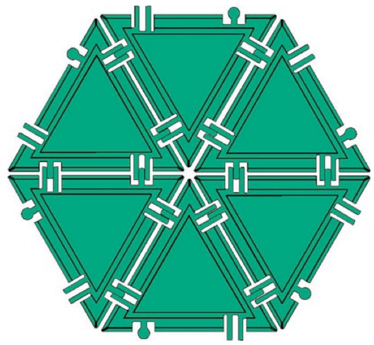
$\frac{1}{4}$
$\frac{1}{6}$
-
=
$\frac{1}{12}$

53.e.



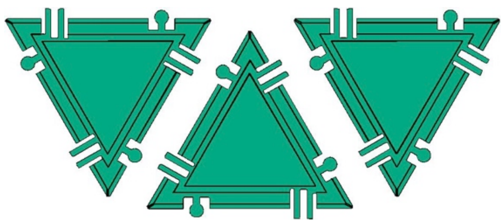
=
$\frac{1}{3}$
+
$\frac{5}{12}$
$\frac{3}{4}$

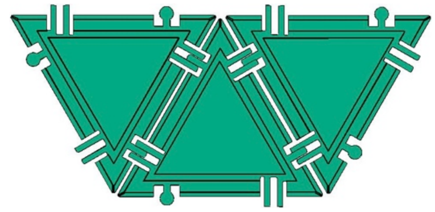
In problem 54 a-c,



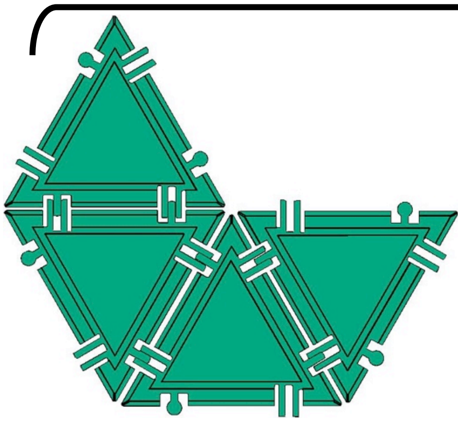
$= 1$

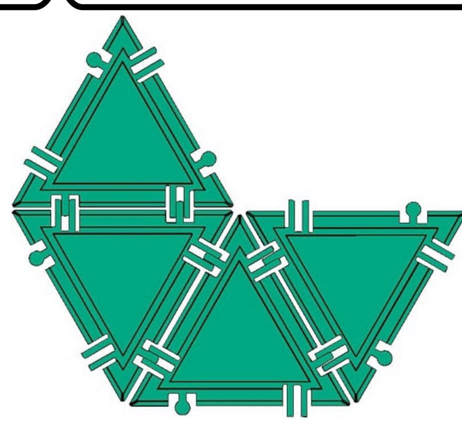
54. a. $3 \times \frac{1}{6}$ = $\frac{1}{2}$



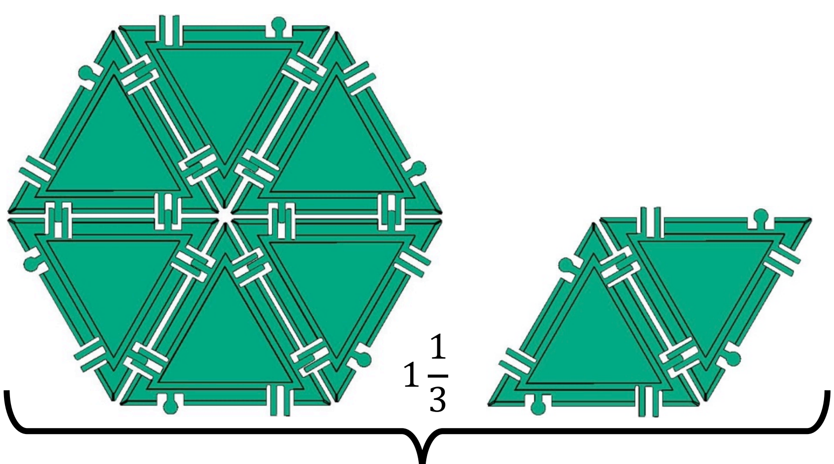


54. b. $2 \times \frac{2}{3}$



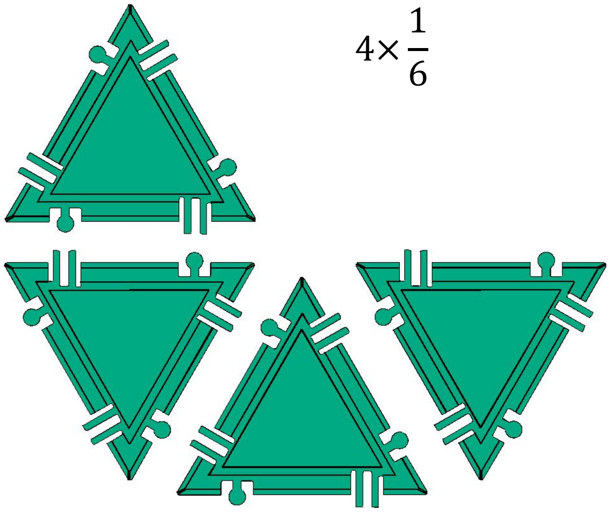


=



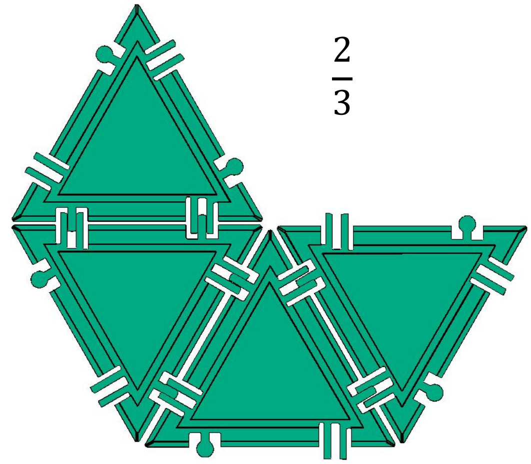
$1\frac{1}{3}$

54. c.



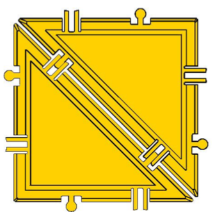
$4\times\frac{1}{6}$

=



$\frac{2}{3}$

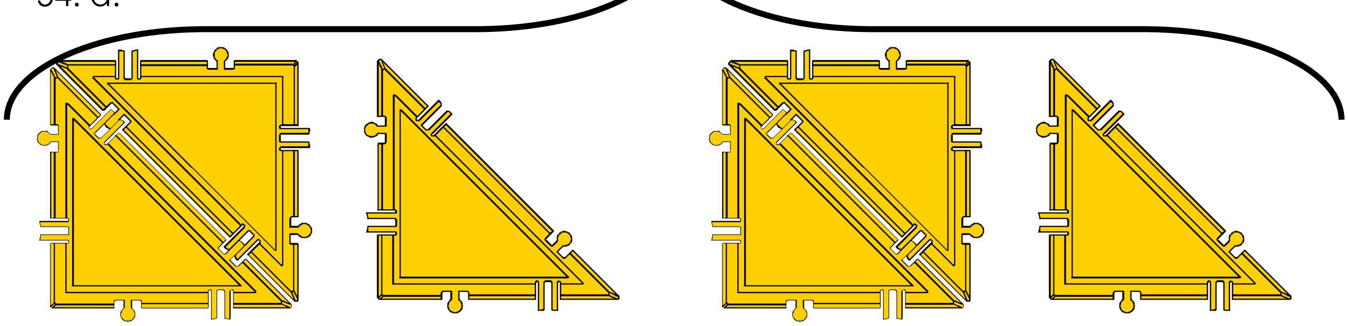
In problem 54d,



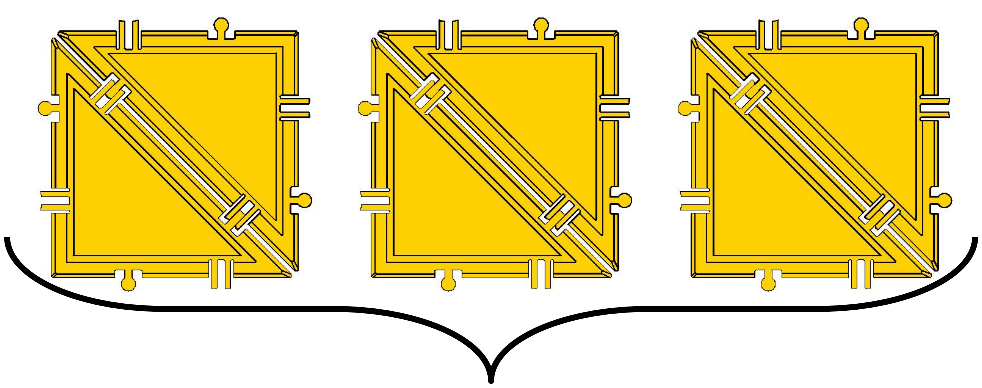
$= 1$

54. d.

$2\times\frac{3}{2}$

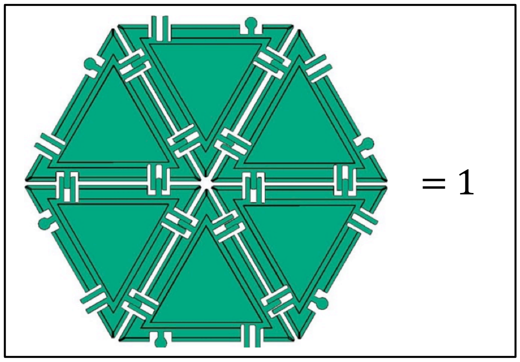


=

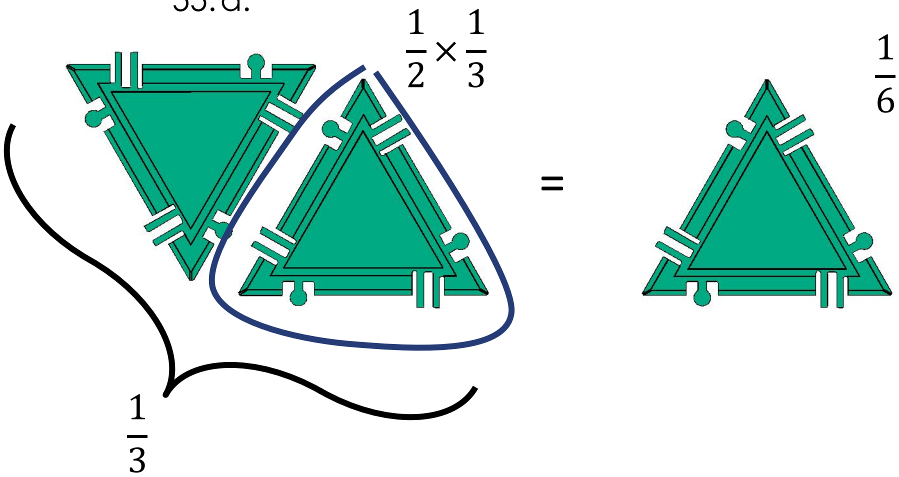


3

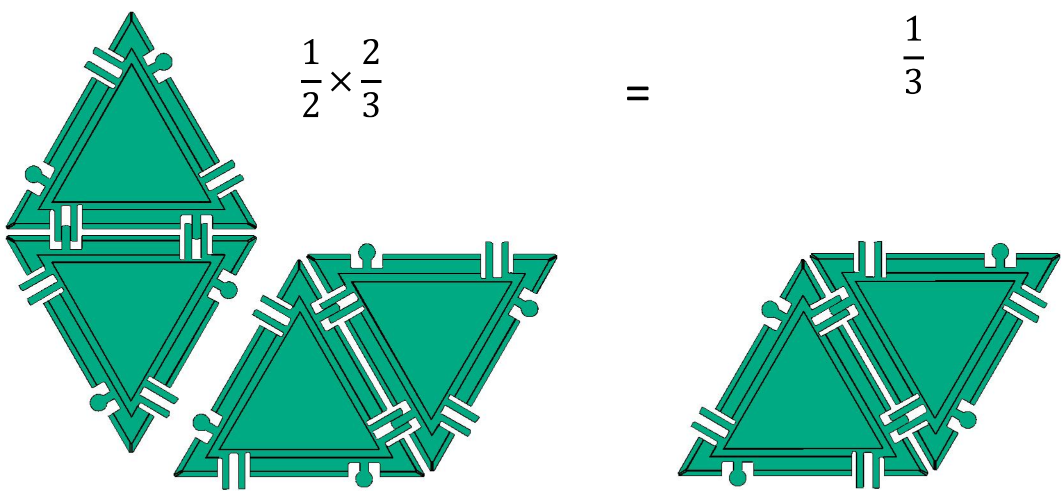
In problem 55 a-b,



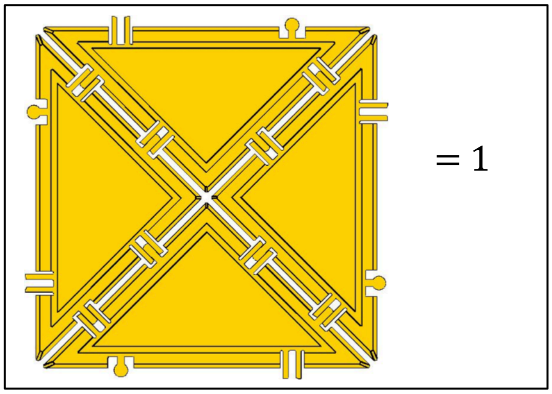
55. a.



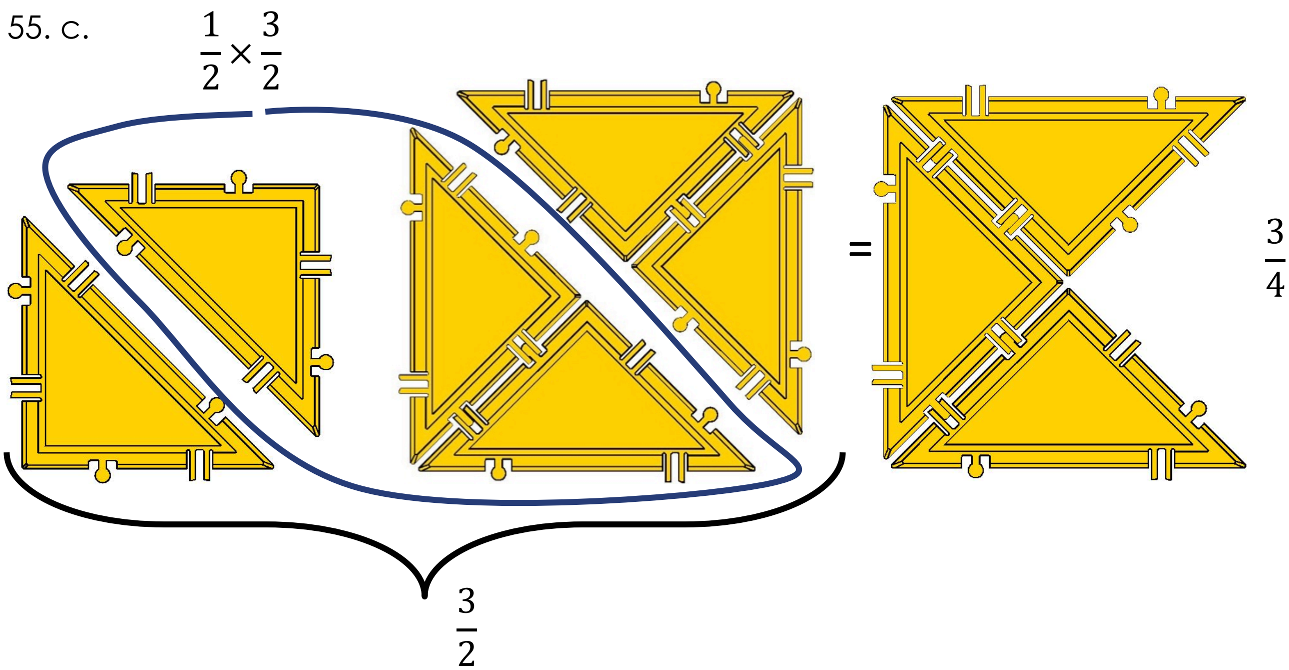
55 b



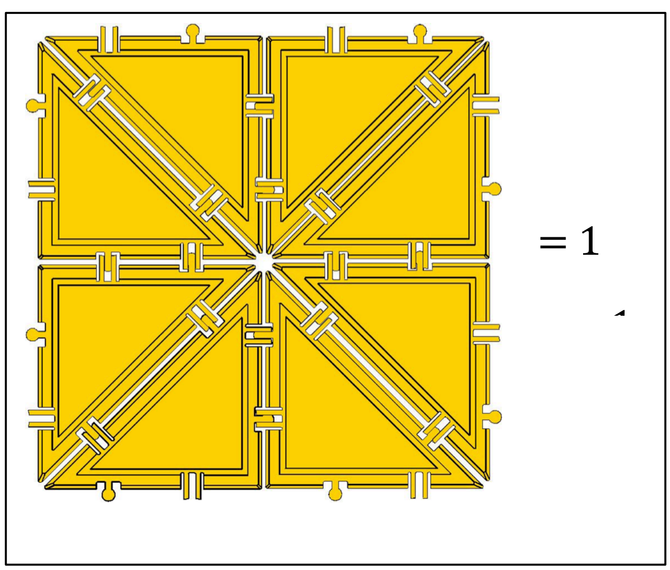
In problem 55c,



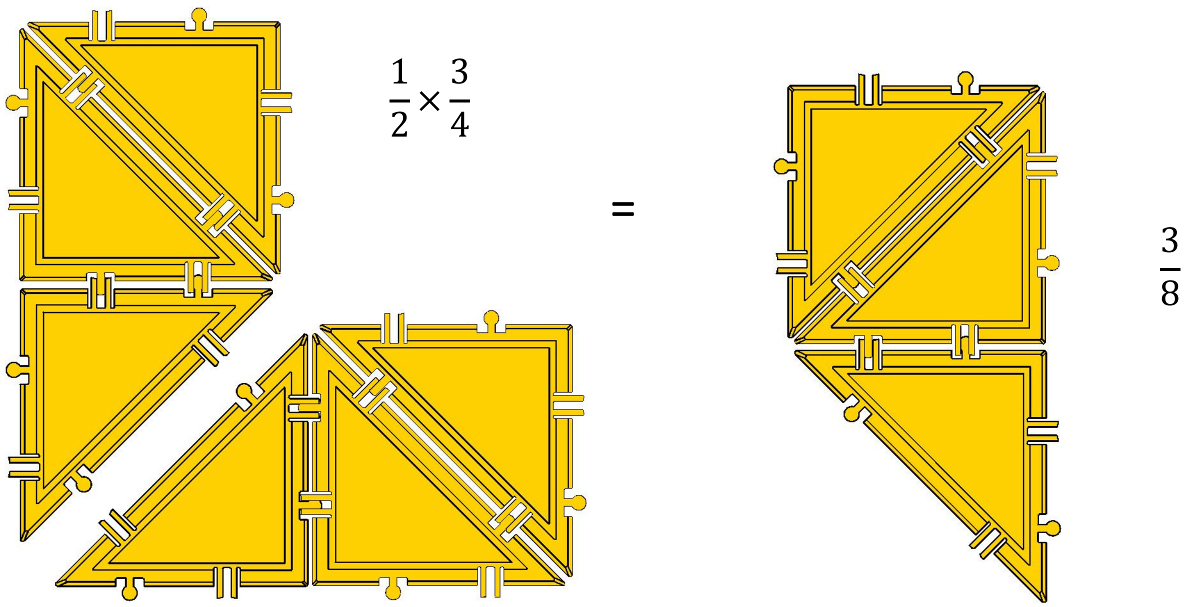
55. c.



In problem 52d,



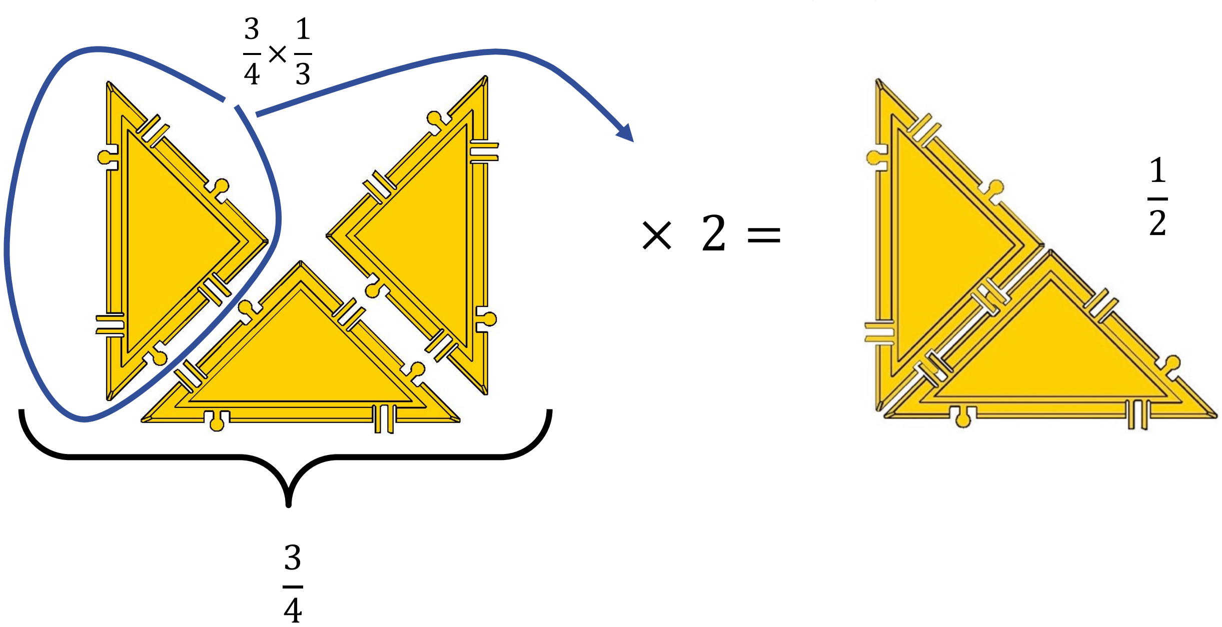
55. d.



In the problems that follow, it will be self-evident what arrangement of tiles represents 1.

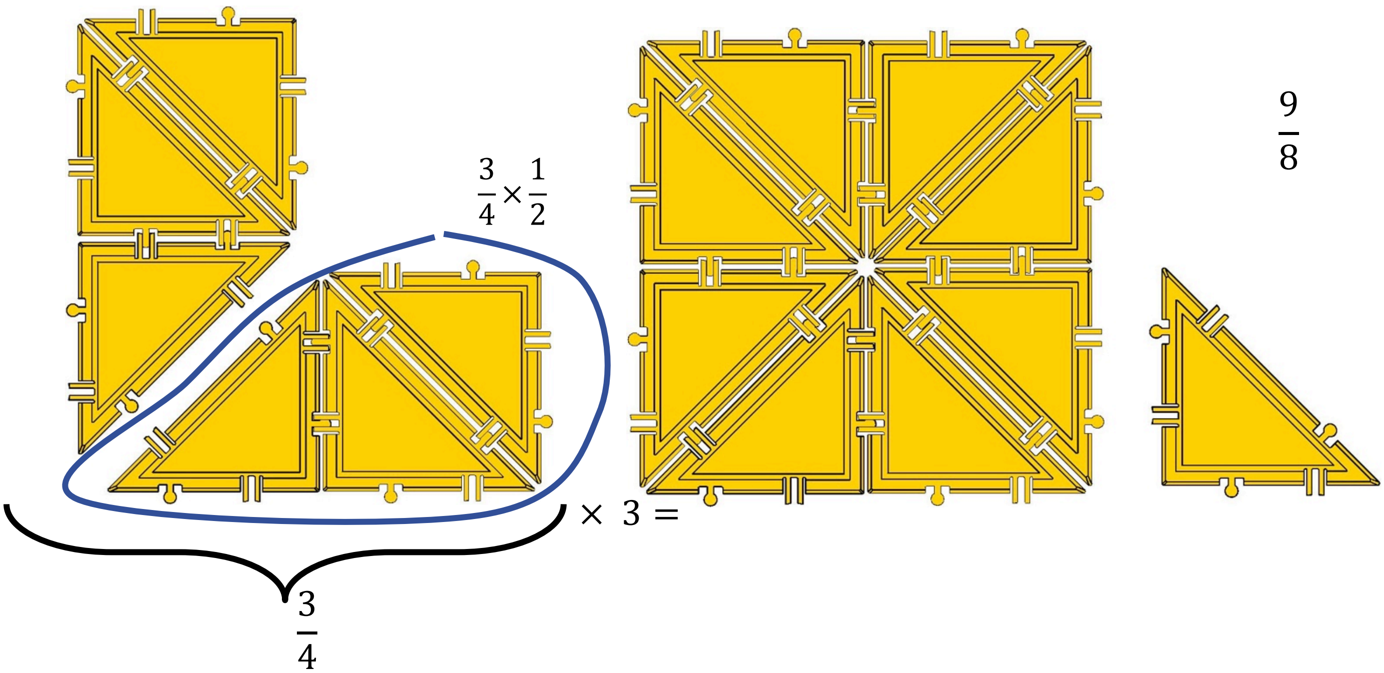
56. a.

We will approach this by thinking of $\frac{3}{4} \times \frac{2}{3}$ as $\left(\frac{3}{4} \times \frac{1}{3}\right) \times 2$.

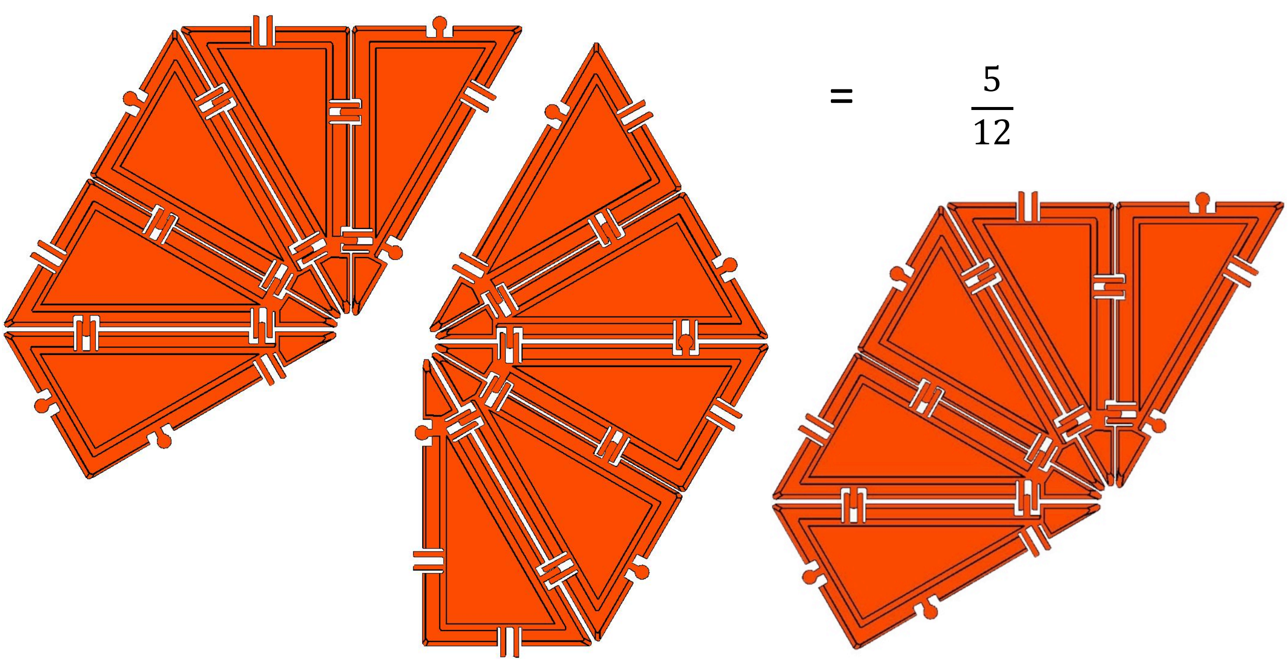


56. b.

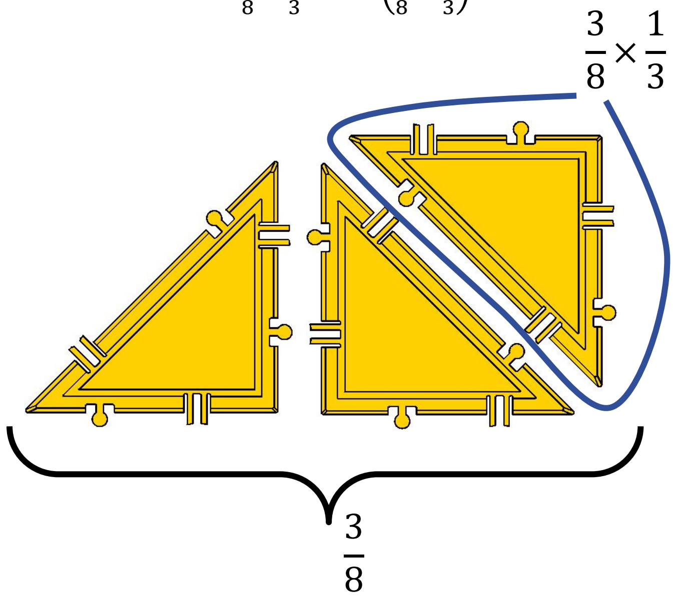
Think of $\frac{3}{4}\times\frac{3}{2}$ as $\left(\frac{3}{4}\times\frac{1}{2}\right)\times 3$.



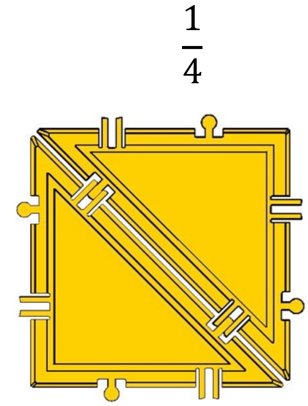
56. c. $\frac{1}{2}\times\frac{5}{6}$



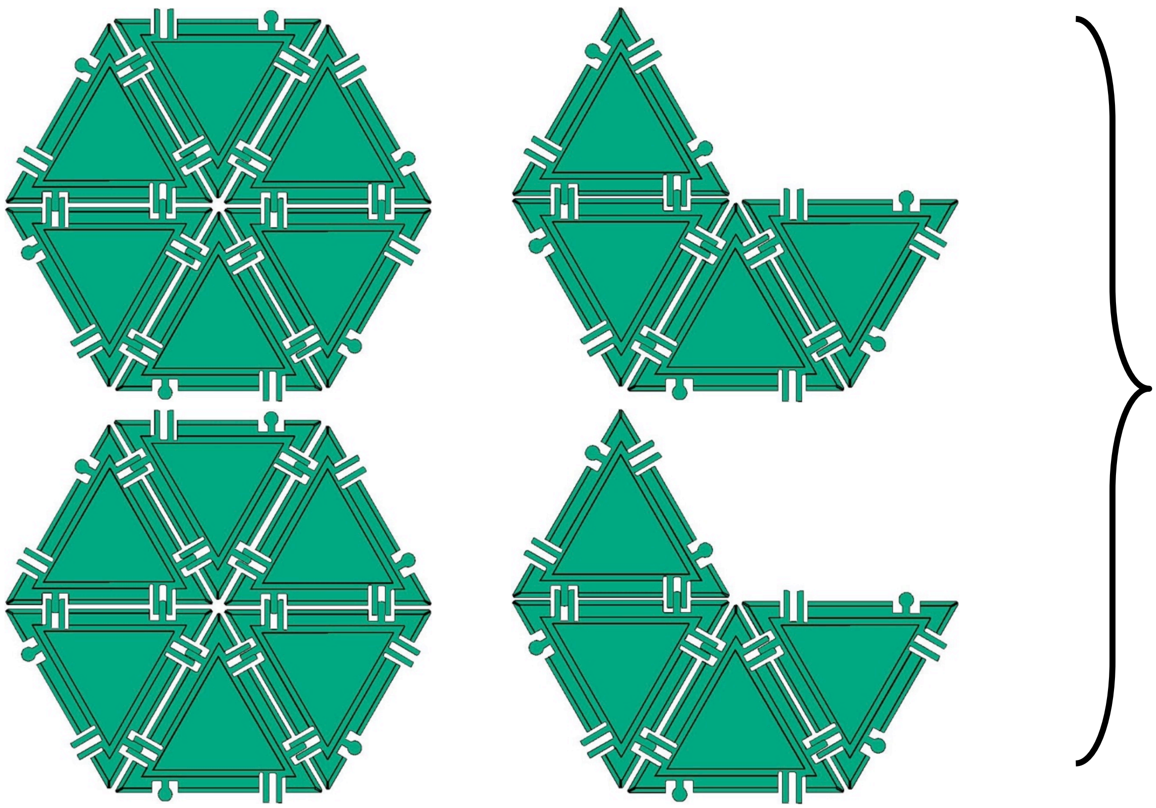
56. d. Think of $\frac{3}{8}\times\frac{2}{3}$ as $\left(\frac{3}{8}\times\frac{1}{3}\right)\times 2$.



$\times\ 2 =$

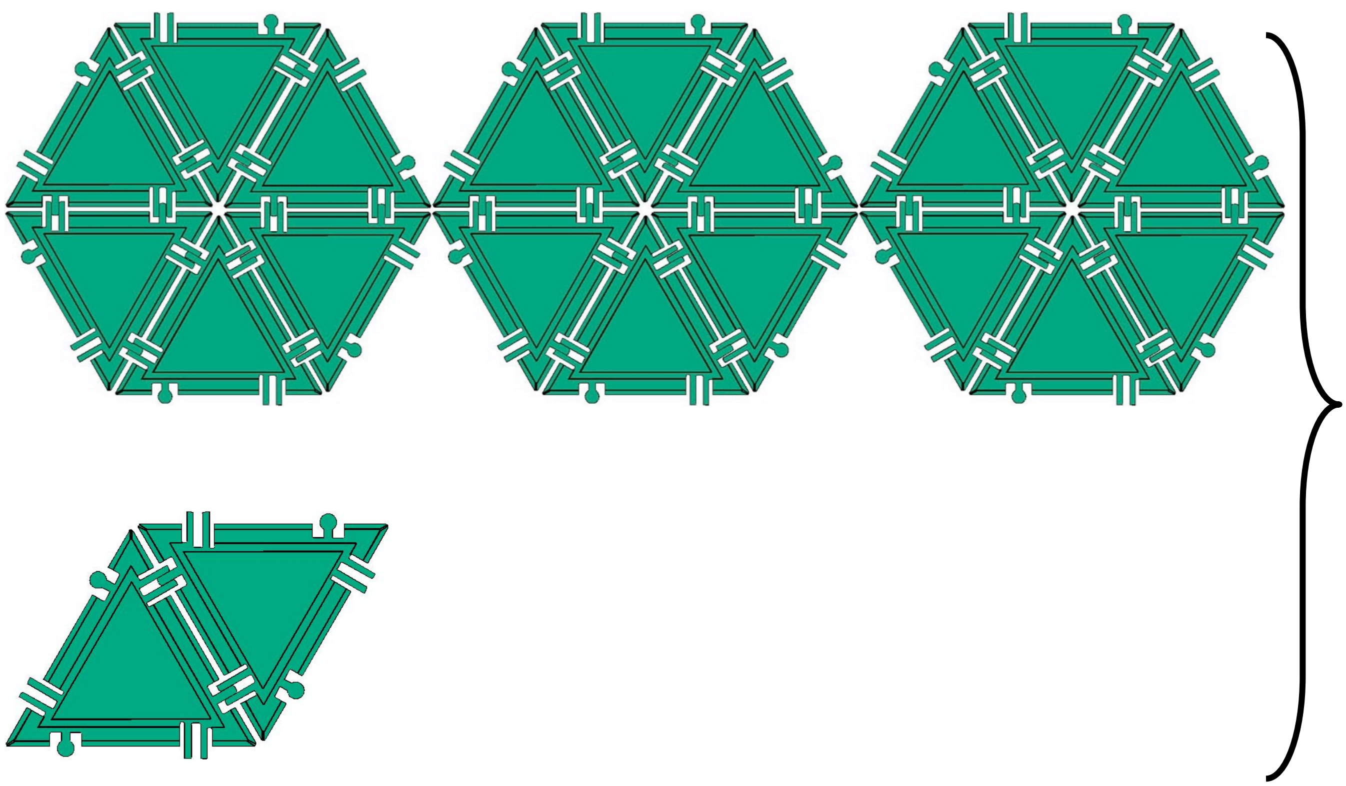


57. a.



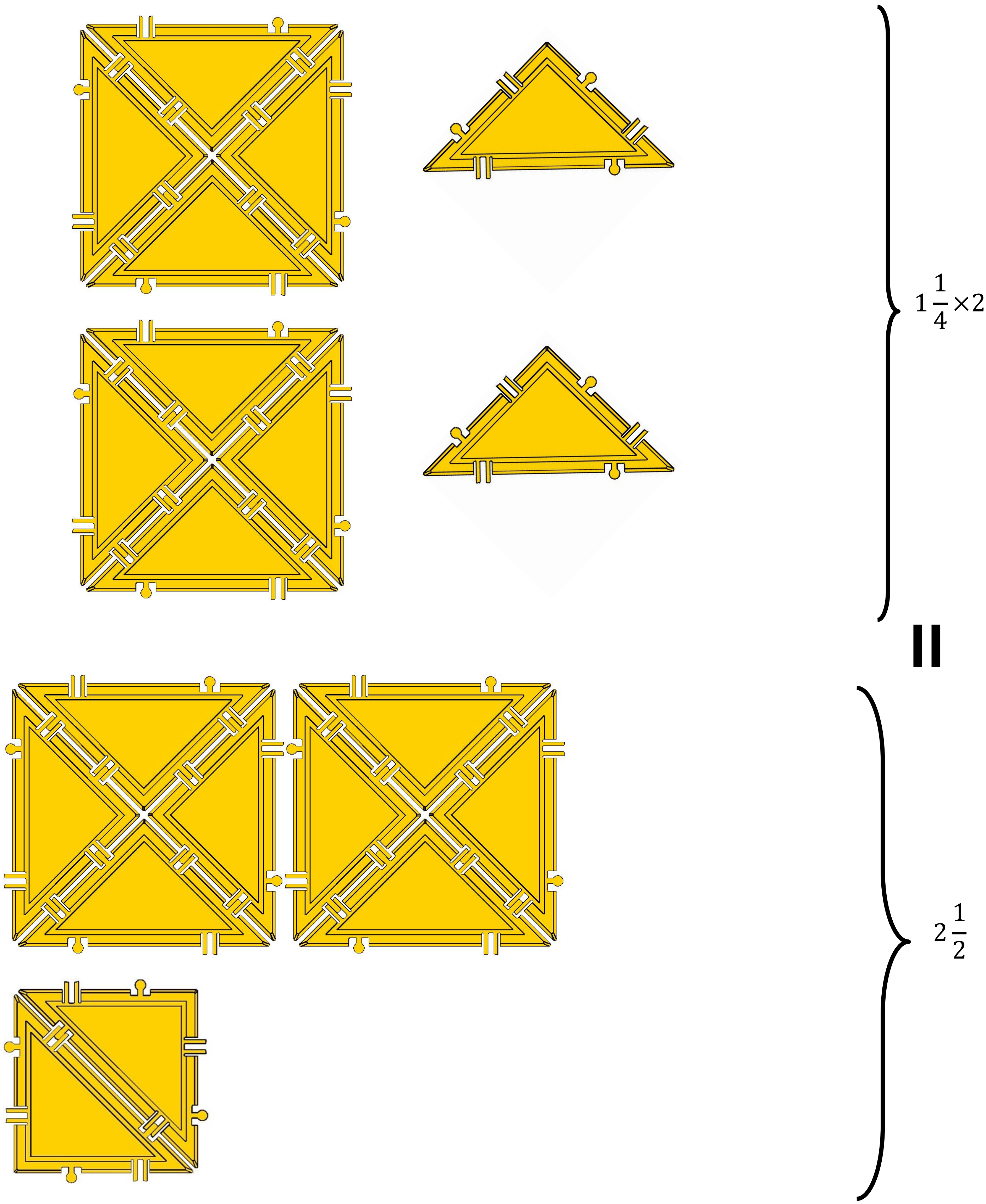
$1\frac{2}{3} \times 2$

=

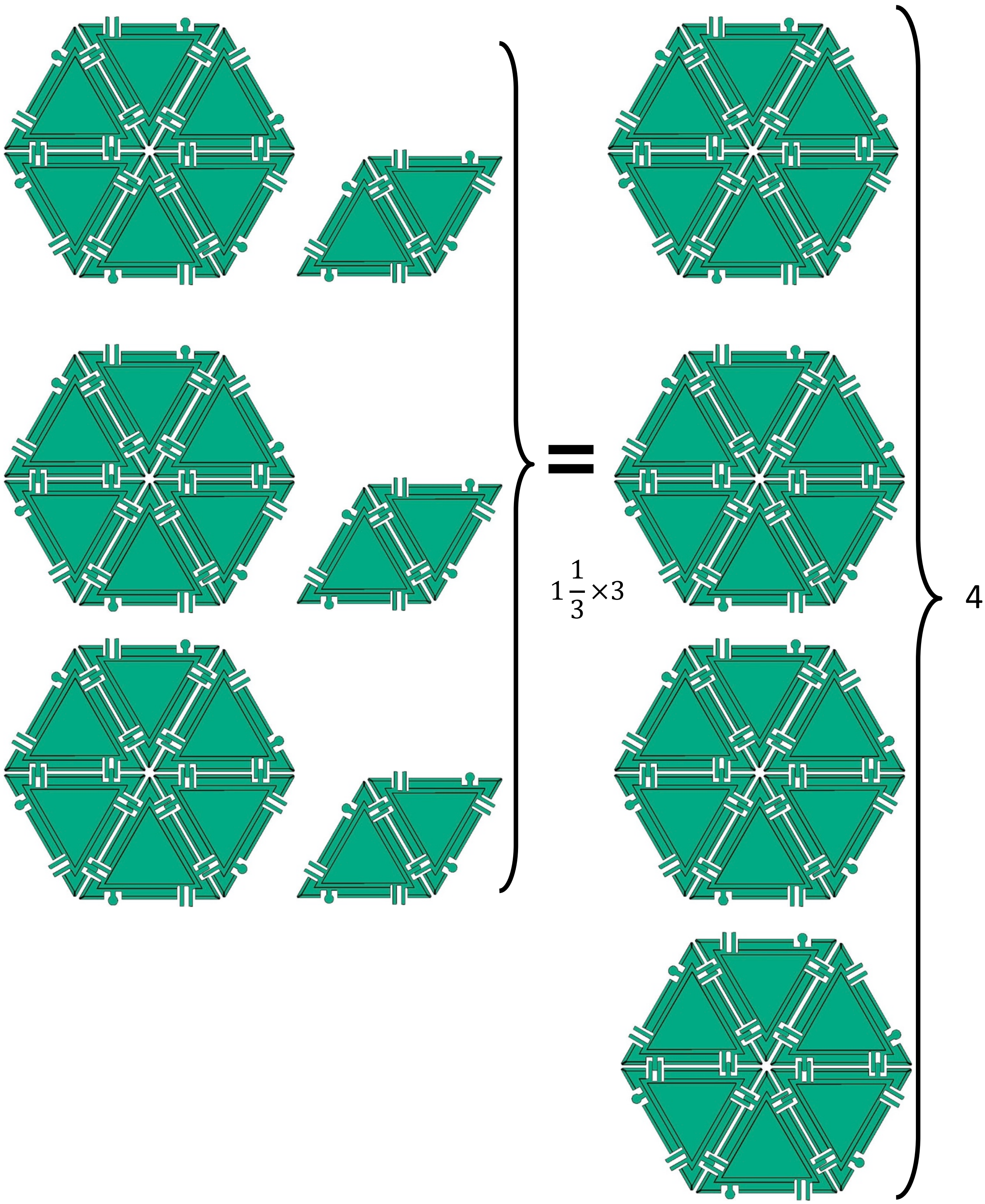


$3\frac{1}{3}$

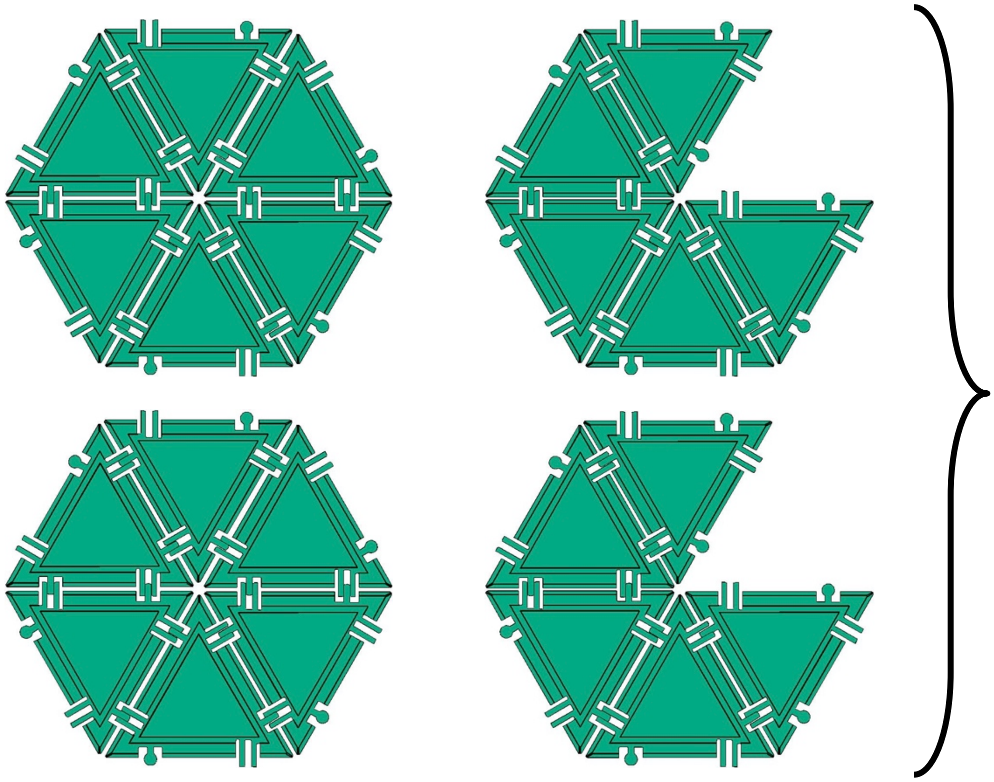
57. b.



57. c.

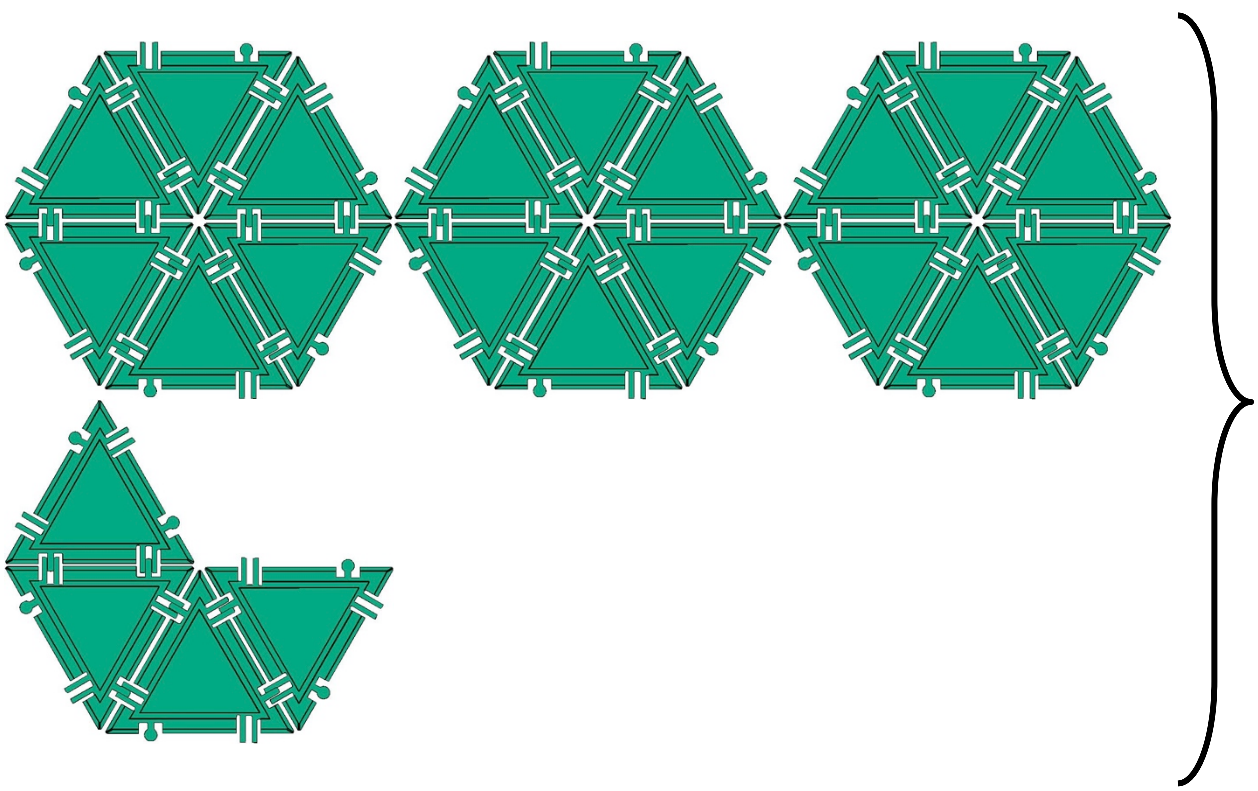


57. d.



$1\frac{5}{6}\times 2$

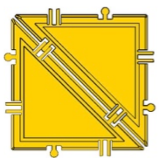
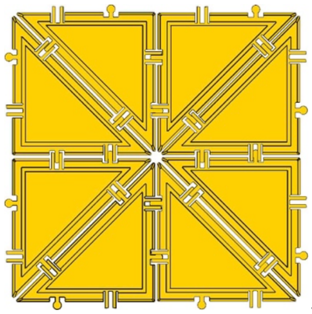
=

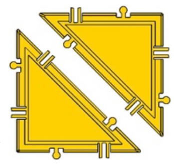


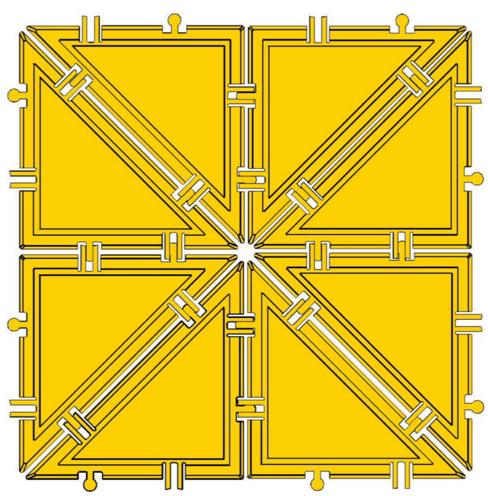
$3\frac{2}{3}$

In problem 58, the following convention will be observed:

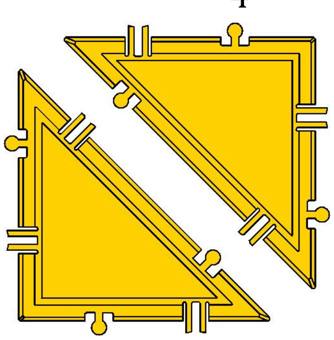
if two tiles are next to each other but not connected, then together they form the divisor. Using Problem 58 as an example,

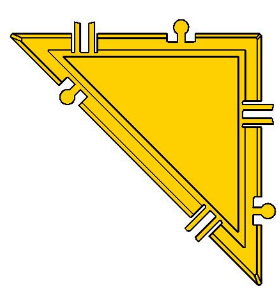
 represents $\frac{1}{4}$ of . Splitting tiles apart represents dividing.

$\frac{1}{4}$ divided by 2 is therefore represented by .

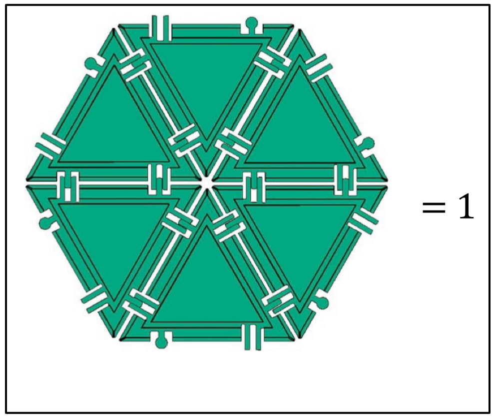
In problem 58 a,  $= 1$

58. a. $\frac{1}{4} \div 2$ $=$ $\frac{1}{8}$



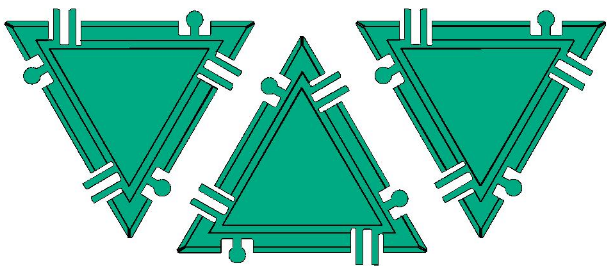


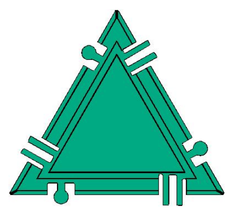
In problems 58 b, c,



$\frac{1}{2} \div 3 \qquad = \qquad \frac{1}{6}$

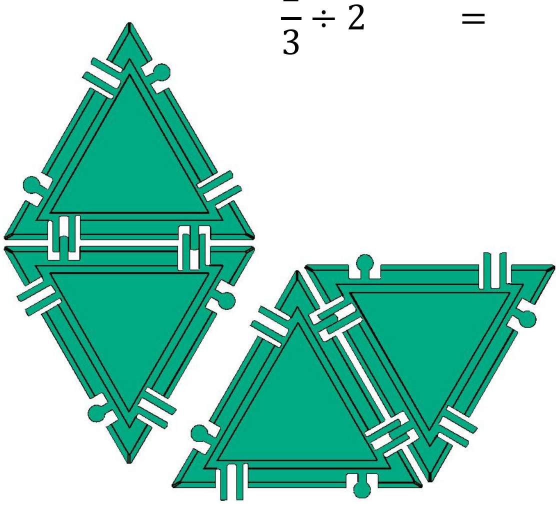
58. b.

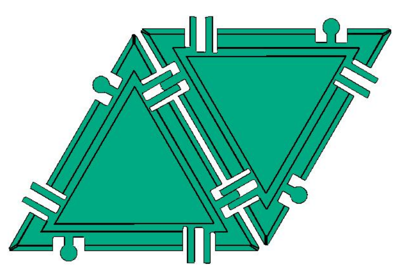




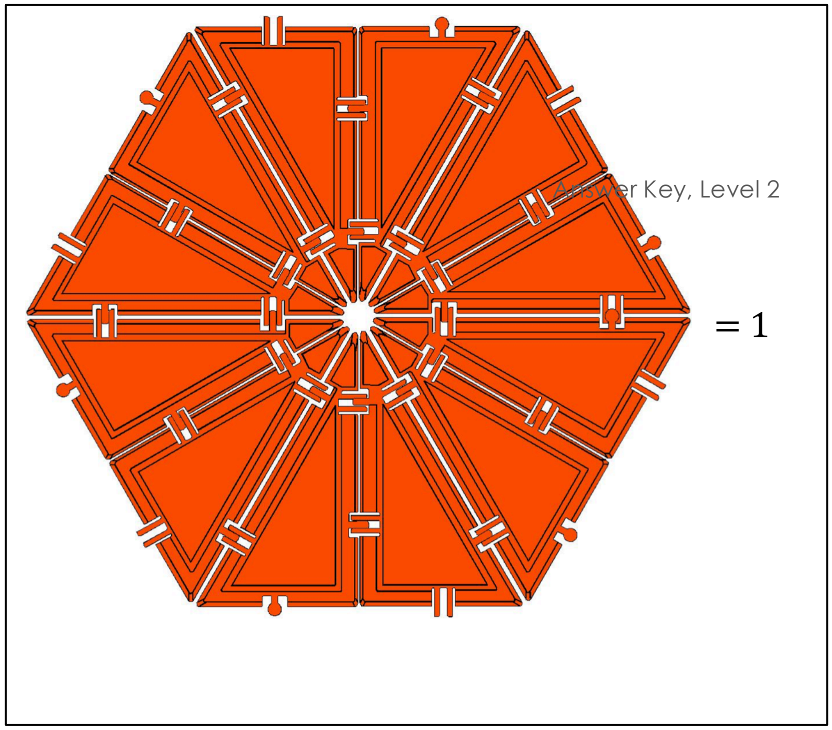
58. c.

$\frac{2}{3} \div 2 \qquad = \qquad \frac{1}{3}$





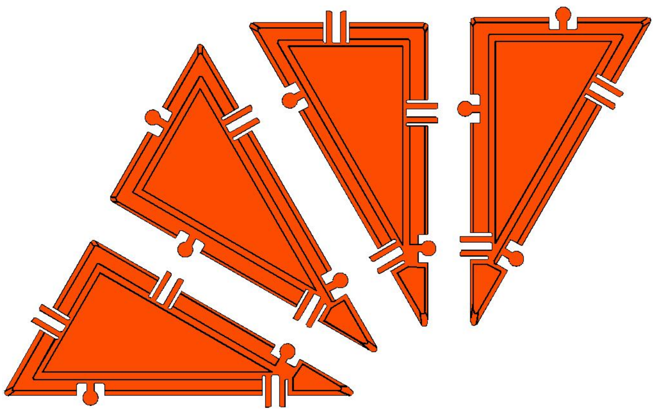
In problem 58. d,

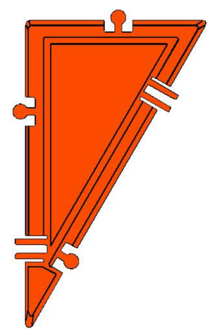


Answer Key, Level 2

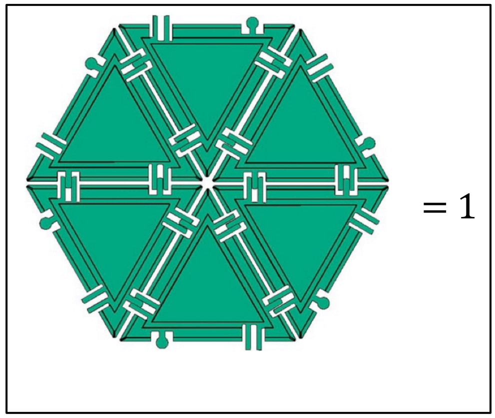
58. d.

$\frac{1}{3} \div 4 \quad = \quad \frac{1}{12}$



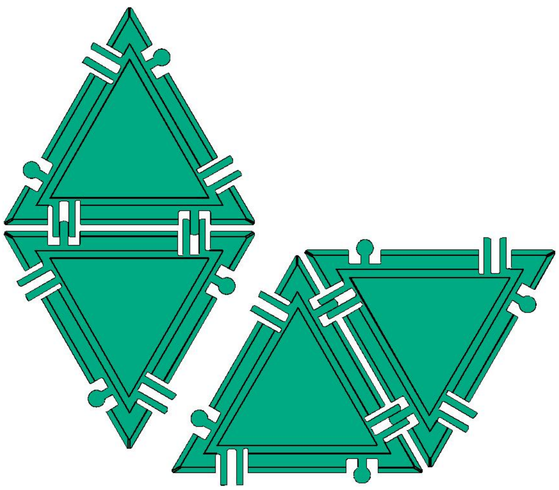


In problems 59 a, b,c,

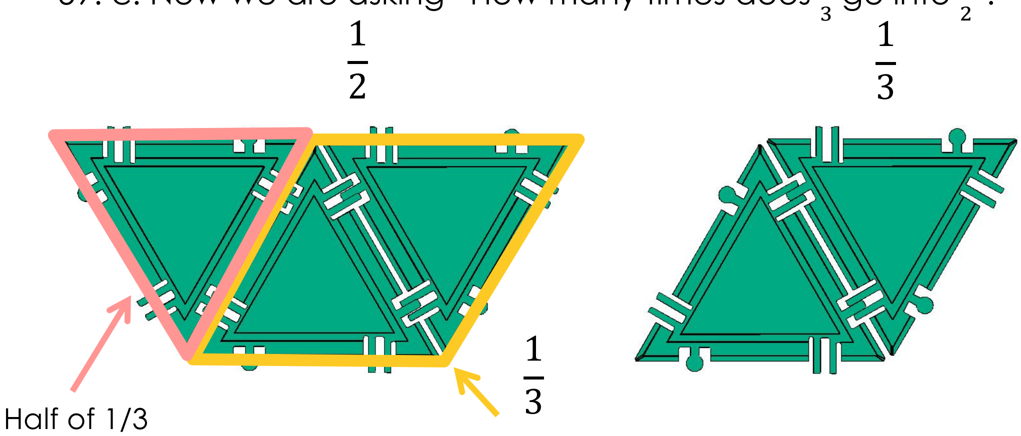


59. a. This problem is the equivalent of asking: "How many times does $\frac{1}{3}$ go into $\frac{2}{3}$?"

The answer is **2**.



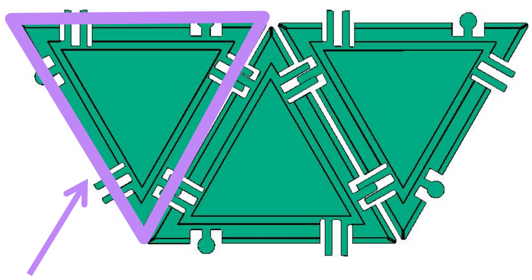
59. c. Now we are asking "How many times does $\frac{1}{3}$ go into $\frac{1}{2}$?"



We see from the above diagrams that a third goes into a half $\mathbf{1\frac{1}{2}}$ times.

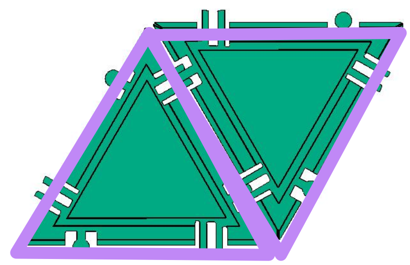
59. c. "How many times does $\frac{1}{2}$ go into $\frac{1}{3}$? We know the answer should be less than 1, since a third is smaller than a half.

$\frac{1}{2}$



1/3 of a half

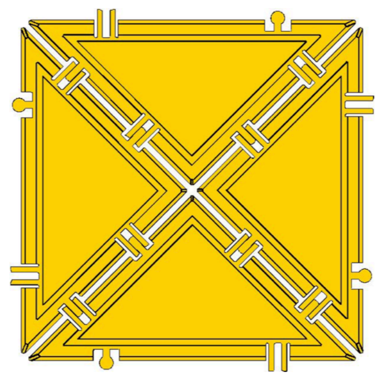
$\frac{1}{3}$



(1/3 of a half) X 2

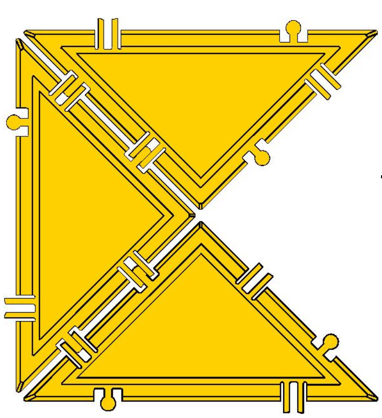
We see from the above diagrams that a **half fits into a third $\frac{2}{3}$ times.**

In problem 59.d,

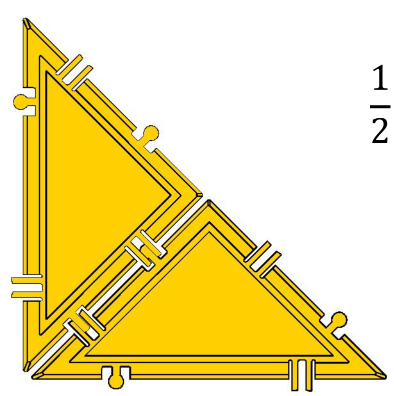


$= 1$

59. d. How many times does a half go into three-quarters?



$\frac{3}{4}$

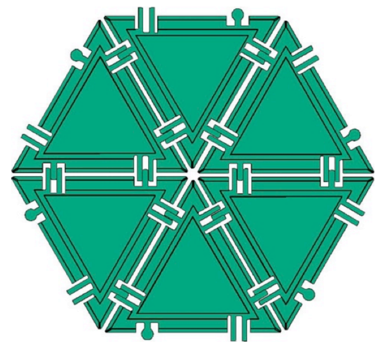


$\frac{1}{2}$

Just as in the previous exercise, a half goes into three-quarters $\mathbf{1\frac{1}{2}}$ times.

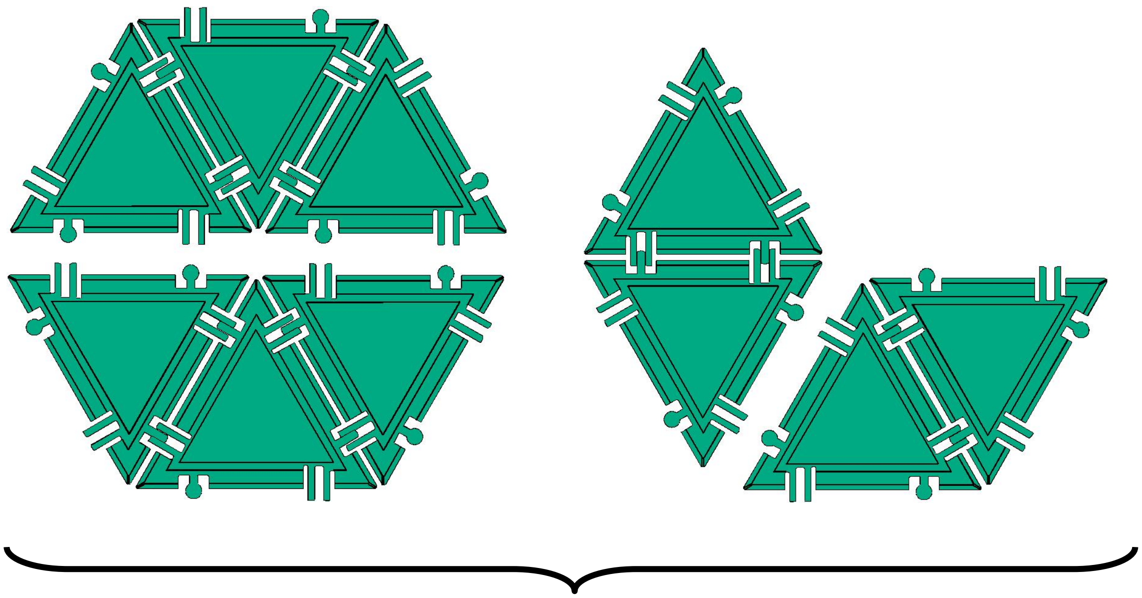
Please look back at the instructions for Problem 58. We will be thinking of multiplying by a unit fraction as dividing by its reciprocal.

For Problem 60. a,



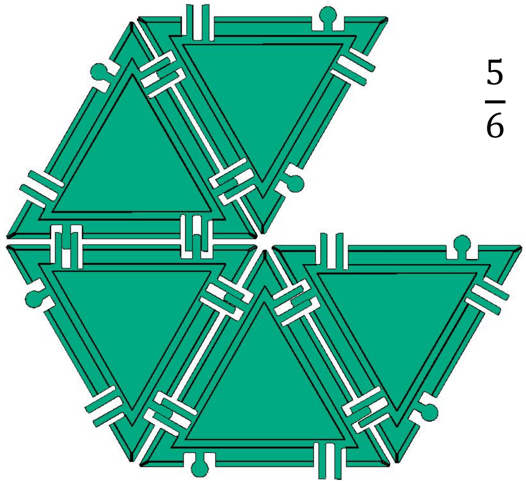
$= 1$

60. a.



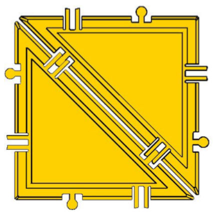
$$1\frac{2}{3}\times\frac{1}{2}$$

$$=$$



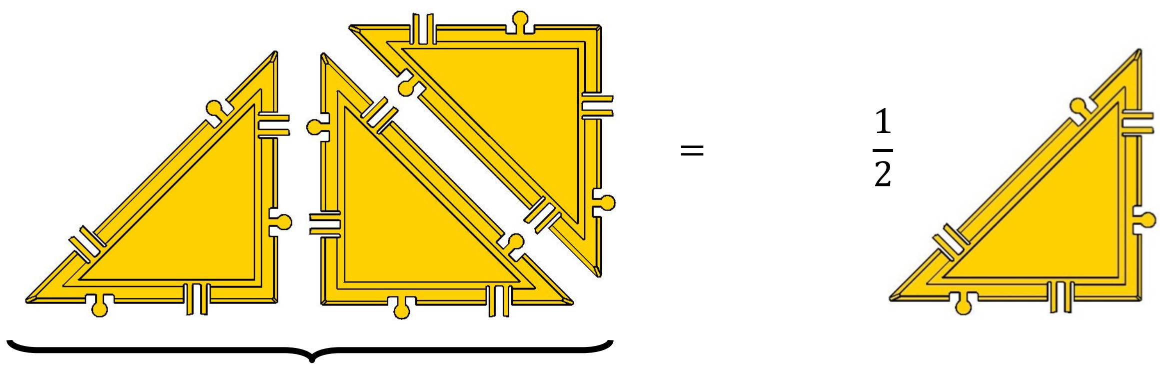
$\frac{5}{6}$

In problem 60. b,

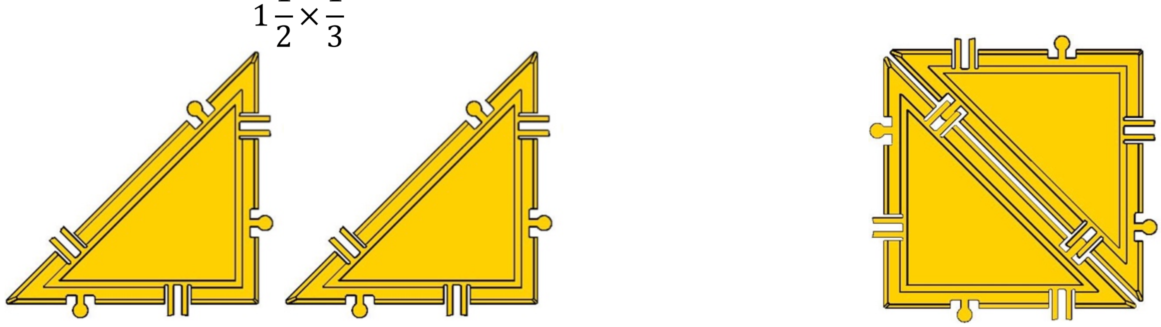


$= 1$

60. b. Think of $1\frac{1}{2}\times\frac{2}{3}$ as $\left(1\frac{1}{2}\times\frac{1}{3}\right)\times 2$.

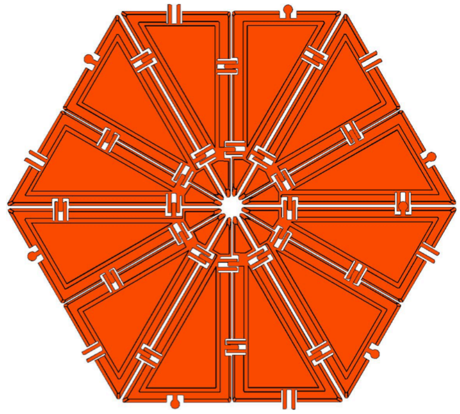


$1\frac{1}{2}\times\frac{1}{3}$ $=$ $\frac{1}{2}$



$\frac{1}{2}\times 2$ $=$ 1

In problem 60. c,

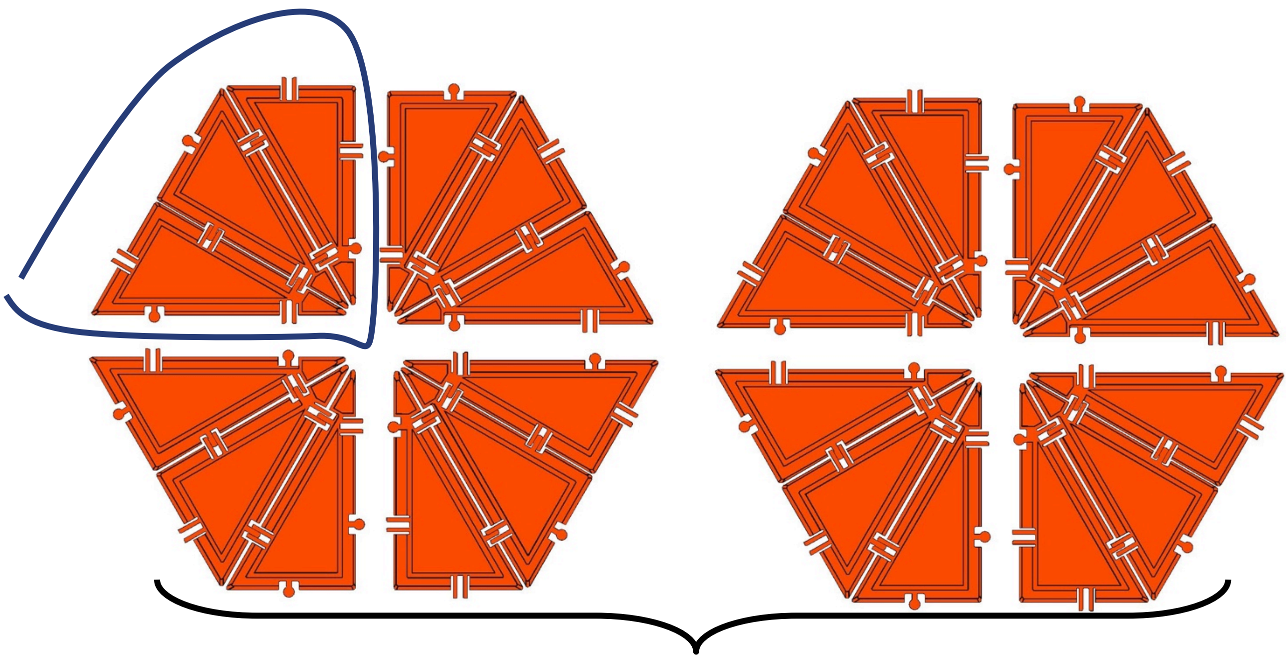


$= 1$

60. c. We will use Distributive Property to solve this problem:

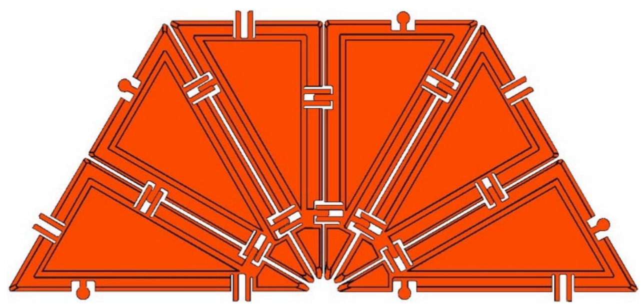
$$2\frac{1}{3}\times\frac{1}{4}=\left(2+\frac{1}{3}\right)\times\frac{1}{4}=2\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{4}$$

$\frac{1}{4}$



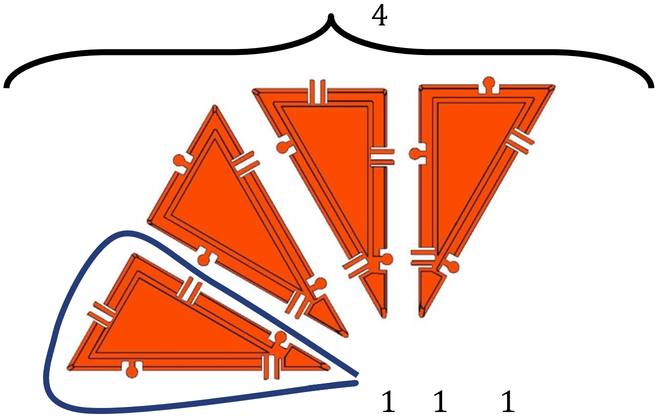
$2\times\frac{1}{4}$

$=$



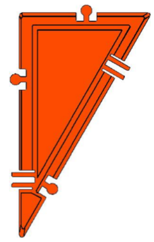
$\frac{1}{2}$

$\frac{1}{4}$

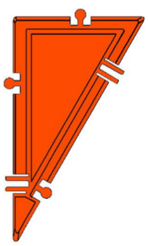


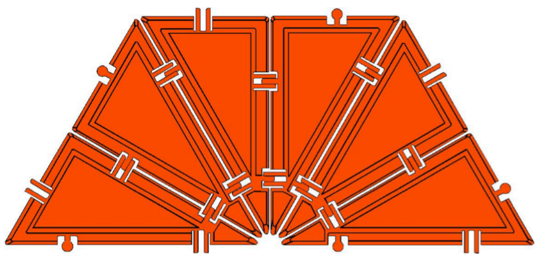
$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

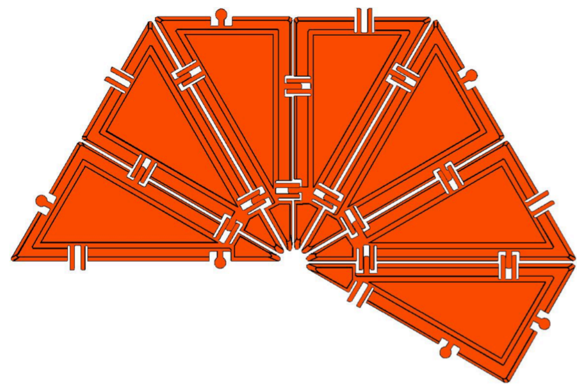
$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



$\frac{1}{12} + \frac{1}{2} = \frac{7}{12}$

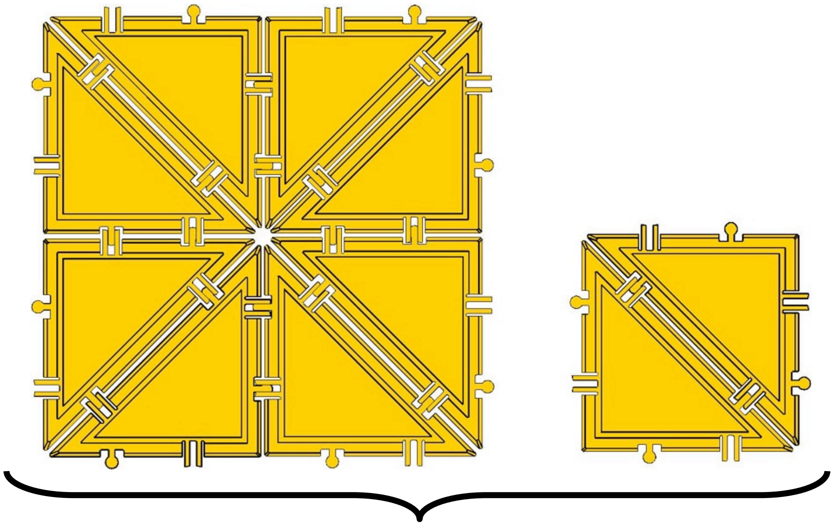






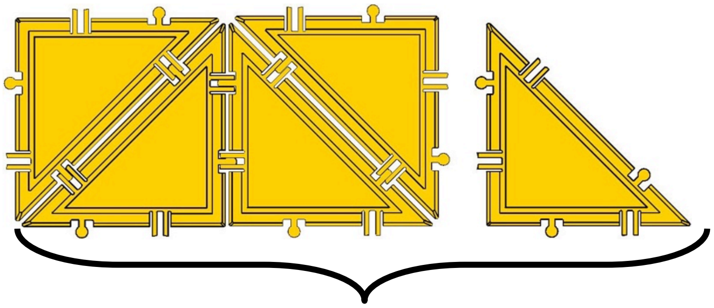
60. d. Once again, we will use the Distributive Property:

$$1\frac{1}{4}\times 1\frac{1}{2} = 1\frac{1}{4}\times\left(1+\frac{1}{2}\right)\times = 1\frac{1}{4} + 1\frac{1}{4}\times\frac{1}{2}$$



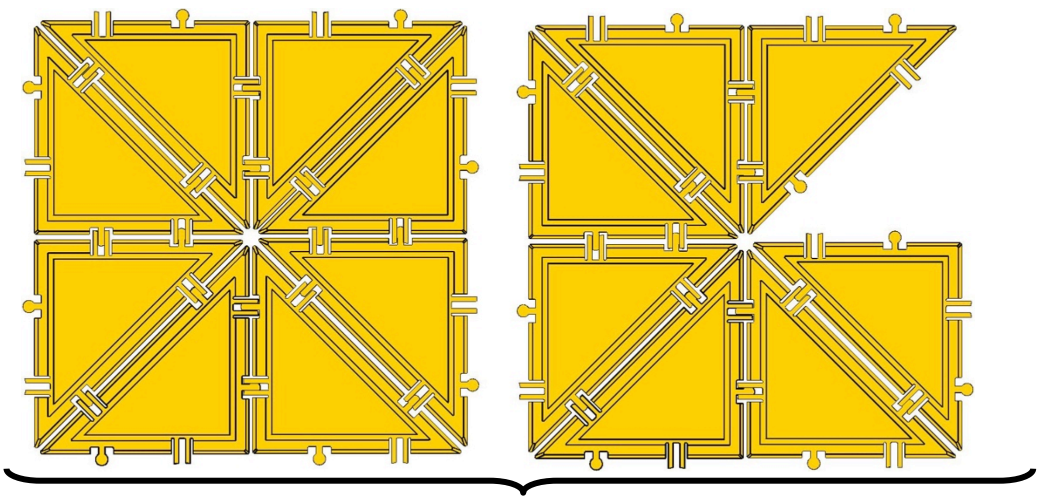
$1\frac{1}{4}$

$+$



$1\frac{1}{4}\times\frac{1}{2}$

=



$1\,{}^{7}/_{8}$