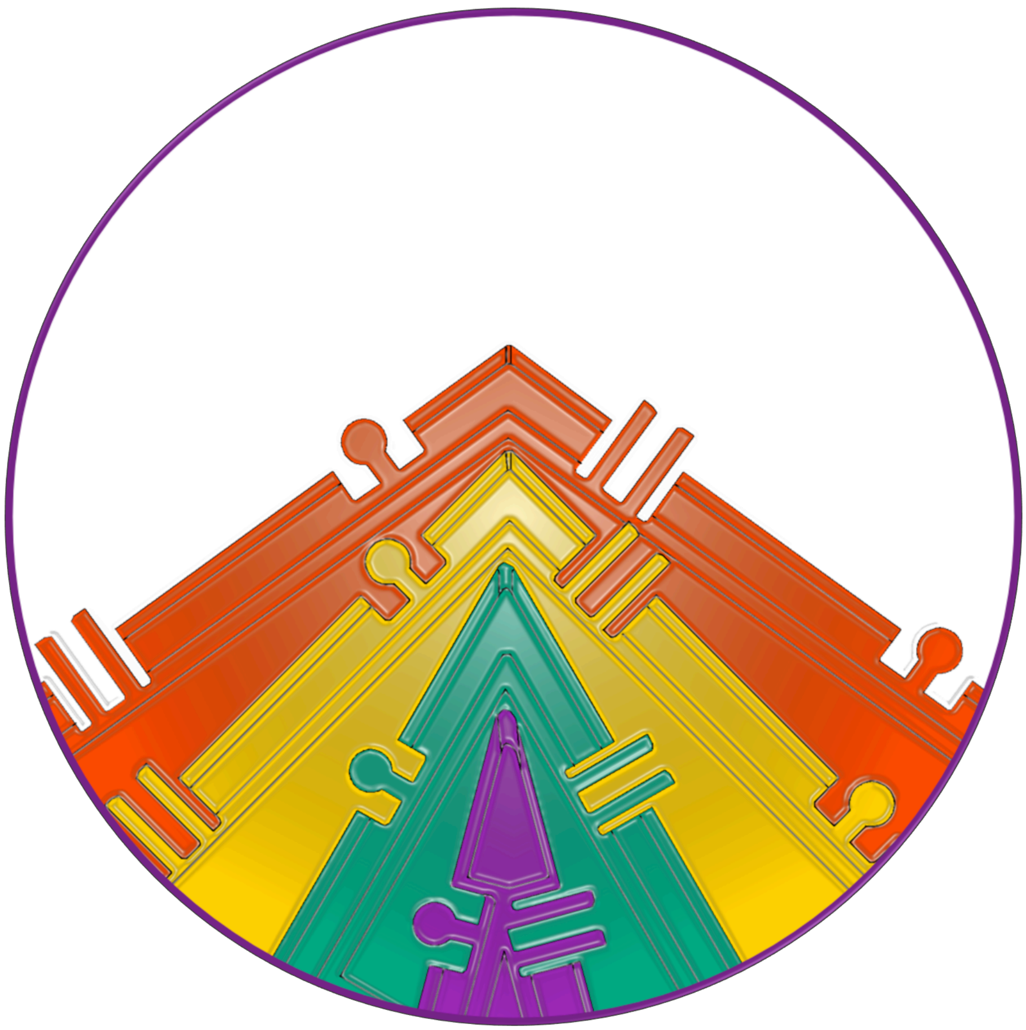
GEOMETILES™

Angle Mania

For grade 4 and up

ANGLE MANIA



Patent Pending

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Introduction

> *Angle measure is a "turning point" in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics.* [1]

This excerpt from the Progressions for the Common Core Math Standards summarizes the motivation behind this workbook. "Angle Mania" helps students connect the study of angles to real life by engaging them in tactile exploration. Students are challenged to construct as well as identify various types of angles. The exercises get students to think "outside the box", thus solidifying their understanding of angle measure.

Activities in this booklet and the Common Core State Standards (CCSS)

CCSS for Mathematical Content supported by activities in this workbook

4.G.A.1 *Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.*

4.G.A.2 *Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.*

4.MD.C.5 *Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.*

4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

CCSS for Mathematical Practice supported by activities in this workbook

MP.1 *Make sense of problems and persevere in solving them.*

These problems challenge students to make sense of problems due to the novel context in which students are asked to reason with angles, and the number of different skills combined in one problem.

[1] http://commoncoretools.me/wp-content/uploads/2012/07/ccss_progression_gm_k5_2012_07_21.pdf, p. 22.

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MP.3 Construct viable arguments and critique the reasoning of others.

These problems lend themselves to students working together. The collaboration inevitably leads to students constructing arguments and evaluating each other's reasoning.

MP.6 Attend to precision

Solving these problems makes it necessary for students to use clear definitions of various polygons in discussion with each other and their teacher.

Learning objectives of this workbook

a. Develop a feel for what a 30°, 45°, 60°, 90°, 120° angle looks like.
b. Reinforce the idea that angle measure is additive through tactile experience with tiles.
c. Develop a solid understanding of angles and angle measure from solving problems in a variety of contexts.
d. Identify different types of angles (acute, right, obtuse) in geometric figures, and learn to classify triangles based on the presence of these angles.
e. Develop tenacity in the face of frustration, as stated in the CCSS MP1: "Make sense of problems and persevere in solving them". In particular, develop resourcefulness in finding non-obvious ways to solve a problem.
f. Learn how to collaborate with one another.

Arguably, point d. is the most important. In light of this, it is ideal if the students can work on each problem for as long as time allows; as long as they are not too frustrated to go on and are trying new ideas, they are spending their time productively.

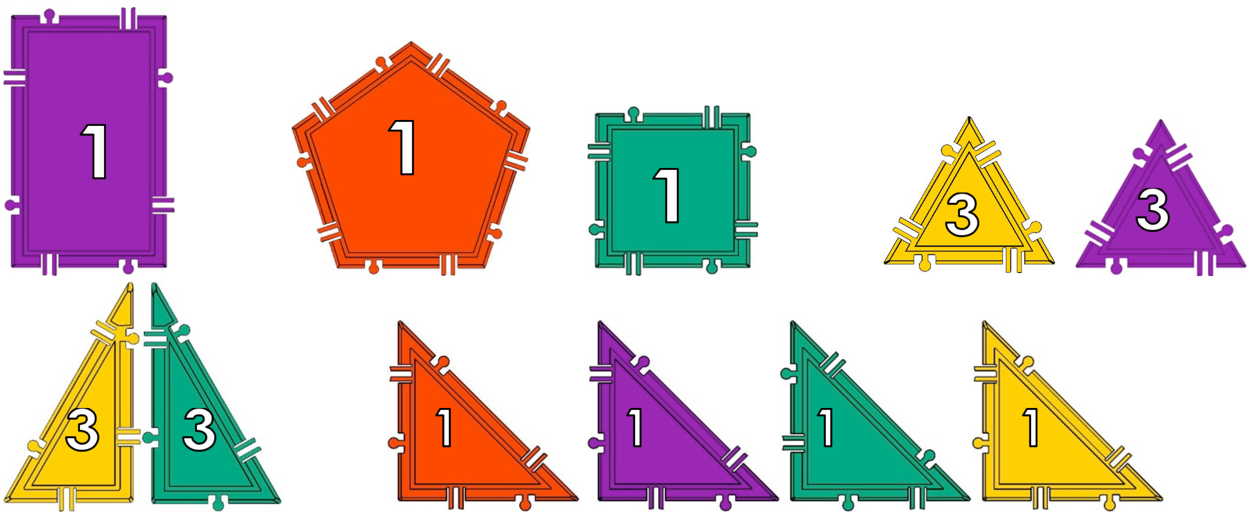
If students are frustrated to the point of disengagement, you can modify the problem so they don't need to construct, say, every single parallelogram possible. They will still get a meaningful learning experience from doing the partial solutions.

Hints are provided as necessary to help the students who are having trouble gaining momentum with solving a problem.

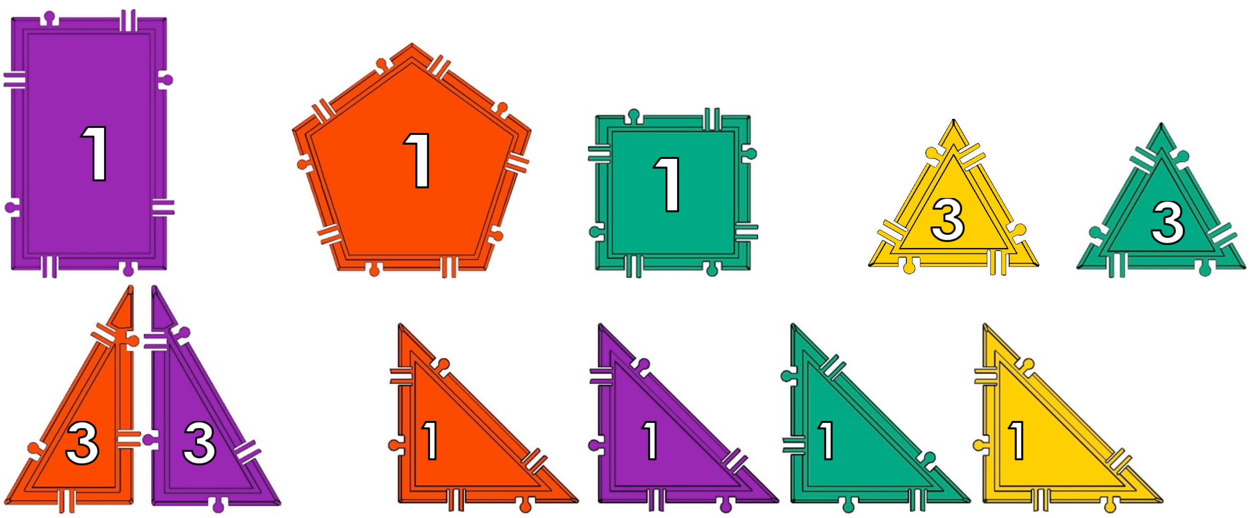
Recommended classroom plan

The exercises are designed to have 3 groups of students work simultaneously with a set of 96 tiles. The set of 96 tiles will be split up into three sets: Set A, Set B and Set C. The three sets are identical to one another, except for the colors of some of the triangles. Each of the three groups can be made of 2-3 students.

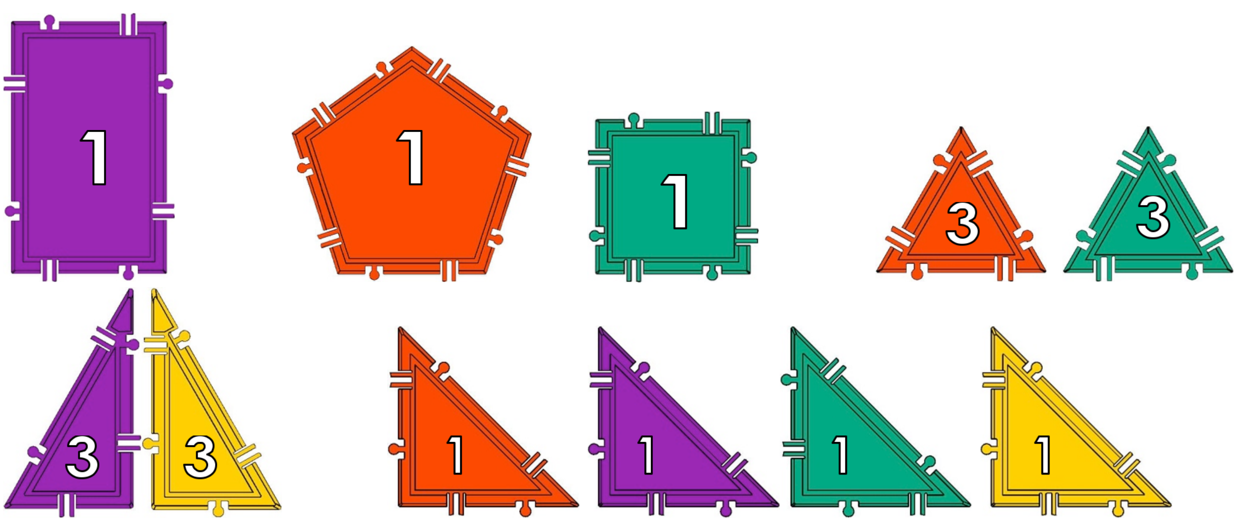
SET A



SET B

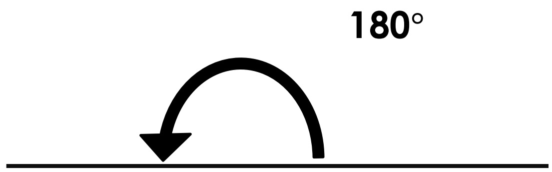


SET C



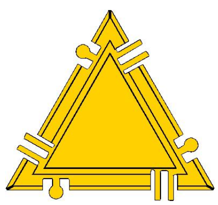
Problems

For questions 1-3, remember that the angles on one side of a straight line add up to 180 degrees.



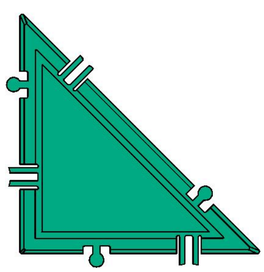
Use only this information (no protractor!) to answer questions 1-3.

1. a. How do you know for sure that the angles in this triangle are equal to each other? Use two identical triangles to compare them.



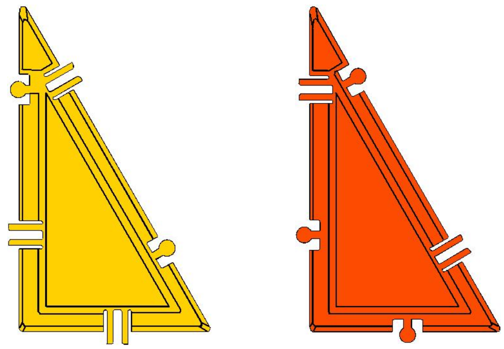
b. Find the measure of each angle in this triangle.

2. a. How do you know for sure that 2 of the angles in this triangle are equal to each other? Use two identical triangles to compare them.



b. Find the measure of each angle in this triangle.

3. a. How do you know that these two triangles have equal angles?



   b. Find the measure of each angle in this triangle.

4. Make an **obtuse triangle** using 2 tiles.

5. Make a **right triangle** using 2 tiles.

6. Make a **right triangle** using 3 tiles.

7. Make a **pentagon** that has right angles. What is the measure of the other angles of your pentagon?

8. Make a **hexagon** that does NOT have all equal angles. What is the measure of each of the angles of your hexagon?

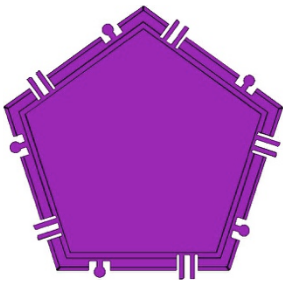
9. What is the measure of each angle in a **regular hexagon** (hexagon with all equal sides and equal angles)?

10. Use 2 tiles to make a **quadrilateral** (not a parallelogram or trapezoid). What is the measure of every angle in your quadrilateral? What is the sum of the angles in every quadrilateral? Check the sum with a protractor.

11. Make a **trapezoid** with two right angles. What is the measure of the other angles in your trapezoid?

12. Make a **trapezoid** with line symmetry. What is the measure of the angles in your trapezoid?

13. Which is the most accurate estimate for the measure of each angle of a regular pentagon?



A. $70° - 90°$ B. $100° - 120°$ C. $120° - 140°$ D. $140° - 160°$

Hint: you can see if your estimate is reasonable by comparing the angle of a pentagon to an angle whose measurement you know.

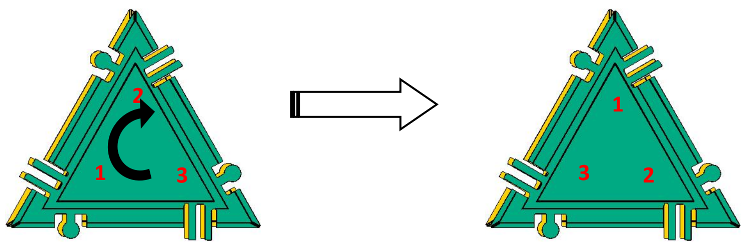
14. Make a **parallelogram** that contains two 45° angles. What is the measure of its other angles? What is the sum of all of its angles?

15. Make a **parallelogram** that contains two 30° angles. What is the measure of its other angles? What is the sum of all of its angles?

Answers

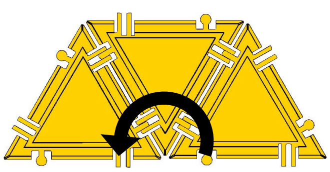
Please note that the colors of the tiles in the answers will be different from the colors in your set.

1. a. To verify that this triangle has equal angles, take one triangle, put an identical one on top of it, and rotate the top one:



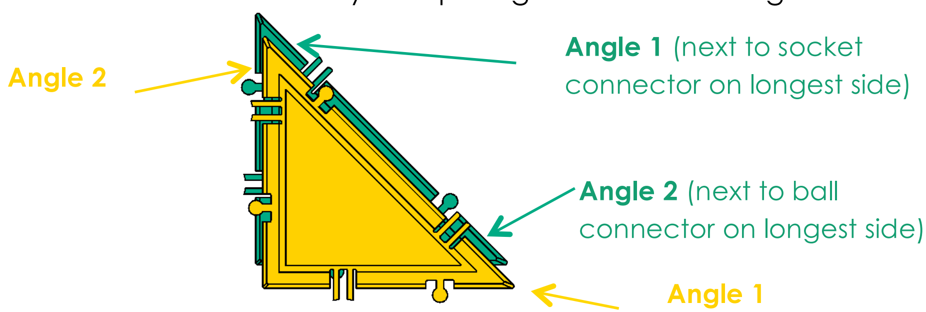
You will see that the angles still line up after rotation.

b. All we need to do now is find out the measure of **one** of the angles of the triangle. We do this by arranging the triangle around a corner like this:



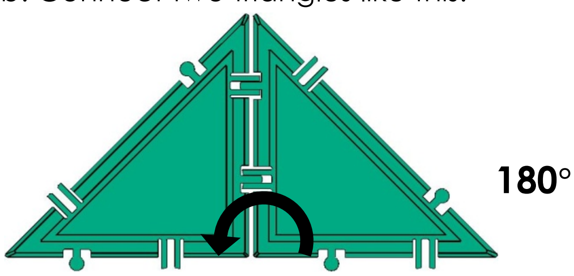
Since $180° \div 3 = 60°$, the measure of the angle is $60°$.

2. a. The angles certainly look the same, but how can we prove they are the same? If this question is too abstract for your students, it is fine to skip it. Take one triangle and put an identical one on top of it. You can keep track of the two different acute angles by noting the type of connector next to the angle on the long side. This way we can be sure we are really comparing two different angles.



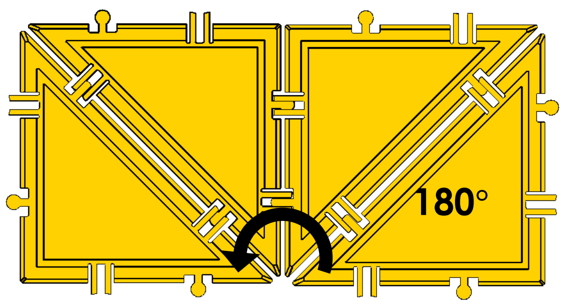
Clearly, the angles are the same.

b. Connect two triangles like this:



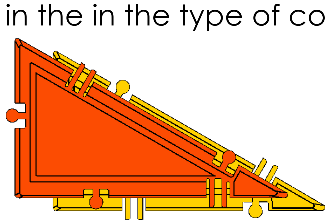
So the measure of each angle is $180° \div 2 = 90°$, .

To find out the measure of each of the equal angles, arrange 4 triangles like this:

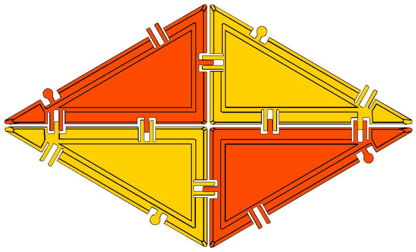


We see from the above diagram that the measure of each angle is $180° \div 4 = 45°$.

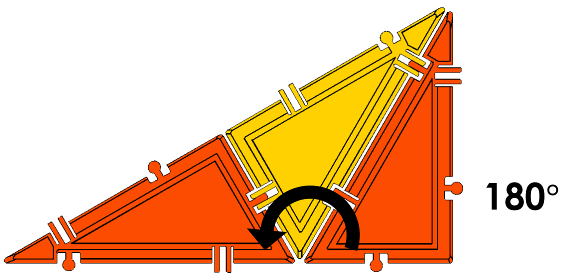
3. a. The triangles are the same when you put one on top of the other. They only differ in the in the type of connectors.



b. Proceeding In the same vein as before, we can prove that the largest angle of the triangles measures $90°$ because 4 of them meet at a corner:

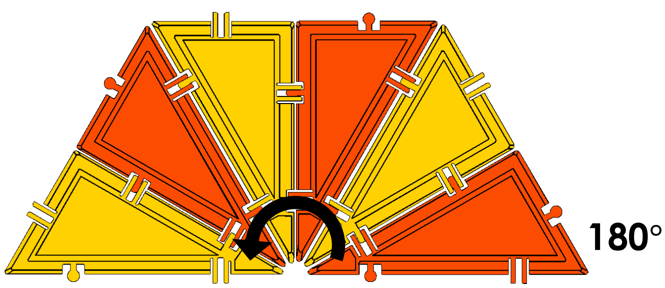


How do we find the measures of the other angles of these triangles? Use the same idea; arrange the triangles so that all the corners with a given angle measure are next to each other:



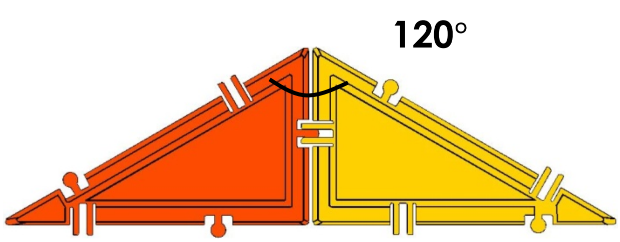
$180° \div 6 = 60°$, so the measure of the angle is $60°$.

Using the same idea, we find that 6 of the triangles fit as shown below:

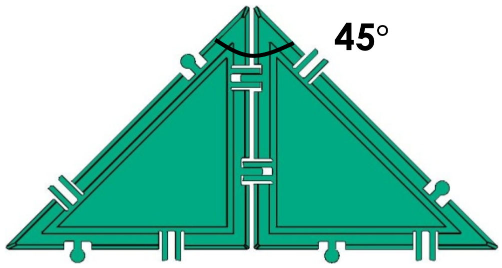


Therefore, the smaller angle of these triangles measures $180° \div 6 = 30°$.

4. We know the angle at the top vertex is obtuse because each of the angles meeting at that vertex measures $60°$, and $60° + 60° = 120°$.



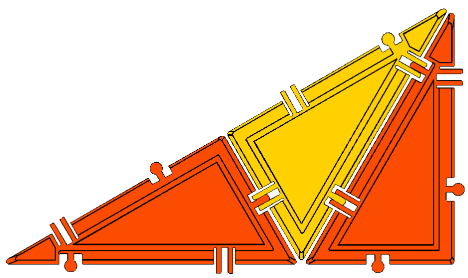
5. We know the angle at the top vertex is a right angle because we know from the previous problem that each of the triangles meeting at that vertex measures $45°$, and $45° + 45° = 90°$.



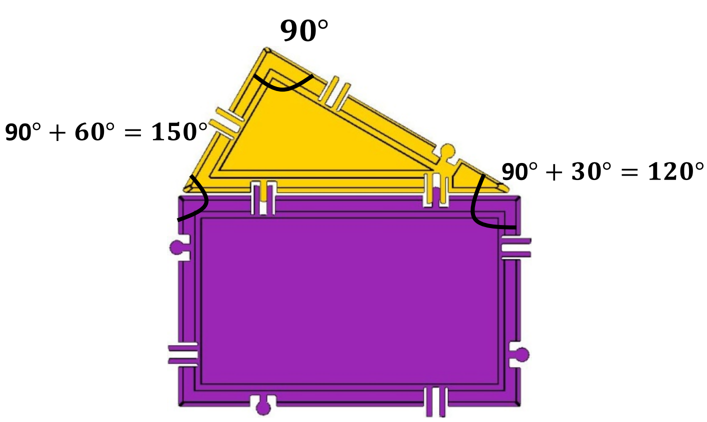
6. This is a challenging problem. You can give students the following incremental hints:

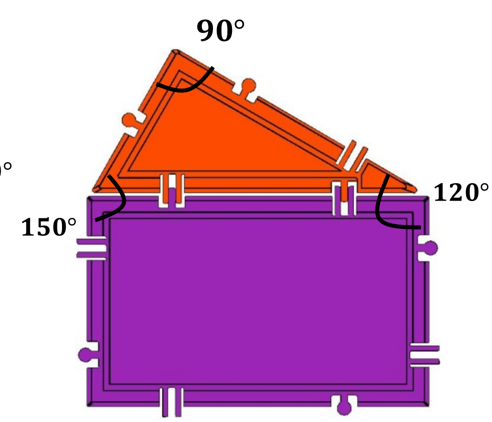
   Hint 1: You will need 3 triangles of the same shape for this. So now they will know that it is either the 3 equilateral, the 3 isosceles, or the 3 scalene triangles.

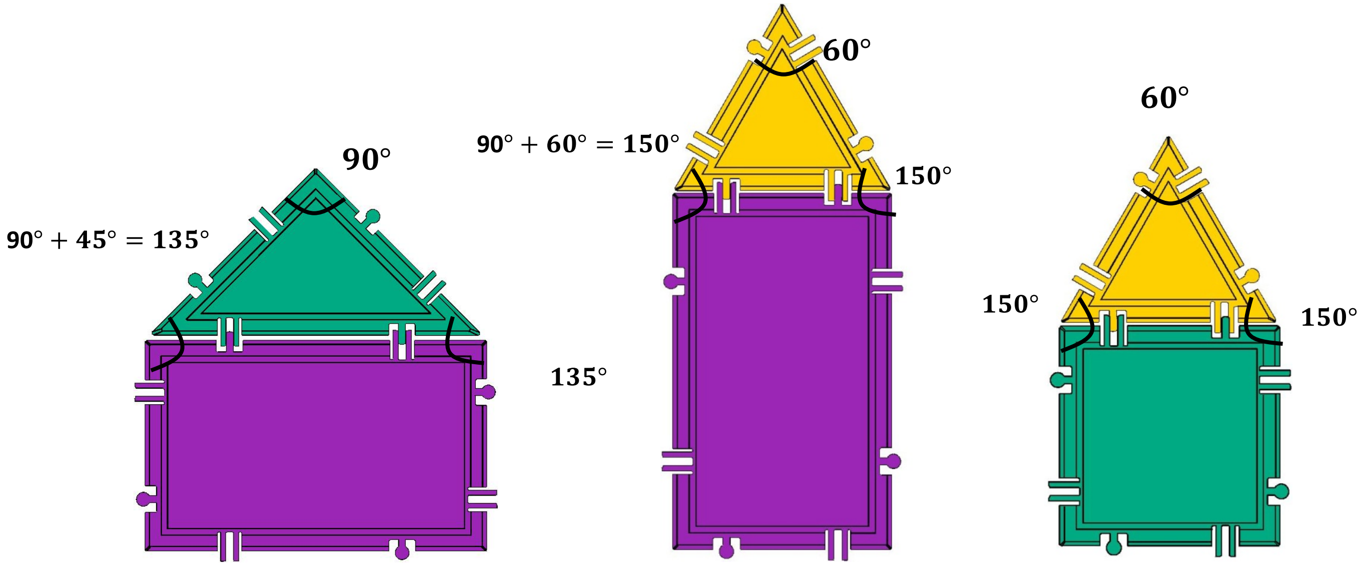
   Hint 2: Is it possible to make <u>any</u> triangle out of the 3 equilateral triangles or the 3 isosceles triangles? No. Therefore, we have got to use 3 scalene triangles.



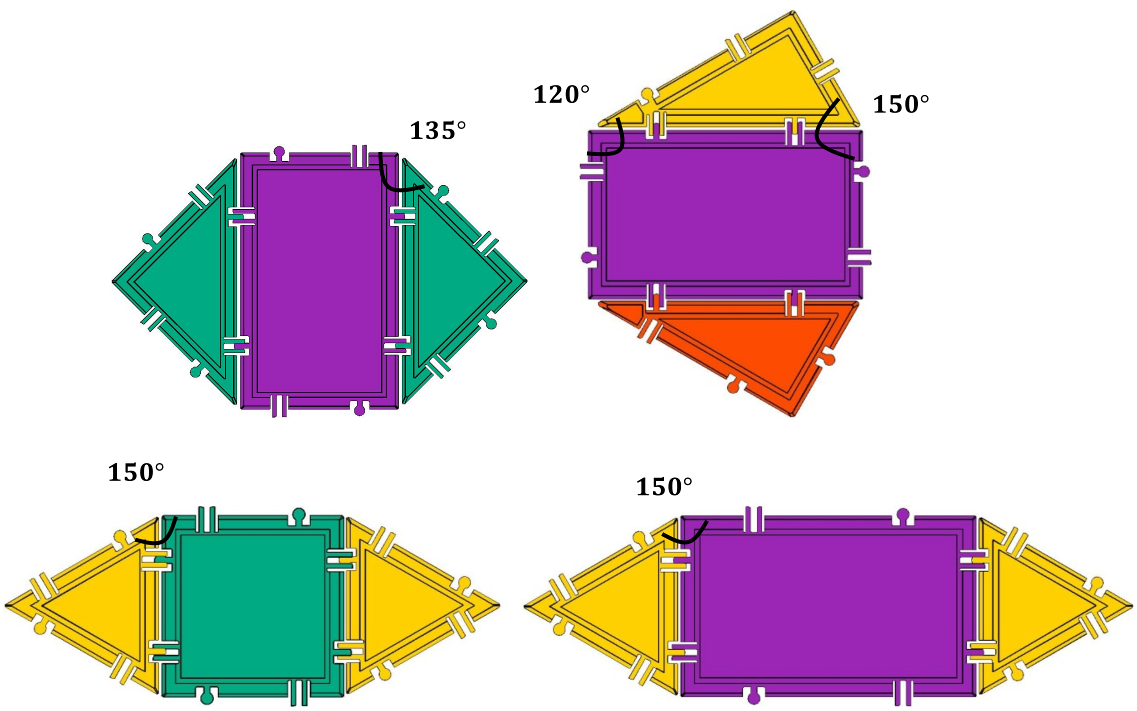
7. We use the angle data we got in the previous problems:







8. Here are some of the possibilities:



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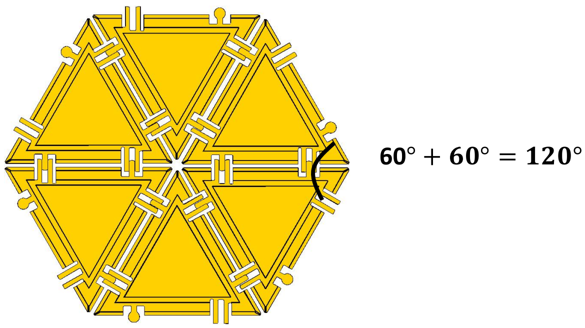
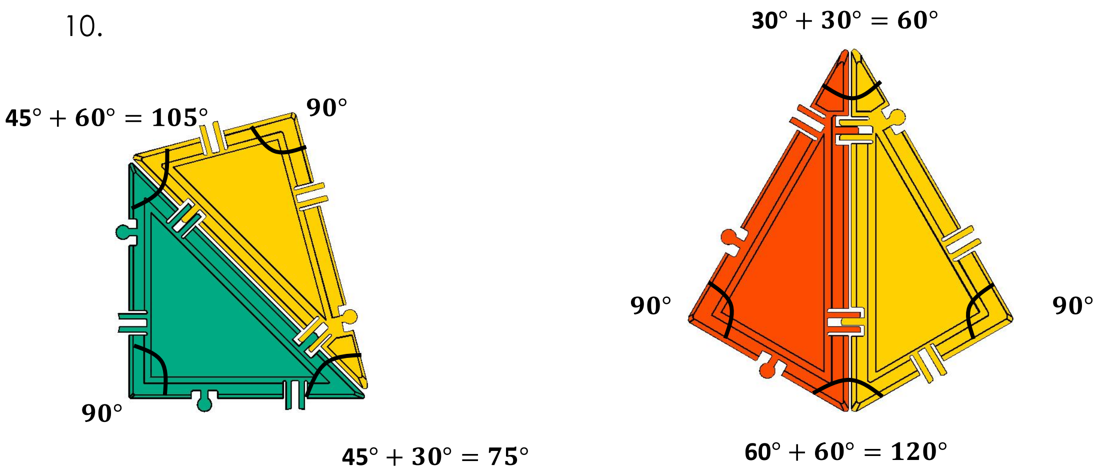
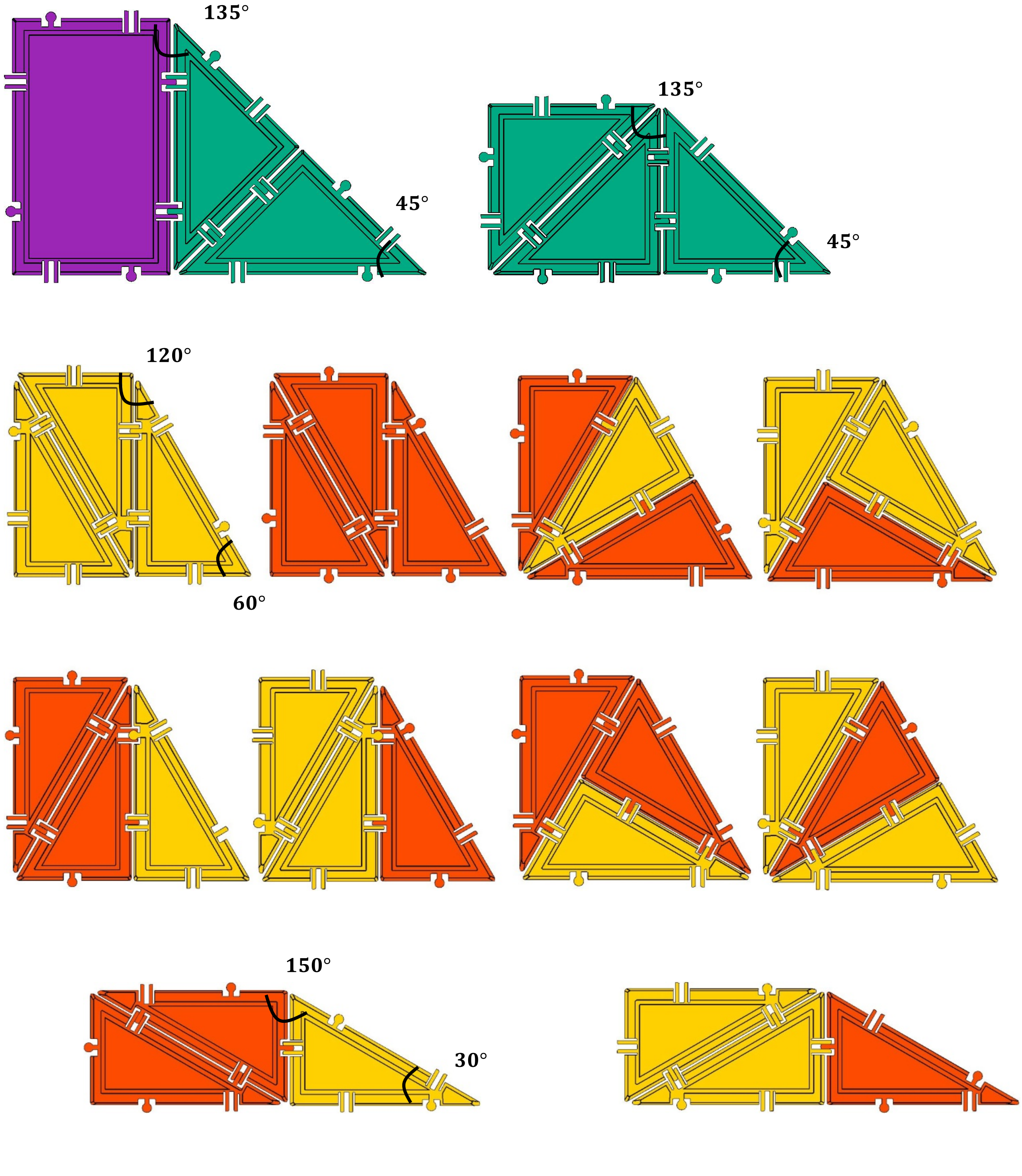


10.



Sum up angles: in the first case we get $90° + 105° + 90° + 75° = 360°$; in the second case we get $90° + 60° + 90° + 120° = 360°$.

11. There are a number of possibilities, all involving a rectangle with a right triangle attached to it.



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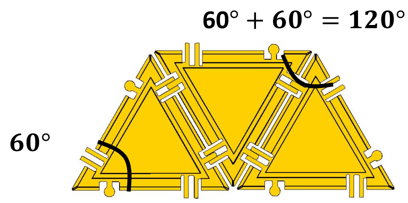
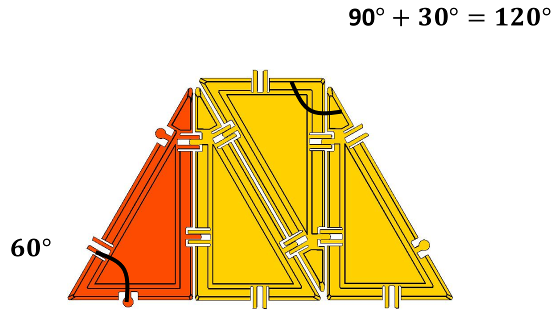
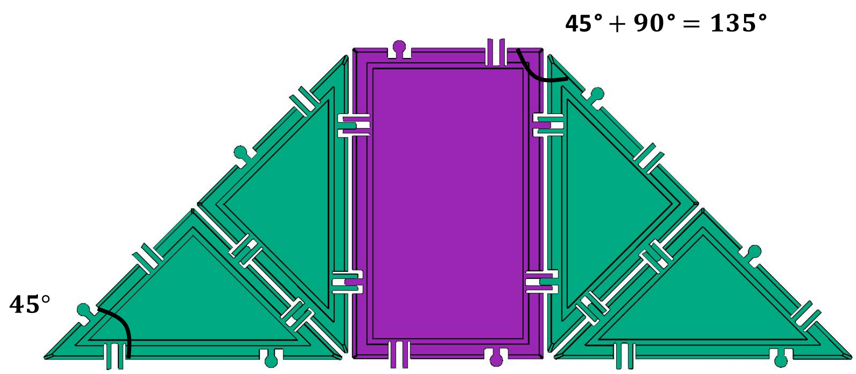
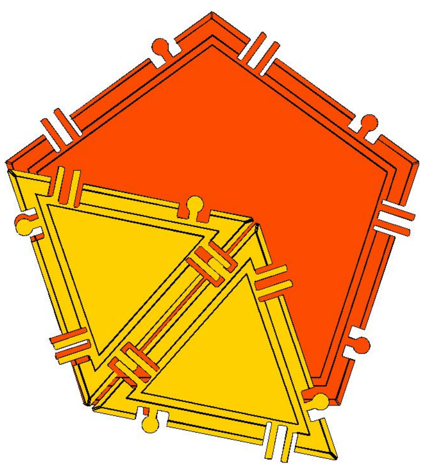






13. Clearly, the angle of a pentagon is obtuse, so A. is out. If you overlay the pentagon with a $120°$ angle (see below), we will see that the angle is less than $120°$. Therefore the answer is B.



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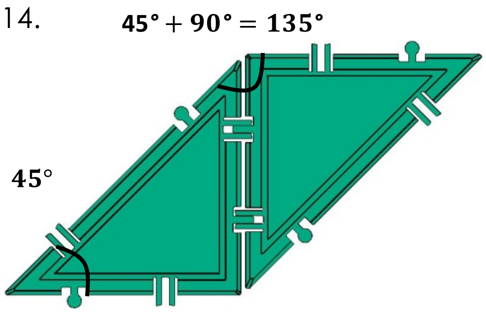
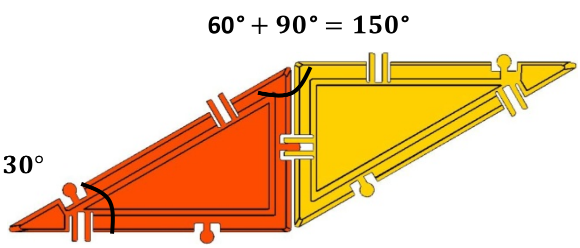


$$2{\times}45° + 2{\times}135° = 360°$$

15.



$$2{\times}30° + 2{\times}120° = 360°$$

