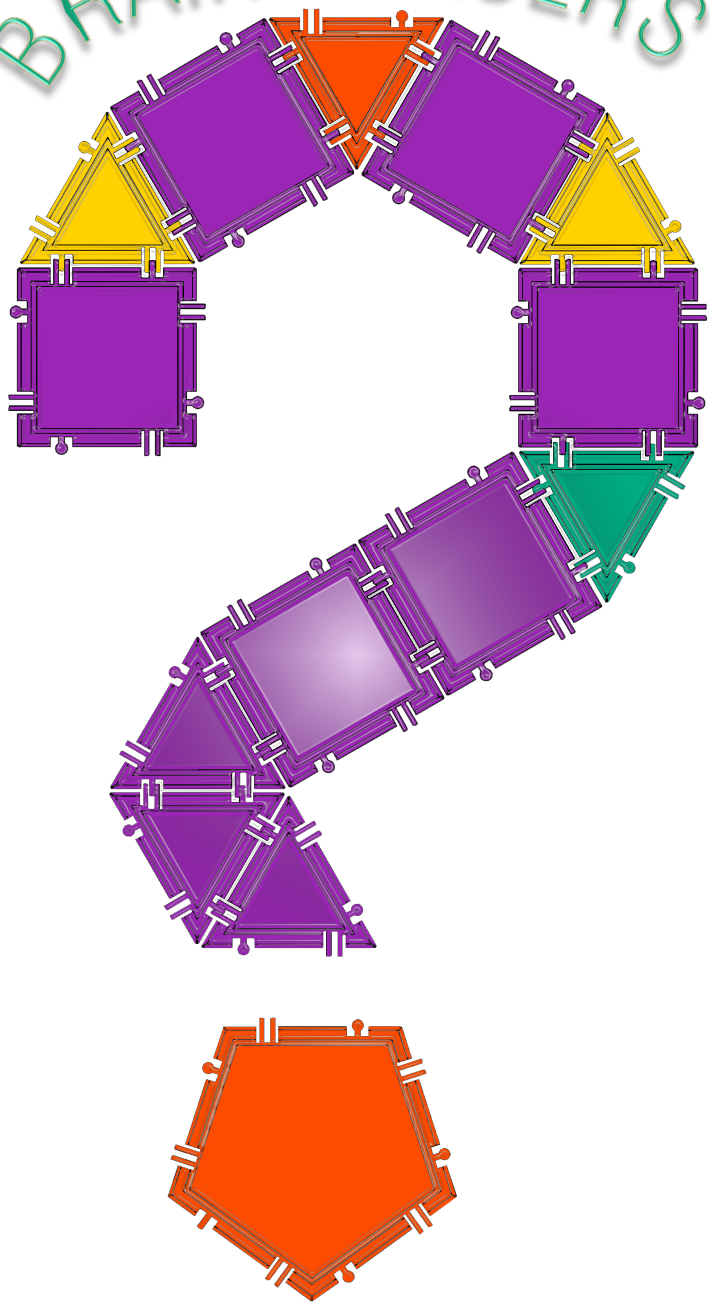
BRAIN TEASERS



Welcome to Geometiles™!

This booklet is all about challenging fun. Work out your neurons by tackling these problems, arranged roughly in order of increasing difficulty. You can get some help on the starred problem in the Hints section.

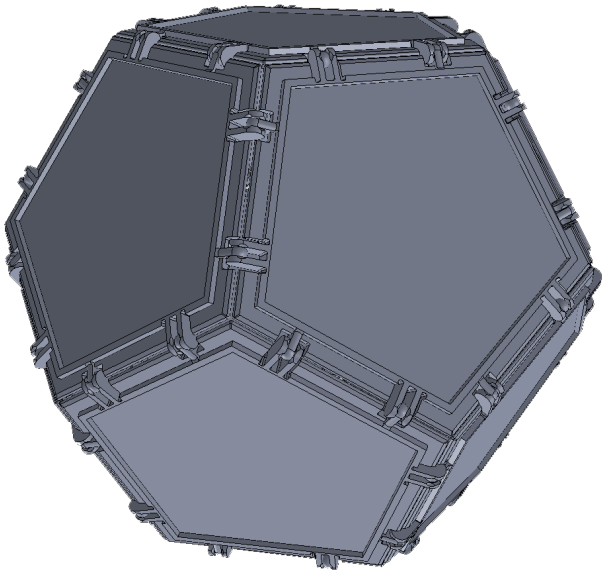
Questions

1. What is the **least** number of colors you need to make a cube so that any two neighboring tiles have different colors?

2. Make a cube out of 12 identically shaped tiles.

3. What is the least number of tiles you need to make a closed solid? Make this solid.

4. *Make a regular dodecahedron out of 3 orange pentagons, 3 yellow pentagons, 3 green pentagons and 3 purple pentagons so that any two neighboring tiles have different colors. How many such arrangements are there?

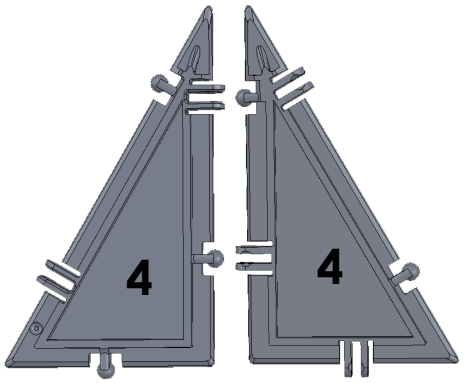


A **regular dodecahedron** is a closed figure made of 12 equilateral pentagonal faces.

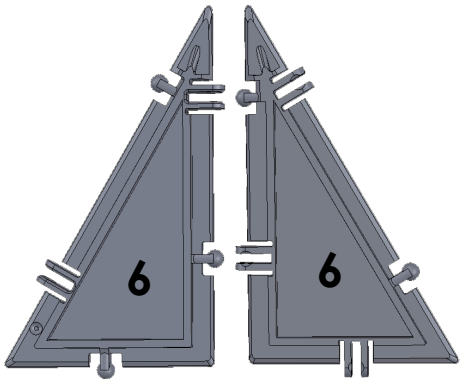
Version: 06/04/16

5. *Build a closed solid having 6 rhombic faces.

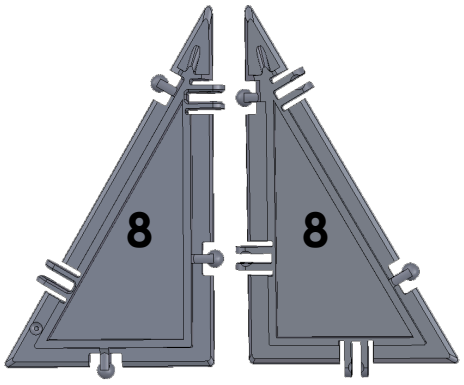
6. *How many closed solids can you create out of the following scalene triangle tiles?



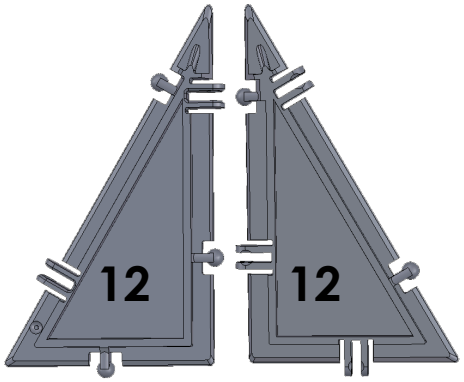
7. *How many closed solids can you create out of the following scalene triangle tiles?



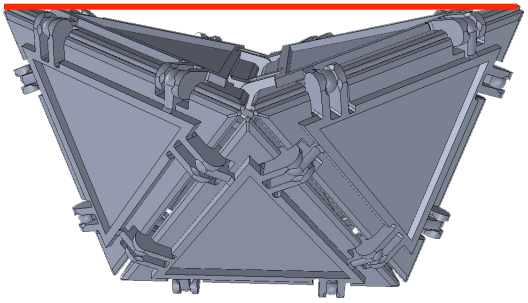
8. *How many closed solids can you create out of the following scalene triangle tiles?



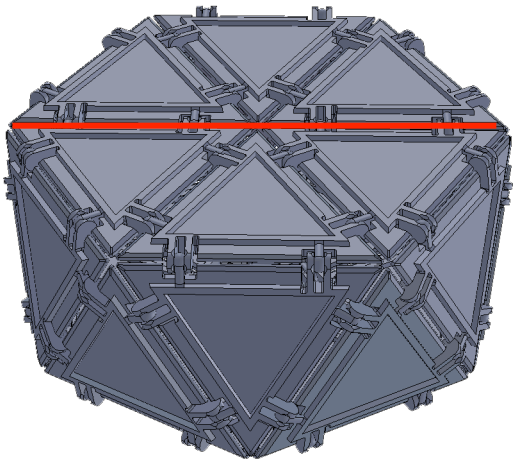
9. *How many closed solids can you create out the following scalene triangle tiles?



10. *A solid is **strictly convex** if a line segment joining any two of its points is entirely contained inside the solid (not on its surface or outside it). Note that a strictly convex solid may not have any coplanar faces. Here are some examples of solids that are NOT strictly convex.



Not strictly convex (or even convex) because red line is not contained inside it.



Convex, but not strictly convex because red line is on the surface. The faces on the top are coplanar.

Here is a question to ponder: How many **strictly convex** solids can you create with just the equilateral triangles?

Hints to selected problems

4. There are 4 ways to do this. Two of them are mirror images of the other two.

5. Think "stretched out cube".

6. One of these is a tetrahedron.

7. One of these is a double tetrahedron.

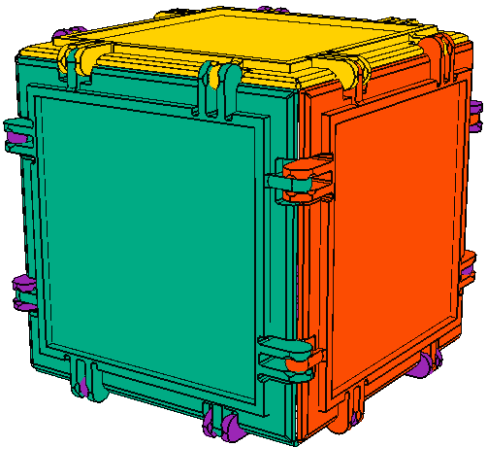
8, 9. Try prisms with various cross sections: triangular, rhombic, parallelogrammatic, trapezoidal.

10. There are only 8 such polyhedra, and they consist of the following number of triangles:
4, 6, 8, 10, 12, 14, 16 , 20.

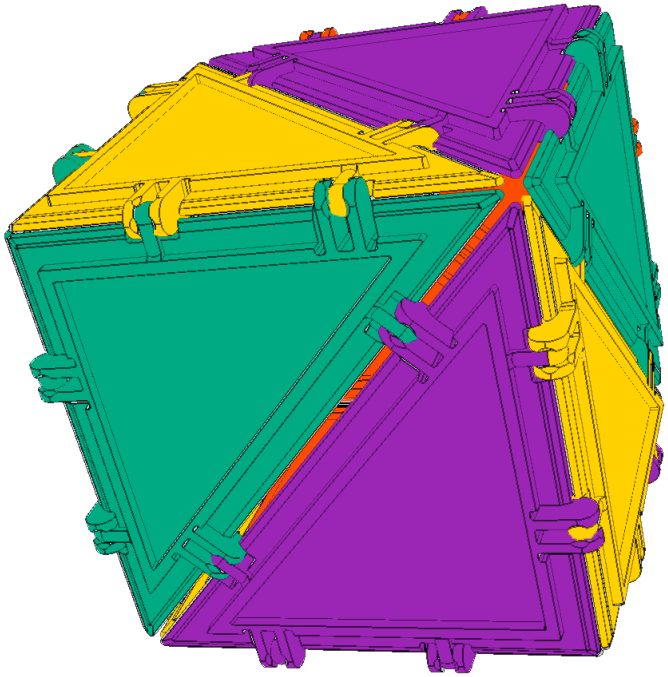
Answers are given on the following page. Don't turn over the page unless you are ready to look at the answers!

Answers

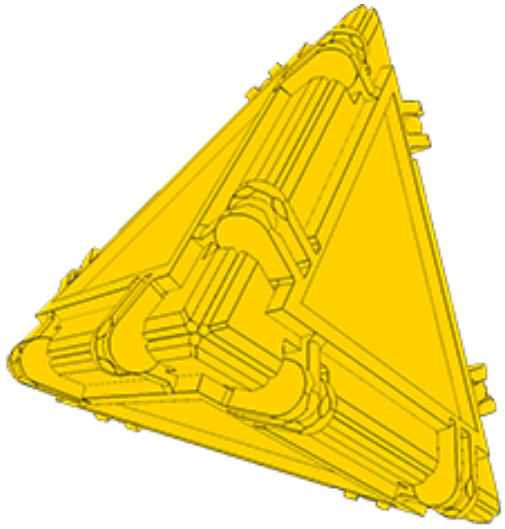
1. 3 colors:



2.

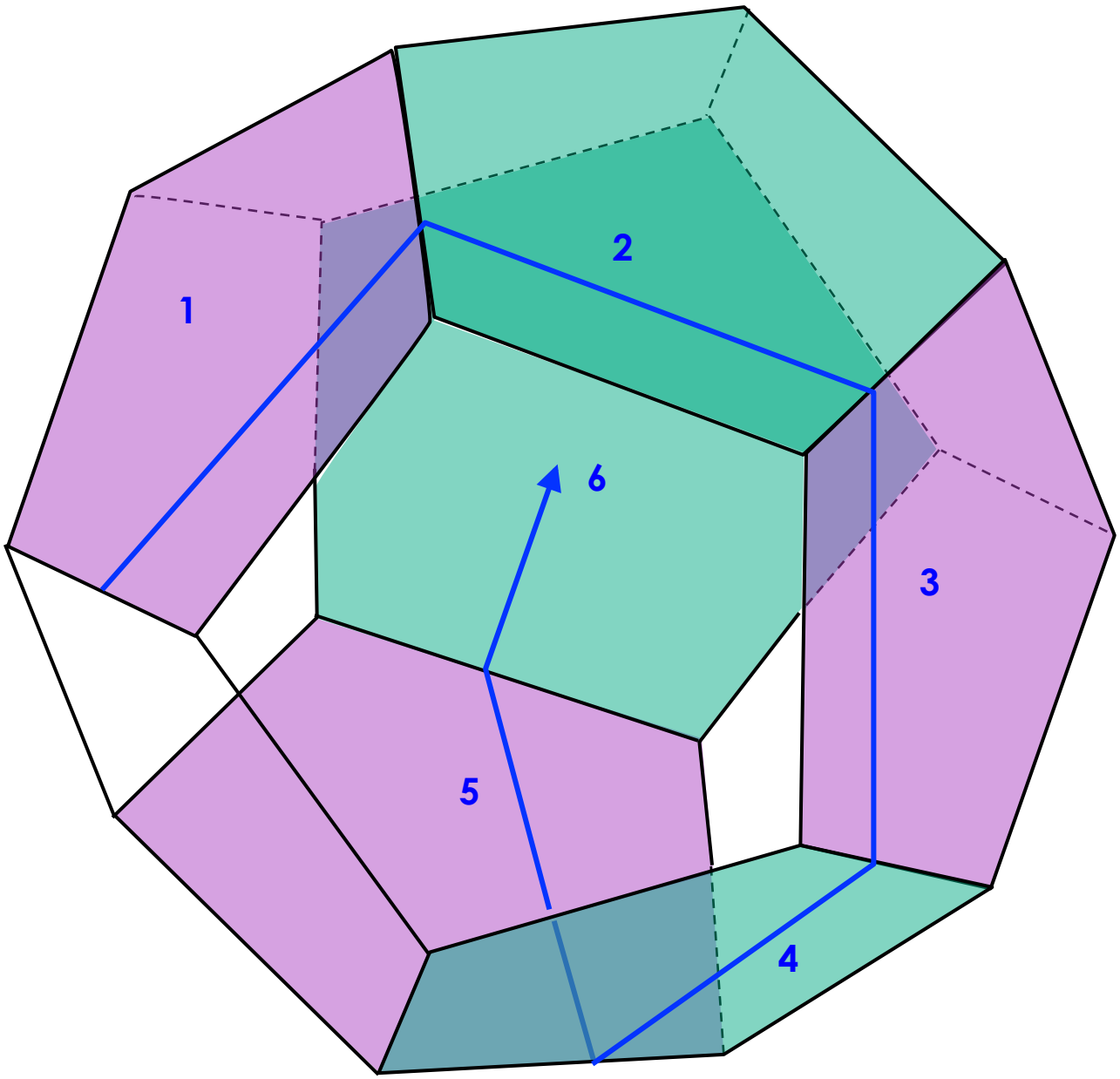


3. You need at least 4 tiles to make a solid. The solid made with 4 tiles is a tetrahedron, otherwise known as triangular pyramid.



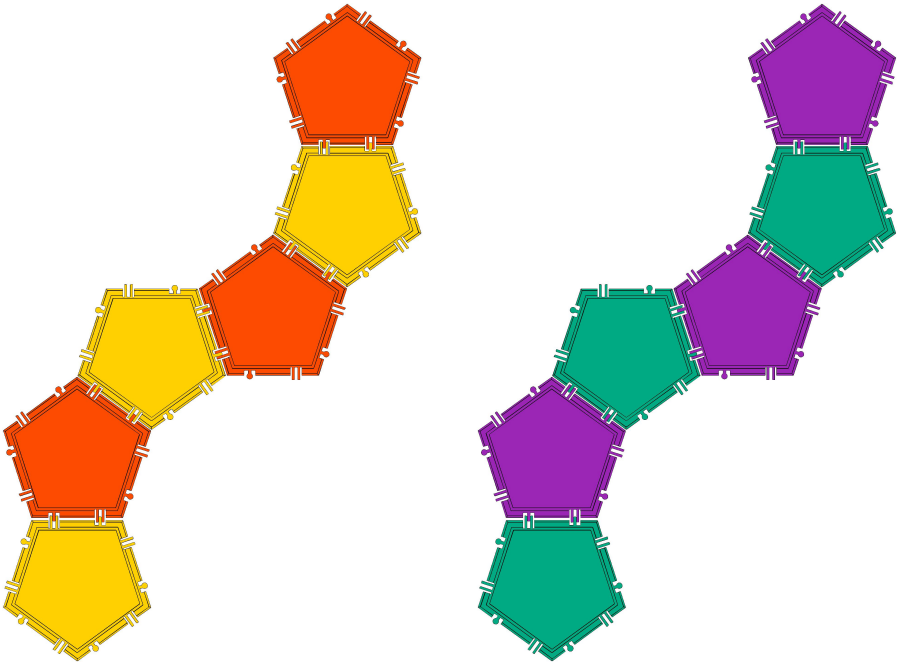
4. We view the dodecahedron as two "S"-like chains of 6 pentagons each, as shown below.

In the picture below, one chain is of 6 pentagons is colored, and its pentagons are numbered. The other chain is comprised of the remaining 6 faces that are left uncolored for clarity.

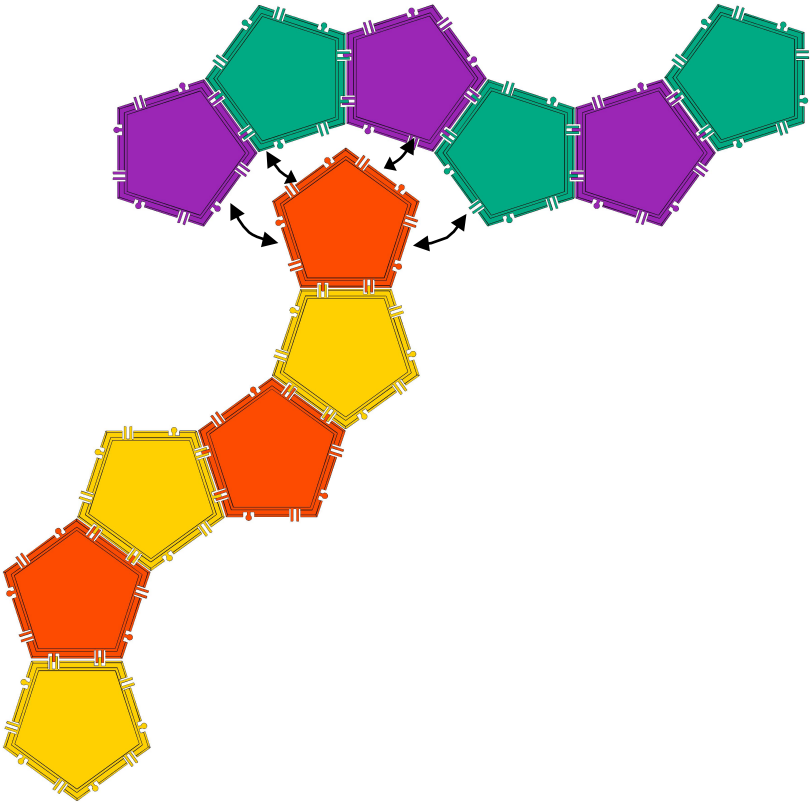


How to construct a solution

Step 1: Make 2 chains of pentagons

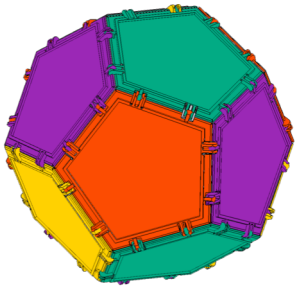


Step 2: Connect 2 chains as follows:

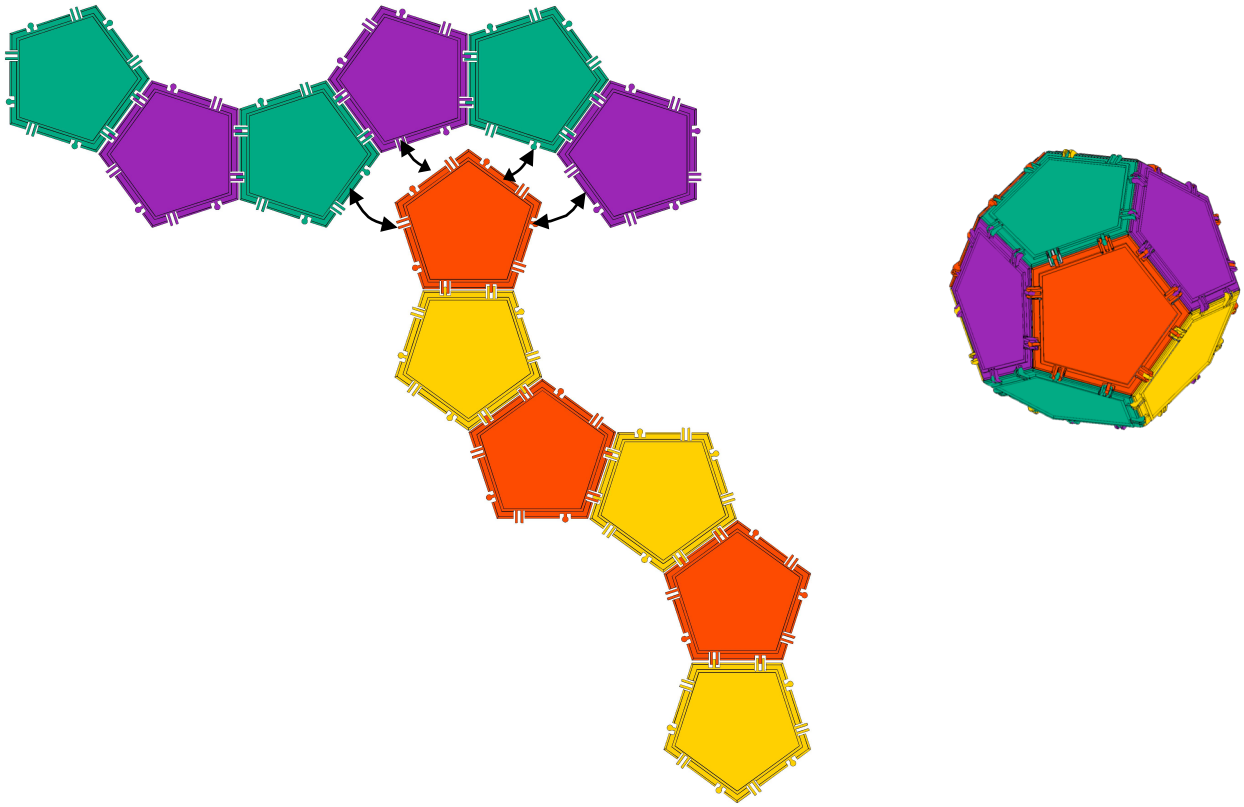


After you connect the pieces shown with arrows, continue with connecting the green/purple chain with the orange/yellow chain. After you finish, you will get a dodecahedron such that any of its neighboring faces have different colors:

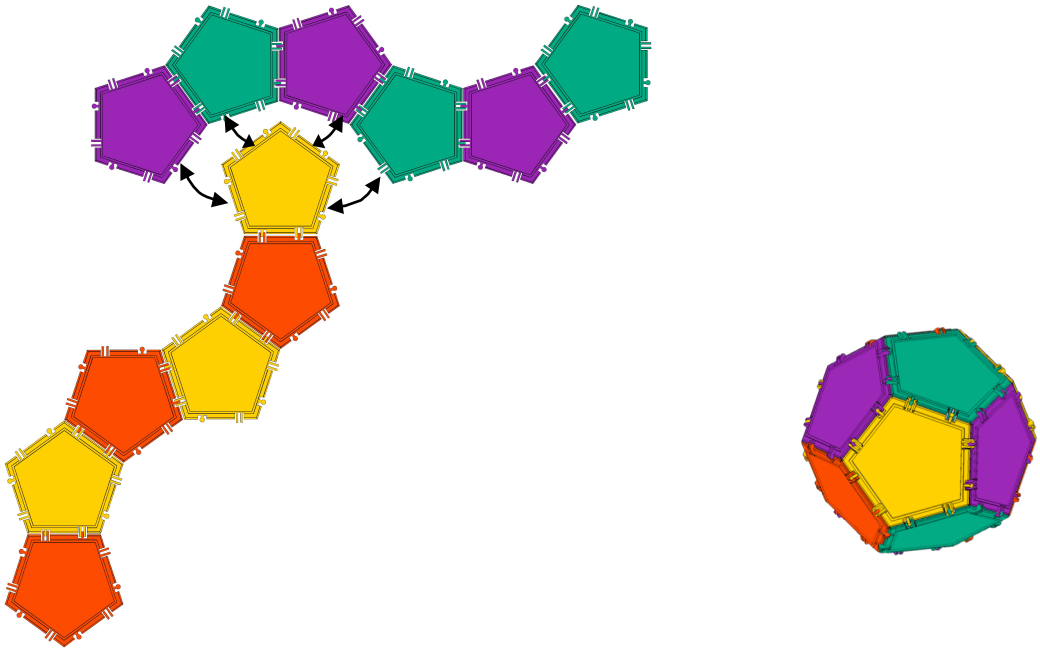
Step 3



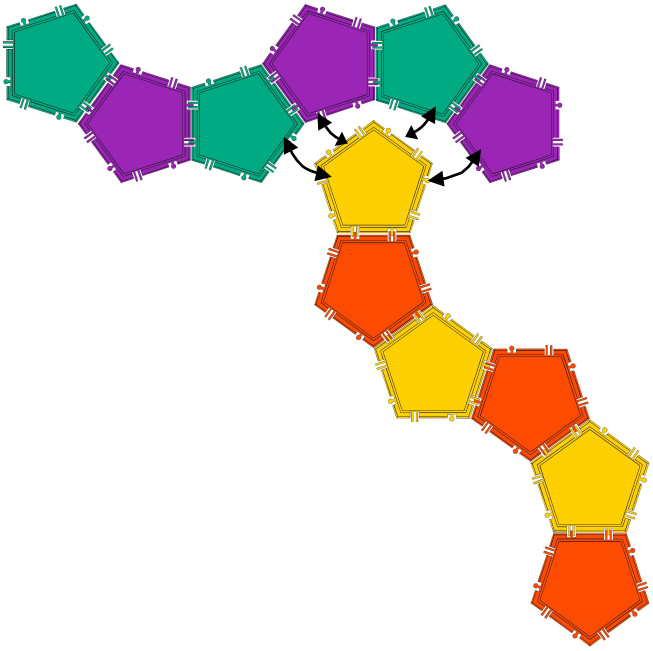
You might ask yourself if there is more than one way to connect the two chains of pentagons together. The answer is "yes". We can simply flip over both of the chains in the above solution and arrive at its mirror image:

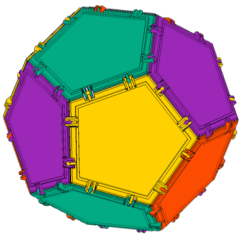


You can also flip the orange/yellow chain upside down and then form the connection:

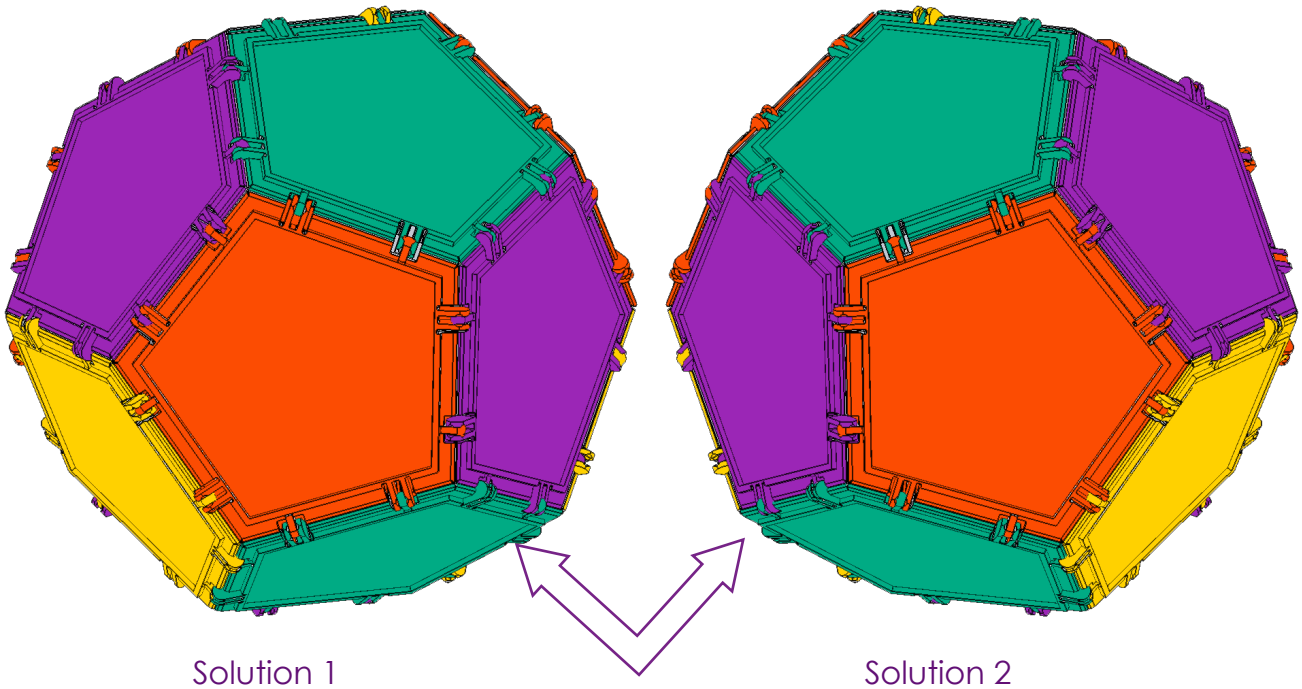


Flip the whole picture over and you'll get a solution that is a mirror image of the previous one:



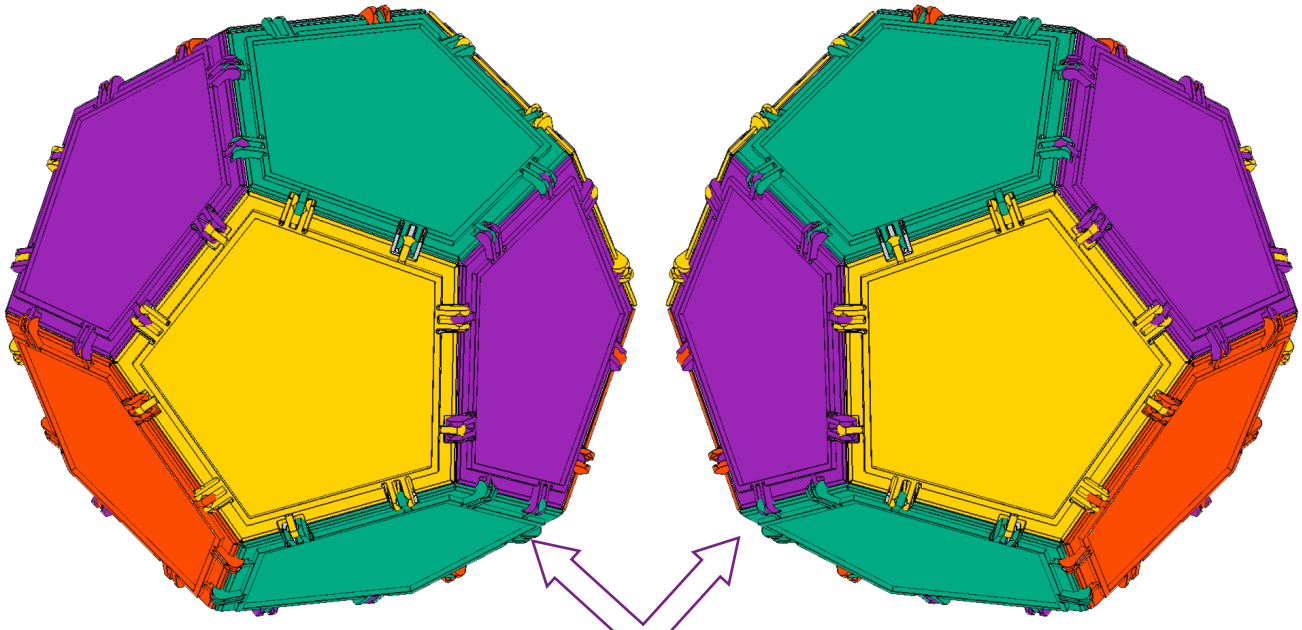


Summary of all answers



Solution 1 Solution 2

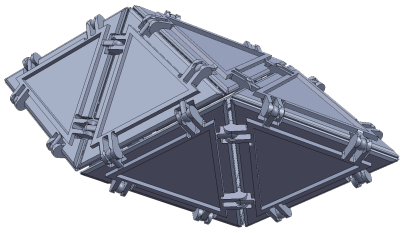
Mirror images



Solution 3 Solution 4

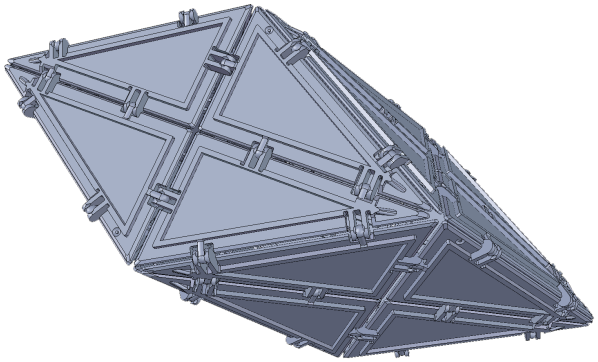
Mirror images

5. This shape is called a **rhombohedron**. It is a special case of a parallelepiped in which each face is a rhombus.



Small rhombohedron

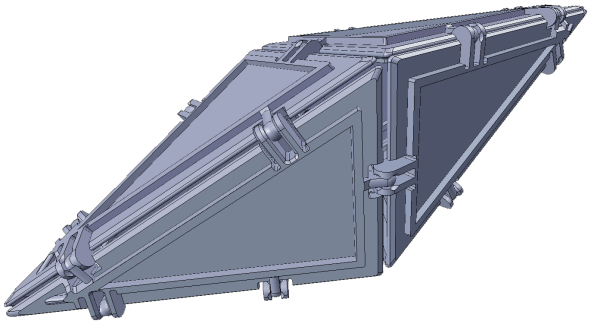
12 equilateral triangles



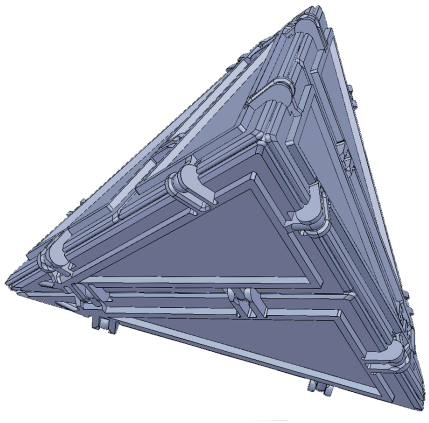
Large rhombohedron

24 scalene triangles

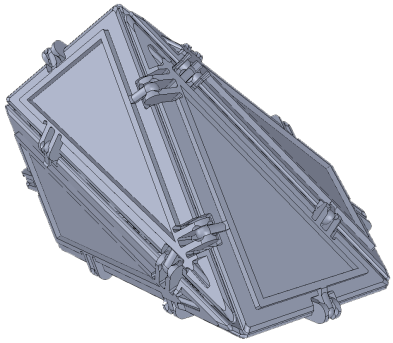
6.



Octahedron

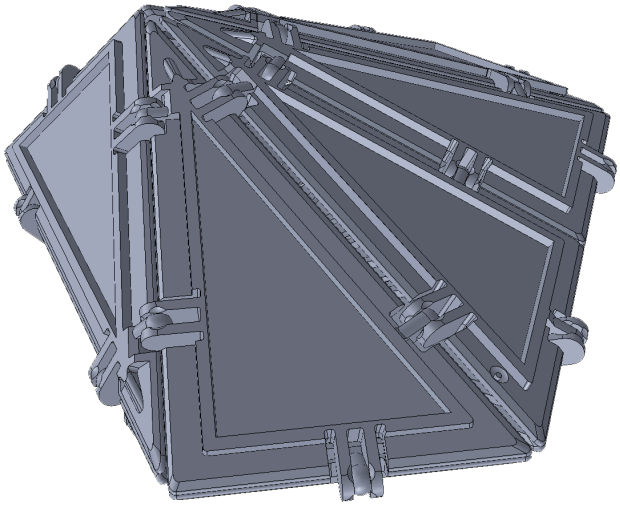


Regular tetrahedron

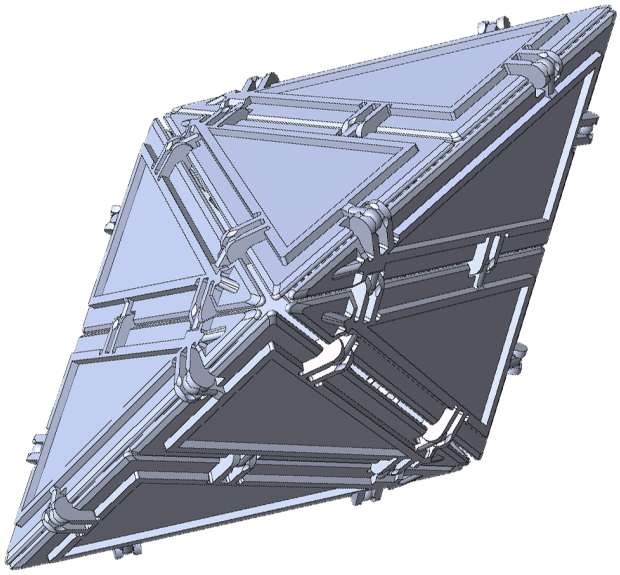


Ocatahedron

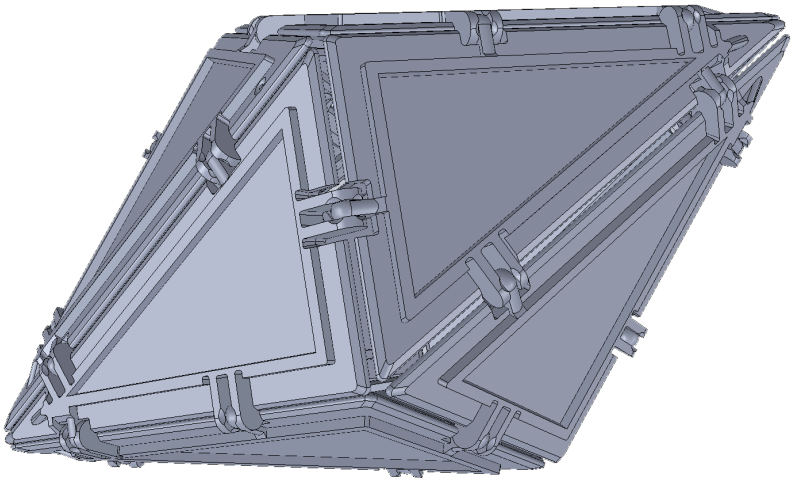
7.



Pentagonal dipyramid

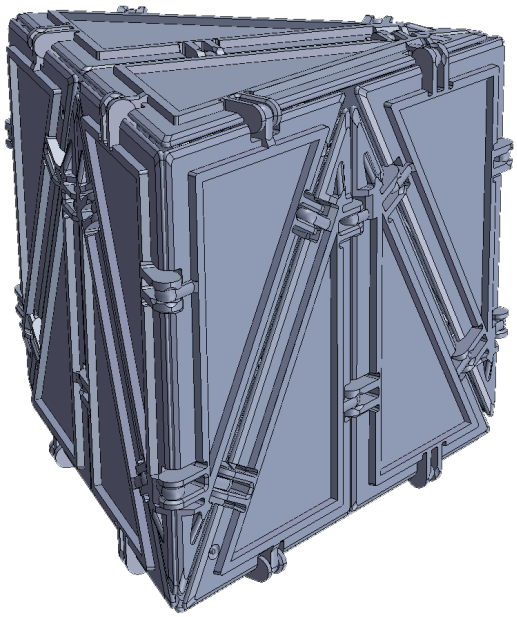


Double tetrahedron

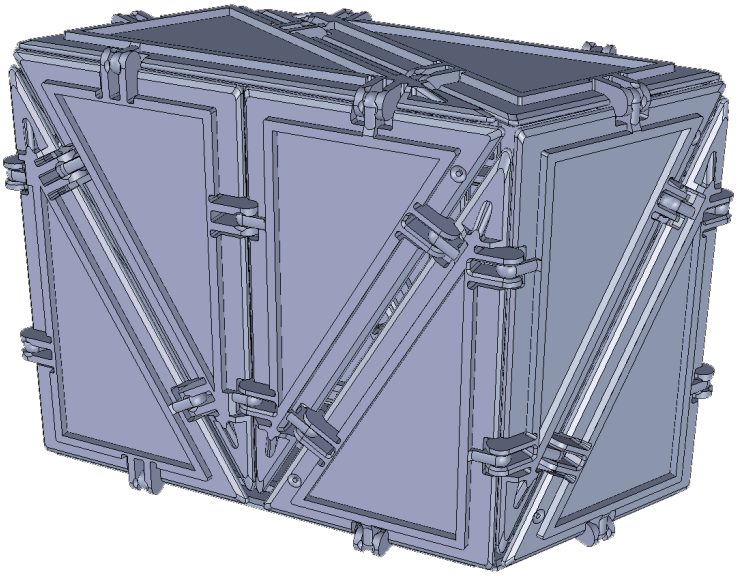


Scalenohedron

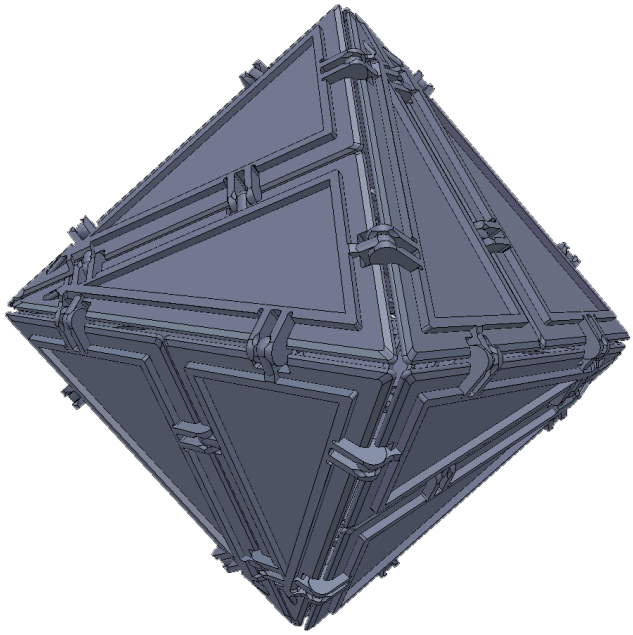
8.



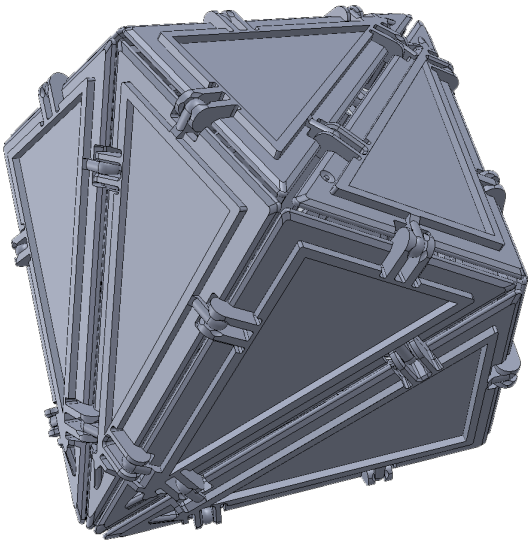
Equilateral triangular prism



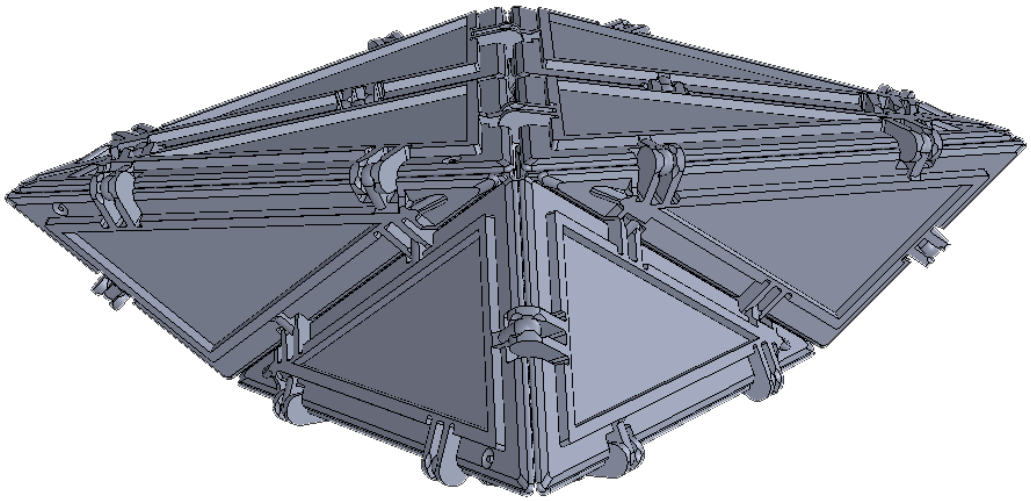
Parallelogrammatic prism



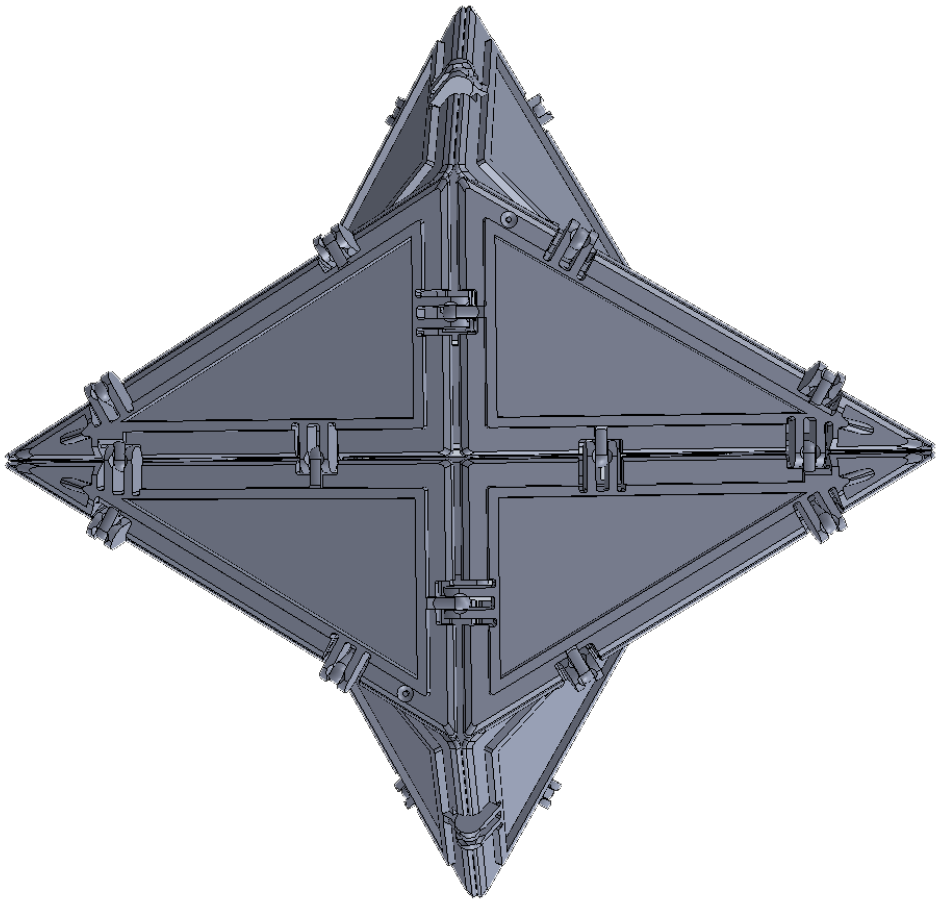
Regular octahedron



Trapezohedron

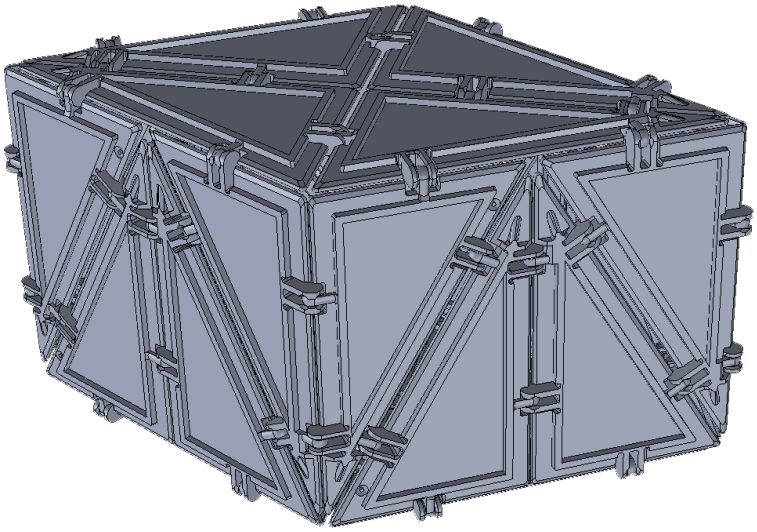


Tetradecahedron, view 1

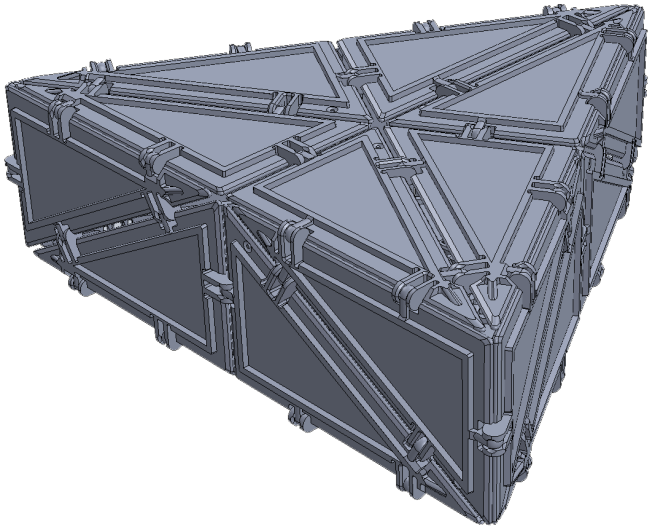


Tetradecahedron, view 2

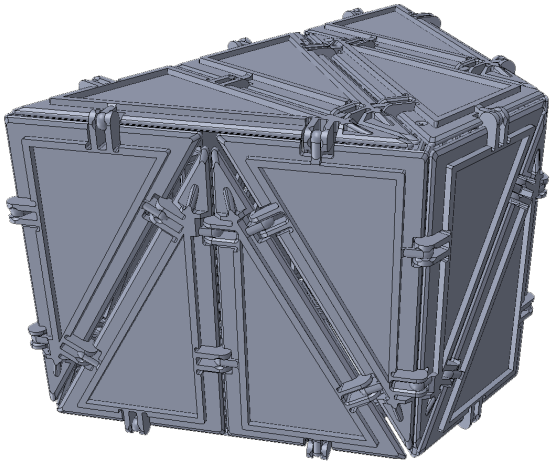
9.



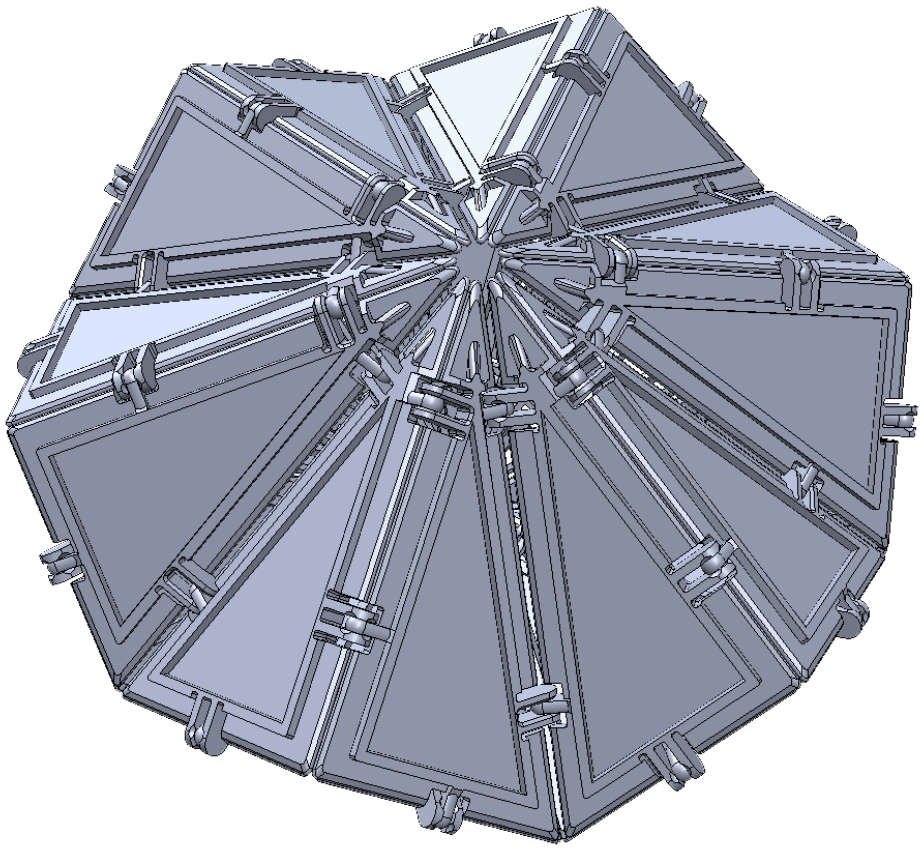
Rhombic prism



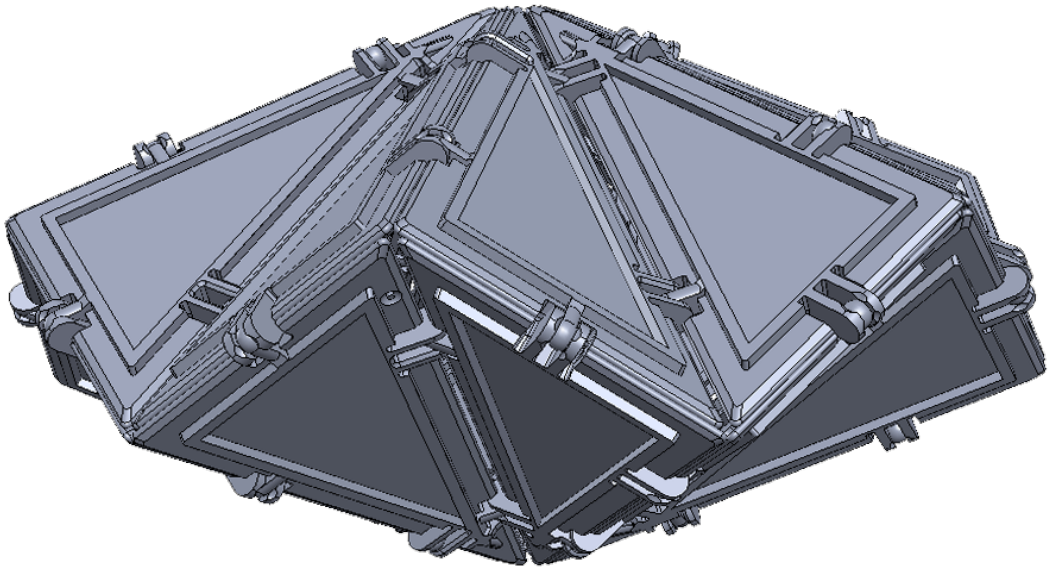
Equilateral triangular prism



Trapezoidal prism

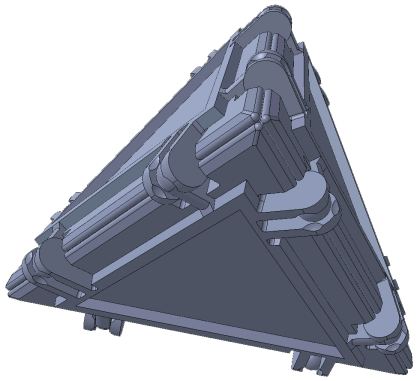
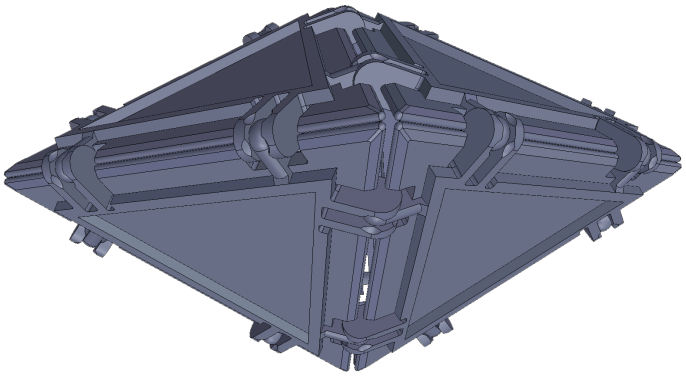
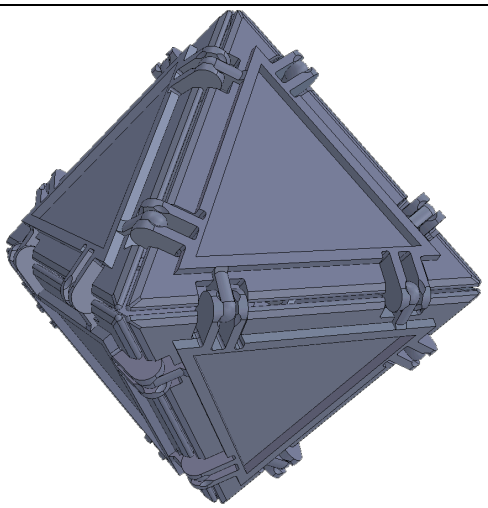


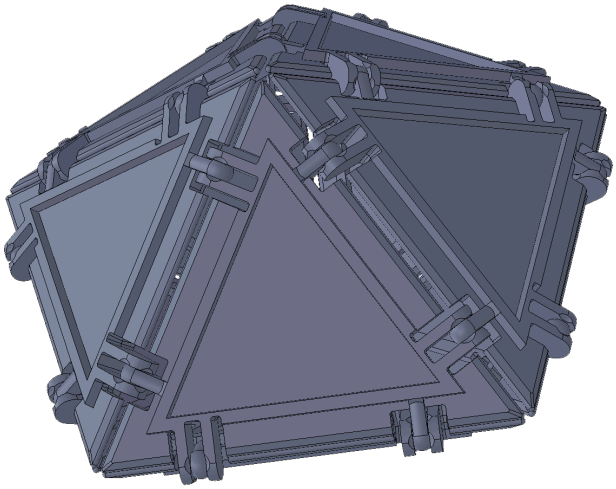
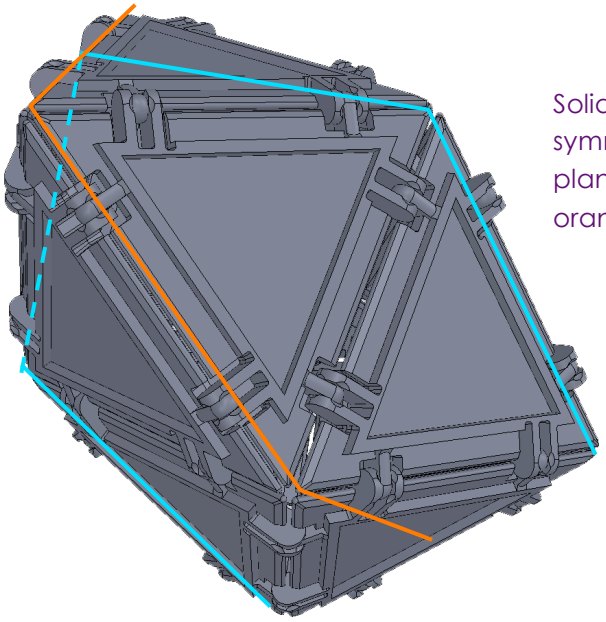
Non-convex scalenohedron (a.k.a. spinning top), view 1

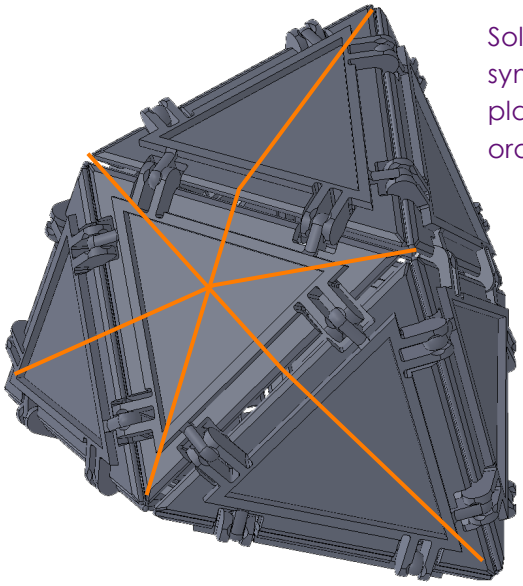
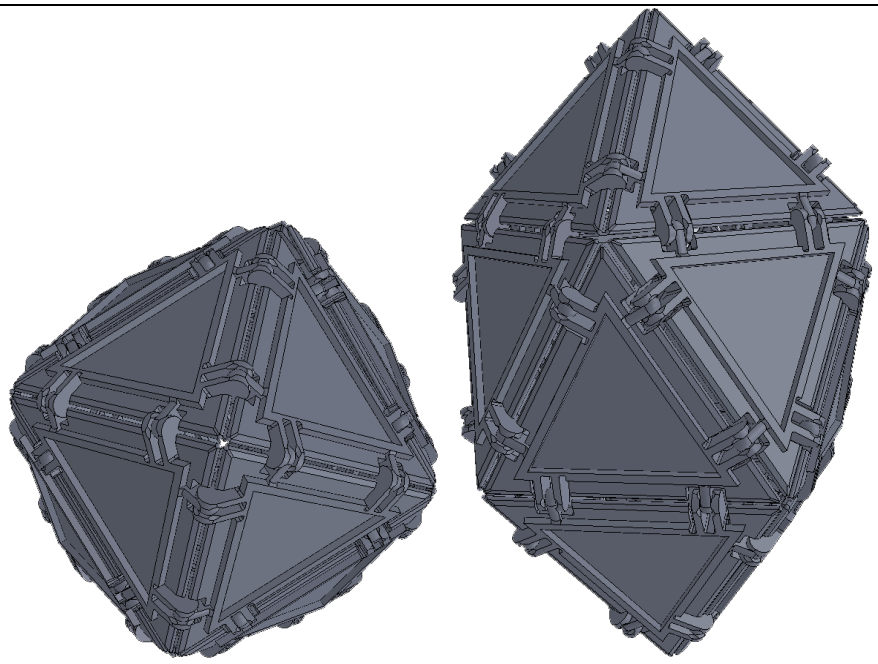


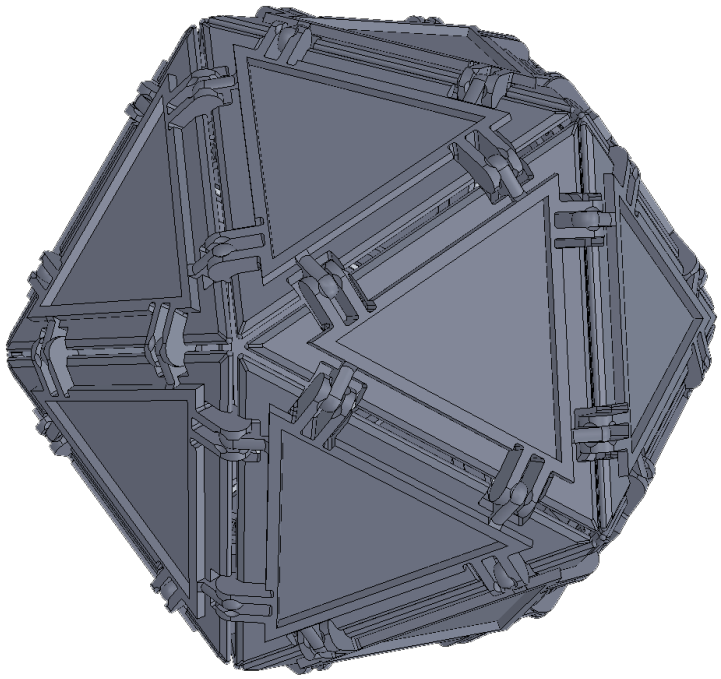
Spinning top, view 2

10. These solids are called the **convex deltahedra**.

Number of equilateral triangles	Solid
4	 tetrahedron
6	 triangular dipyramid
8	 octahedron

Number of equilateral triangles	Solid
10	 pentagonal dipyramid
12	 Solid is mirror symmetric about planes shown in orange and blue. snub disphenoid

Number of equilateral triangles	Solid
14	 Solid is mirror symmetric about all planes shown in orange. triaugmented triangular prism
16	 top view side view gyroelongated square pyramid

20	 icosahedron